8th Advanced Training Course on Radar Polarimetry

Ljubljana, 2026

SAR BASICS & SAR TOMOGRAPHY THEORY

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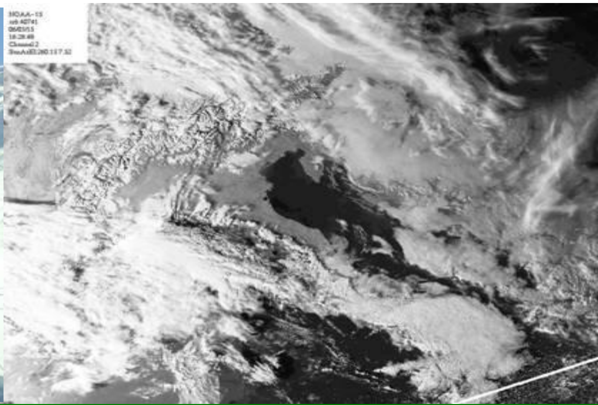
Politecnico di Milano

Radar Imaging

RADAR (***Radio Detection And Ranging***) is a technology to detect and study far off targets by transmitting EM pulses at radiofrequency and observing the backscattered echoes

Some relevant features:

1. **Active instrument: ⇔ *no need for external illumination source***
2. **Delay-based measurement ⇔ *target distance is obtained based on pulse two-way travel time***
3. **Microwaves penetrate through rain and clouds ⇔ *visibility in all weather conditions***
4. **Microwaves can penetrate into some natural media, like forests, snow, ice, sand ⇔ *sensitivity to the 3D structure of illuminated media***

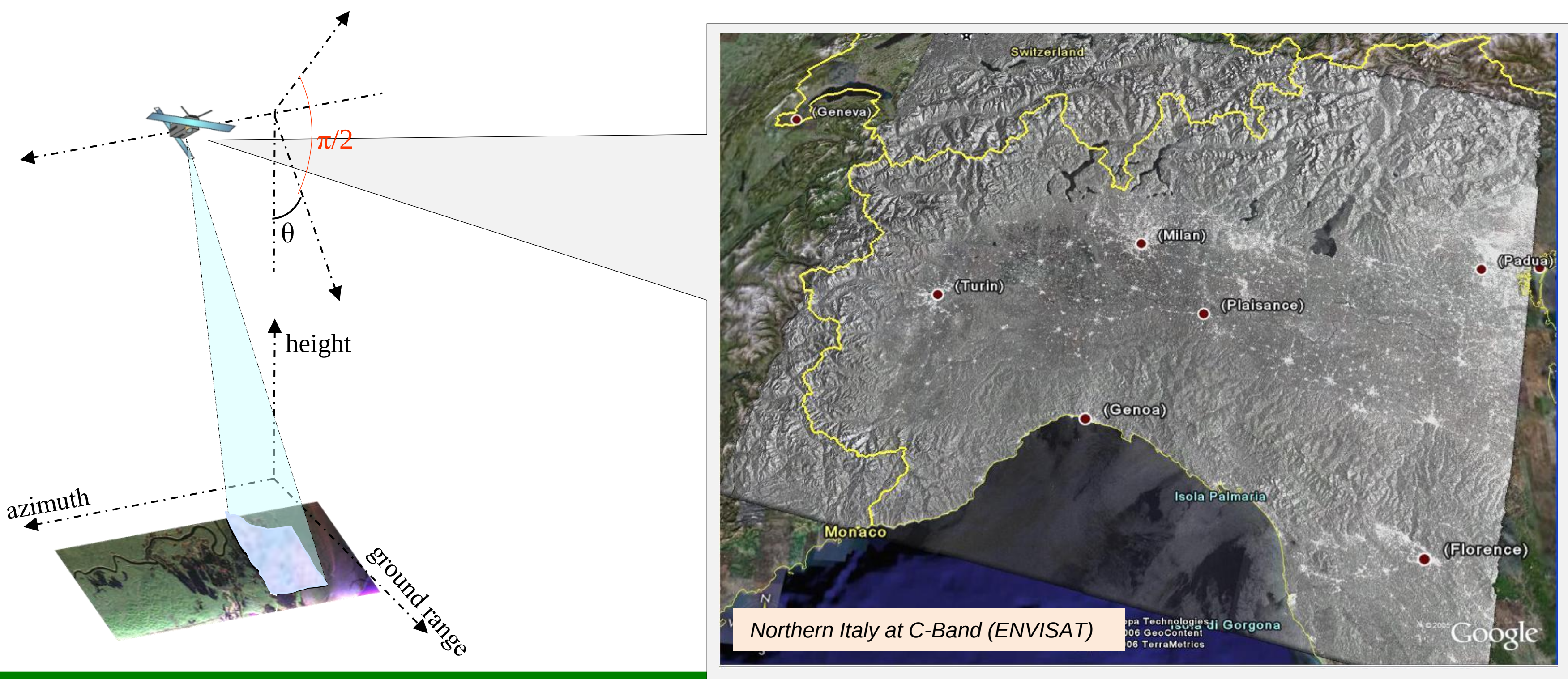


SAR Imaging

SAR systems employ a RADAR sensor flown onboard a satellite platform to synthesize an antenna aperture as long as several kilometers

- Accurate measurement of Radar echoes backscattered from the targets as the system is flown along the satellite trajectory
- Image formation by Digital Processing techniques

⇒ The result is a high resolution **two-dimensional** map of the imaged scene



Northern Italy at C-Band (ENVISAT)

SAR Imaging

Key features:

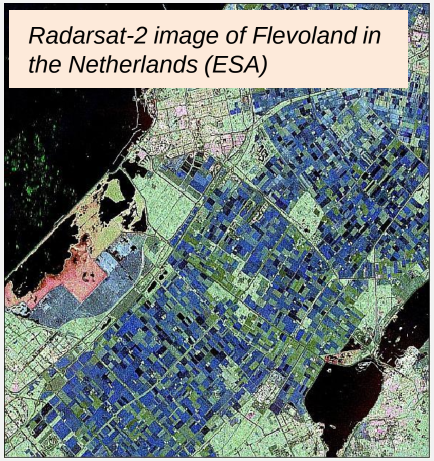
- Microwaves penetrate through rain and clouds ⇔ *visibility in all weather conditions*
- Aperture Synthesis⇔ *fine spatial resolution*
- Phase information ⇔ *millimeter accuracy about distance variations*

⇨ Spaceborne SARs provide *accurate* and *continuous* information about the Earth's surface and its evolution over time

Topographic mapping

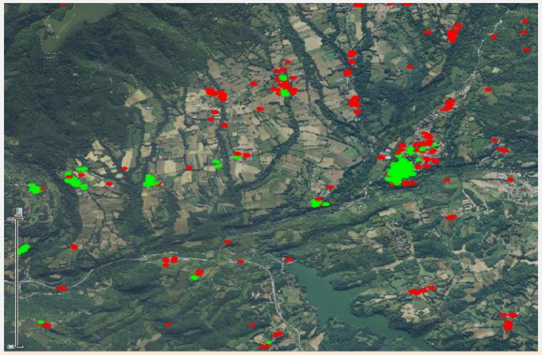
DEM of Mount Etna, Sicily, derived from ERS-1 (ESA)

Land mapping



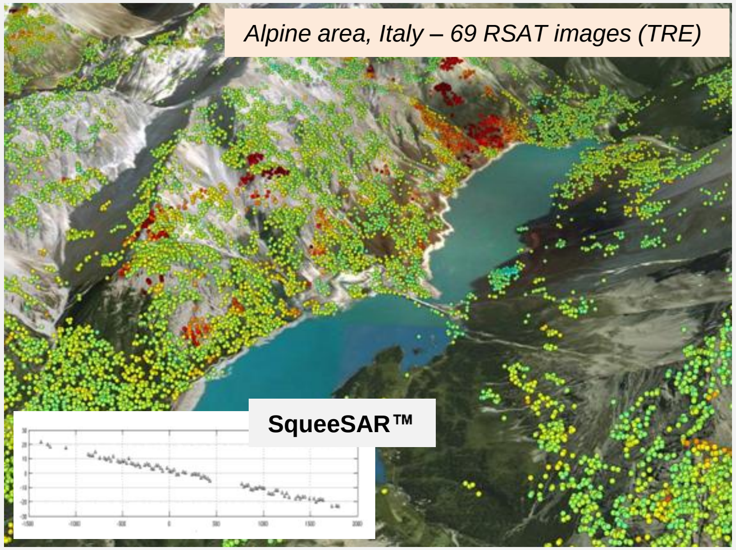
Radarsat-2 image of Flevoland in the Netherlands (ESA)

Change detection



Post-earthquake change detection map in Amatrice, Italy, derived from Sentinel-1A (PoliMi)

Deformation monitoring



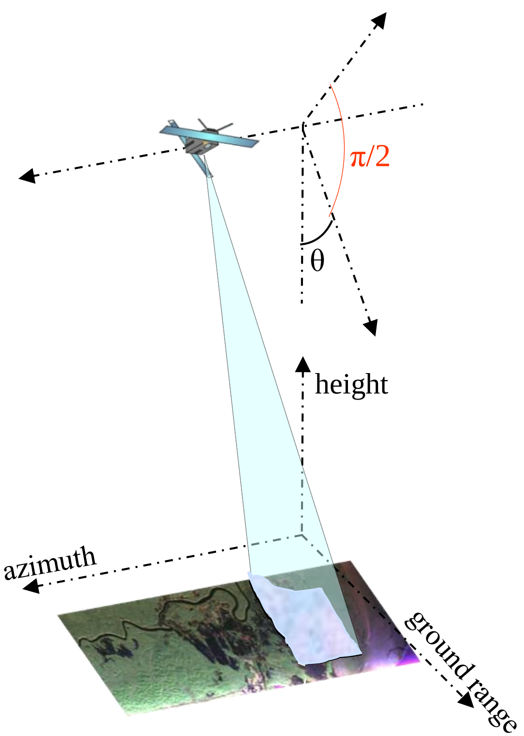
Alpine area, Italy – 69 RSAT images (TRE)

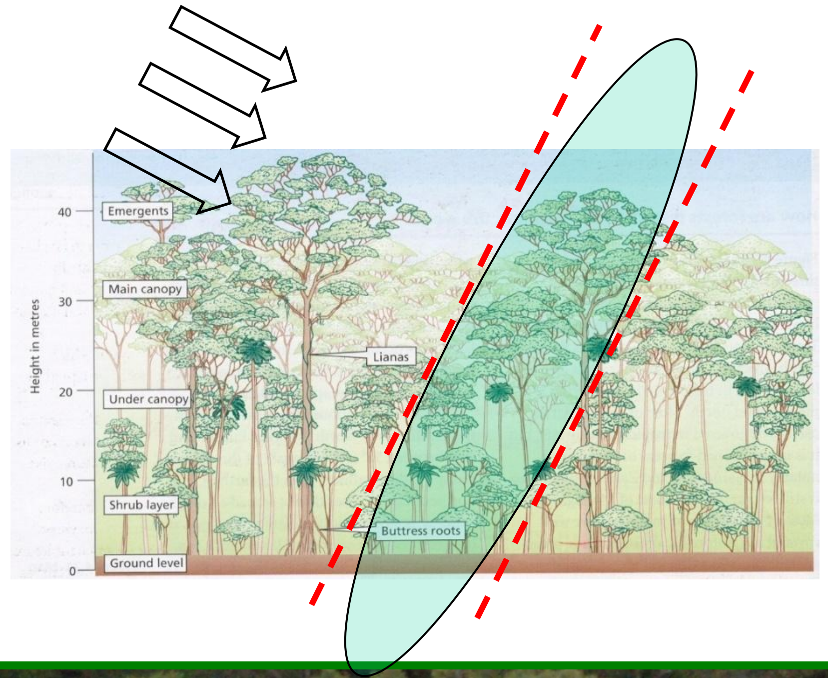
SAR Imaging

Another key feature:

- Microwaves **penetrate** into natural media, like forests, snow, ice, sand ⇔ *sensitivity to the three-dimensional structure of illuminated media*

⇒ A single pixel within a SAR image is actually a mixture of different scattering mechanisms distributed over height



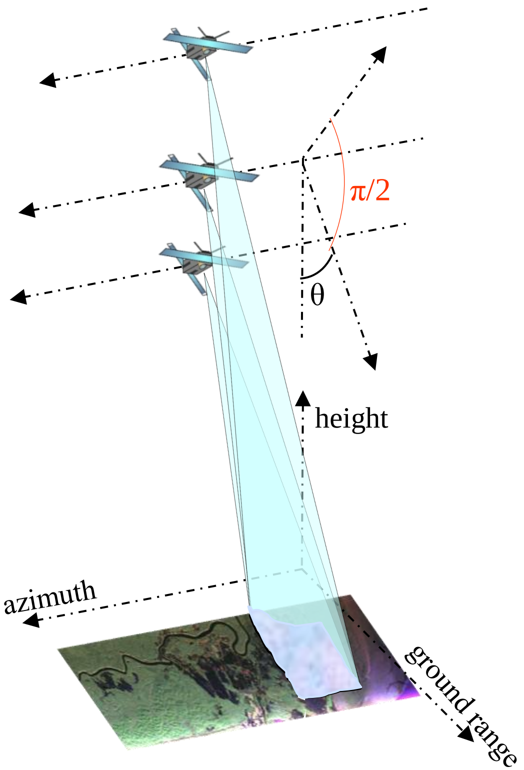


Tomographic SAR Imaging

TomoSAR systems employ a RADAR sensor flown along **multiple** trajectories

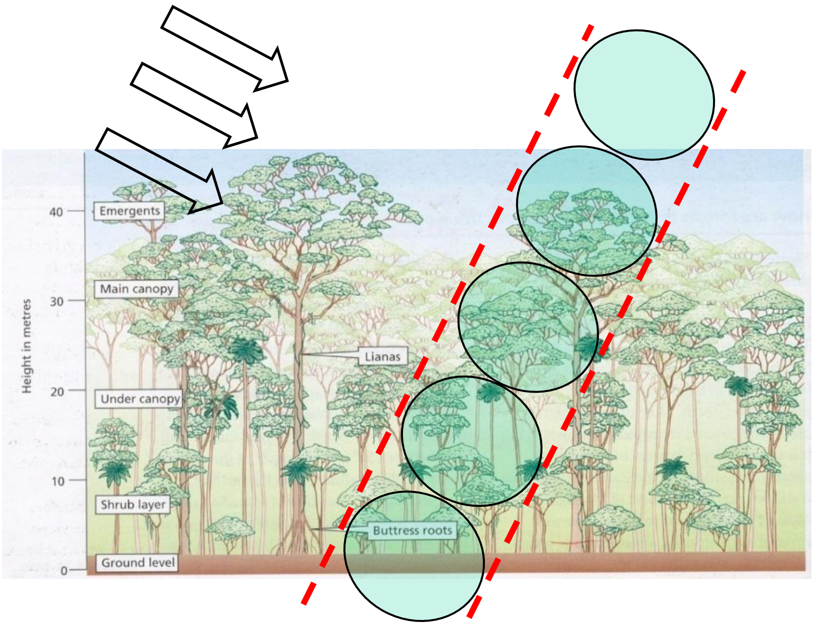
- Image formation by Digital Processing techniques

$\Rightarrow$ ***Three dimensional representation*** of Radar intensity at a given wavelength



SAR produces pixels

TomoSAR produces voxels !!!



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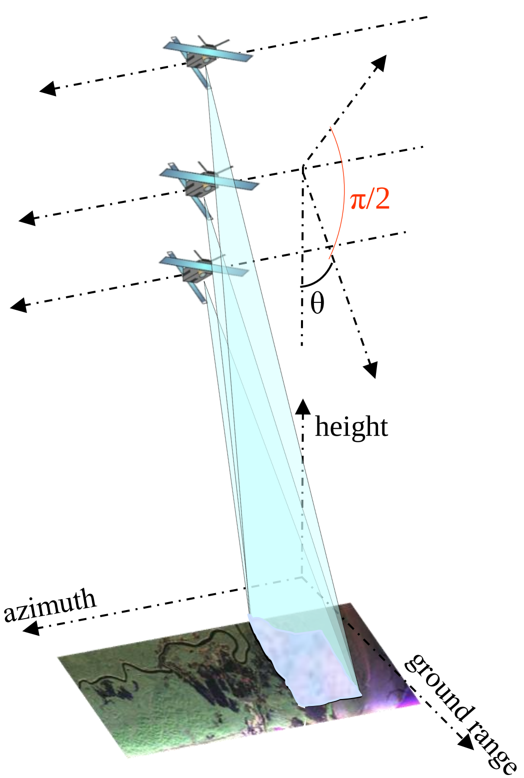
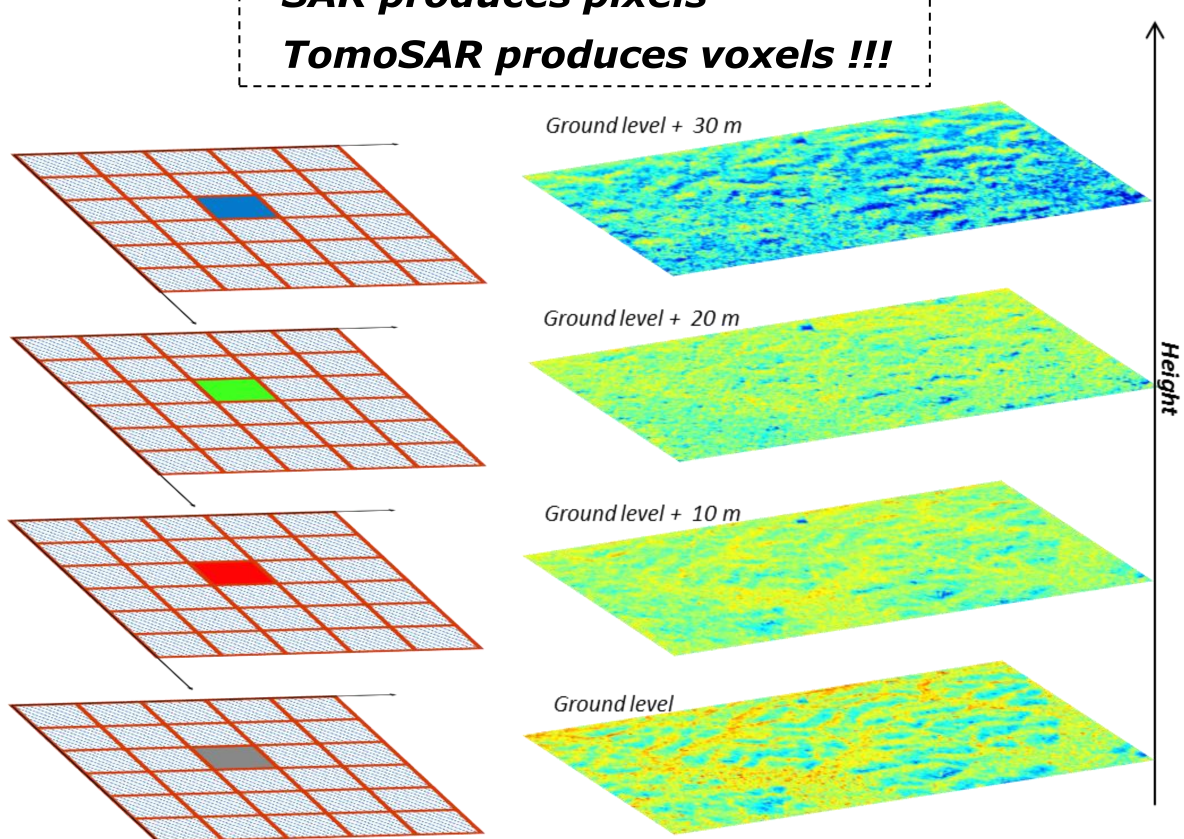
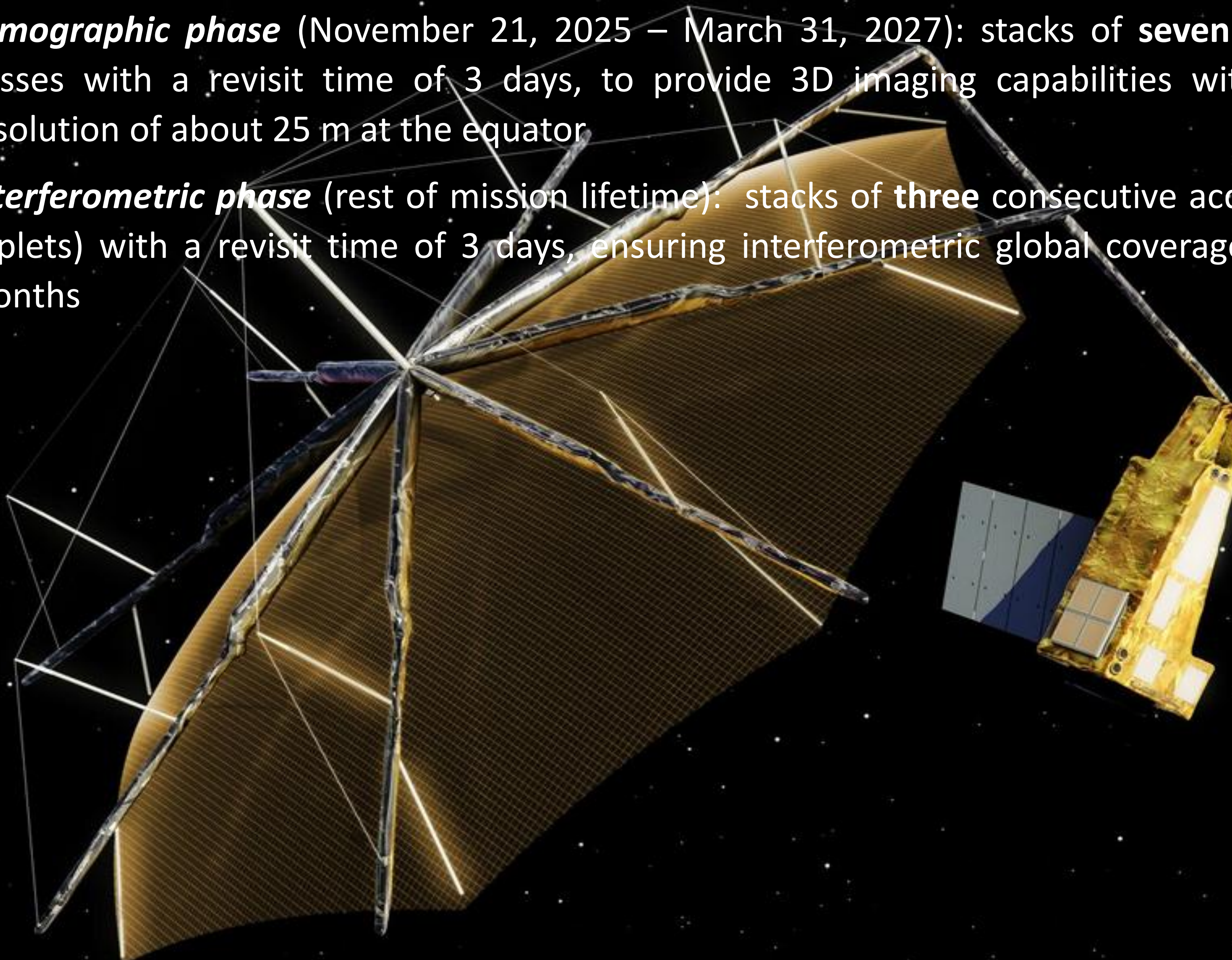




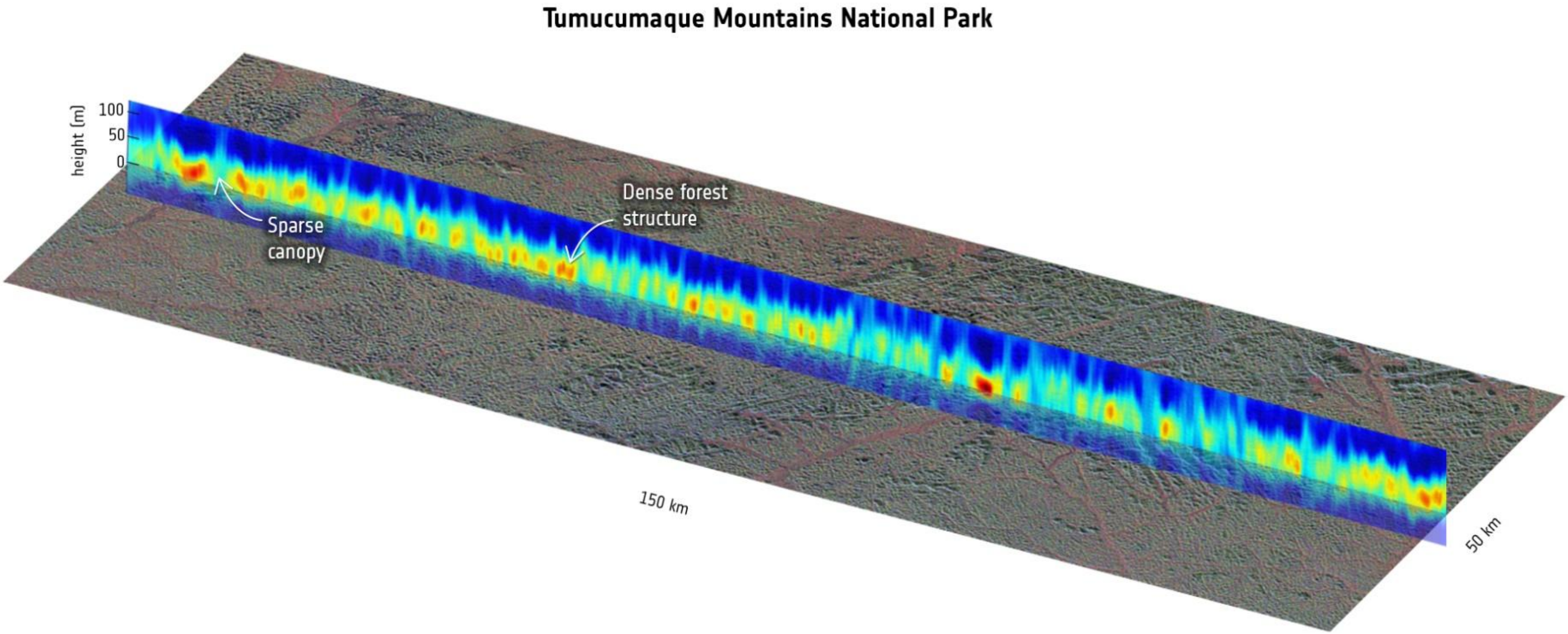
BIOMASS will implement two Mission phases

- ***Tomographic phase*** (November 21, 2025 – March 31, 2027): stacks of **seven** consecutive passes with a revisit time of 3 days, to provide 3D imaging capabilities with a vertical resolution of about 25 m at the equator
- ***Interferometric phase*** (rest of mission lifetime): stacks of **three** consecutive acquisitions (or triplets) with a revisit time of 3 days, ensuring interferometric global coverage every nine months



Tomographic SAR Imaging

The first BIOMASS forest tomography was successfully produced October 31, 2025, at the Tumucumaque Mountains National Park, Northern Brazil



TomoSAR & Forested areas

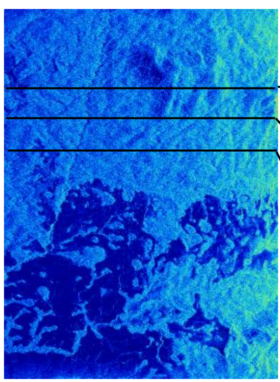
***Forest scenarios**: separation of backscatter from different heights within the vegetation*

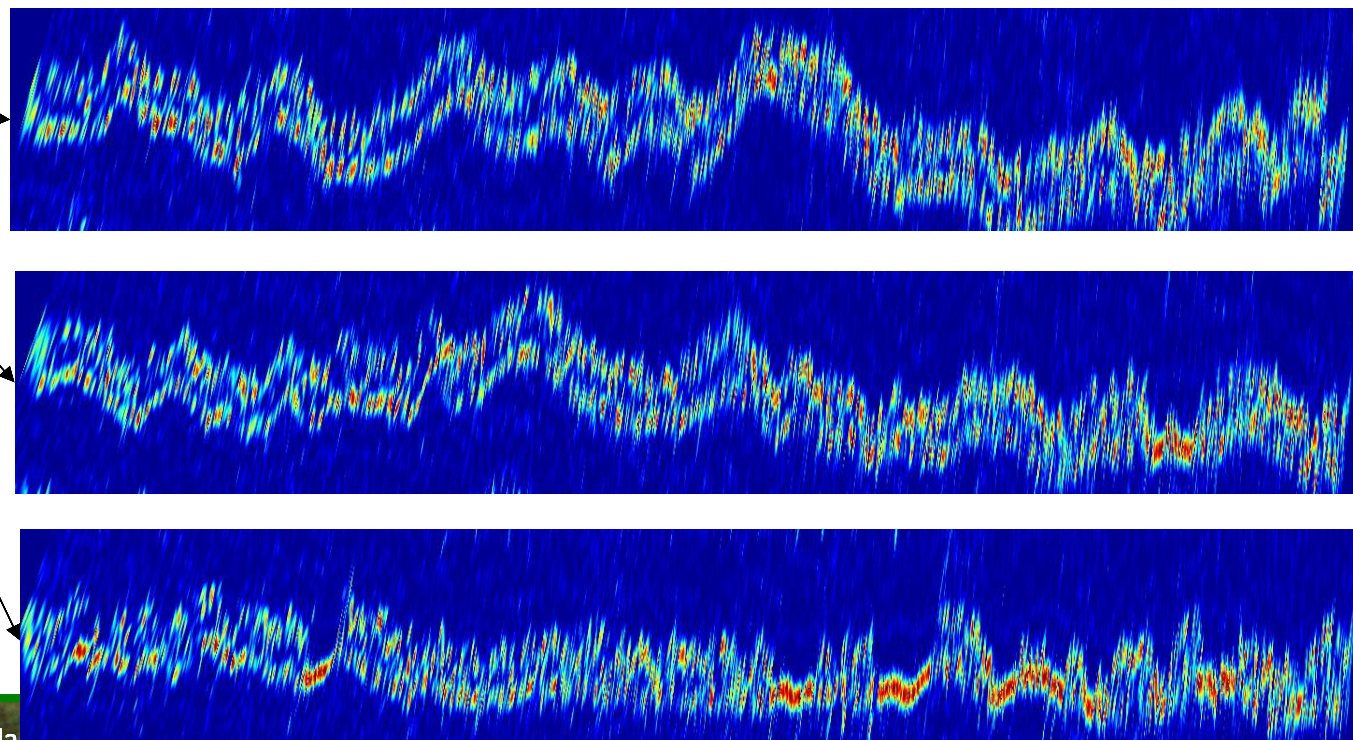
⇒ *Forest height*

⇒ *Sub-canopy terrain topography*

⇒ *Classification of forest structure*

⇒ *Improved forest biomass retrieval*





Tomographic data from AfriSAR 2016 (ESA)

Site: Gabon

Acquisition by DLR & ONERA

TomoSAR & Forested areas

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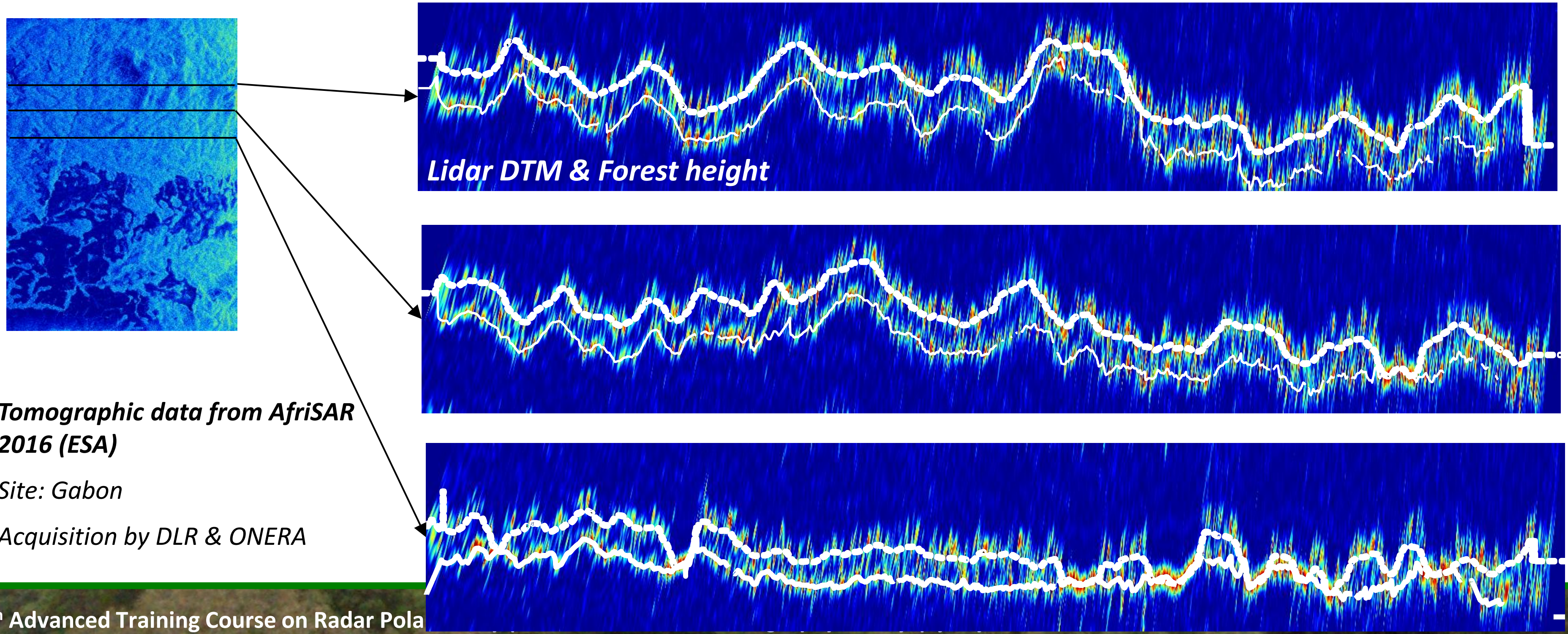


Tomographic data from AfriSAR 2016 (ESA)

Site: Gabon

Acquisition by DLR & ONERA

TomoSAR & Forested areas

Forest height

Site: Lopé, Gabon

Data-set: AfriSAR (ESA)

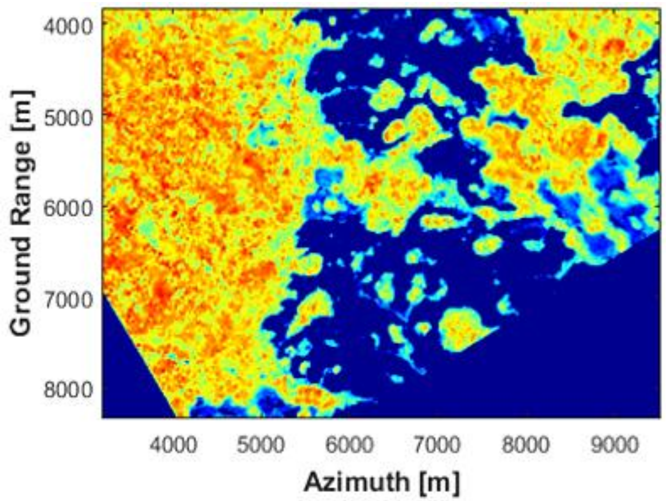
Frequency: P-Band

$\sigma_{SAR\text{-}LIDAR} \approx 3$ *m*

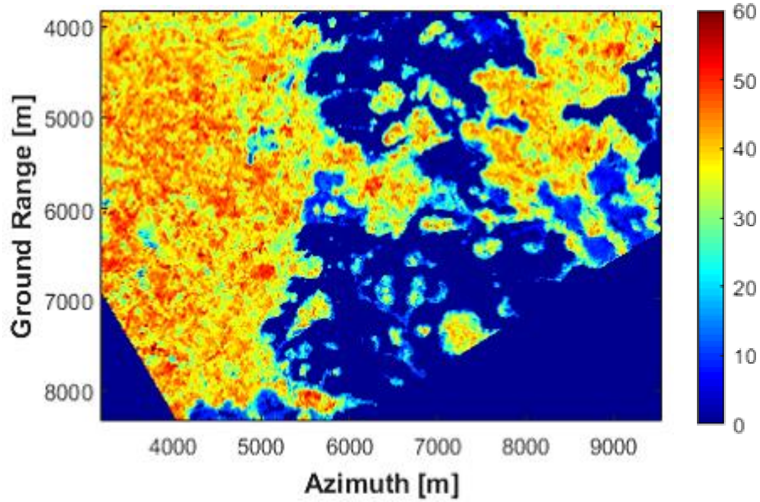
@ 25 m

Yang et al., GRSL, 2020

Lidar Canopy height



TomoSAR Canopy height



Sub-canopy terrain topography

Site: Mondah, Gabon

Data-set: AfriSAR (ESA)

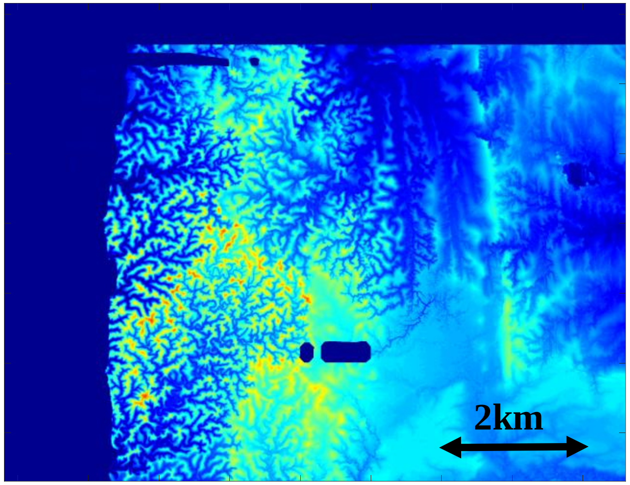
Frequency: P-Band

$\sigma_{SAR\text{-}LIDAR} \approx 2.8$ *m*

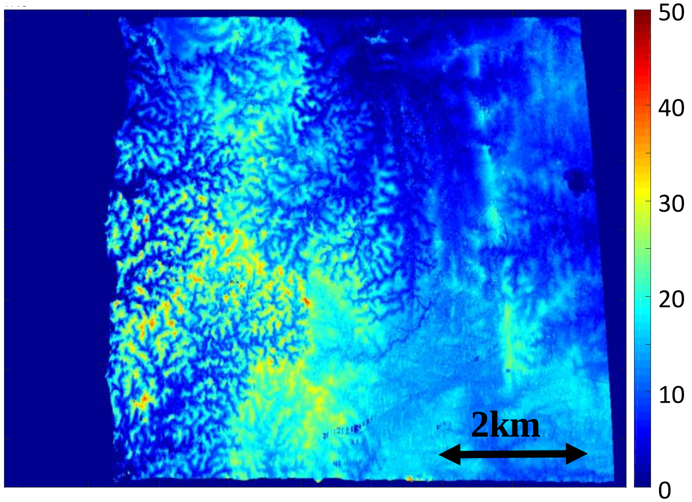
@ 15 m

Mariotti et al., 2019
Pardini et al., 2018
Wasik et al., 2018

Lidar Terrain Model

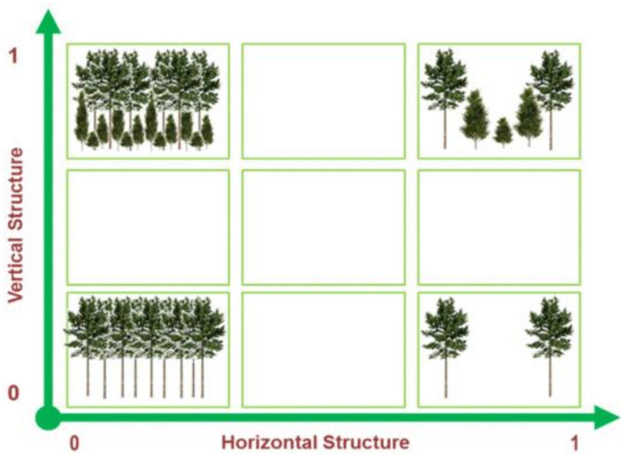


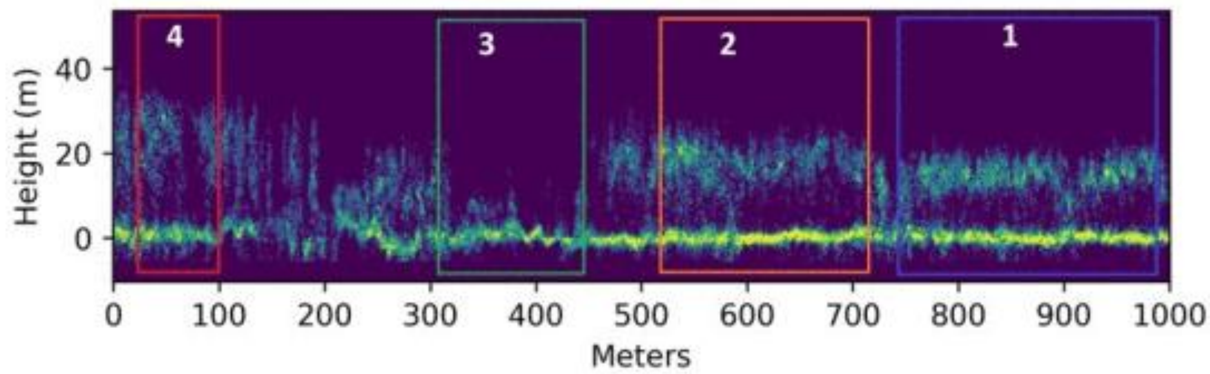
TomoSAR Terrain Model

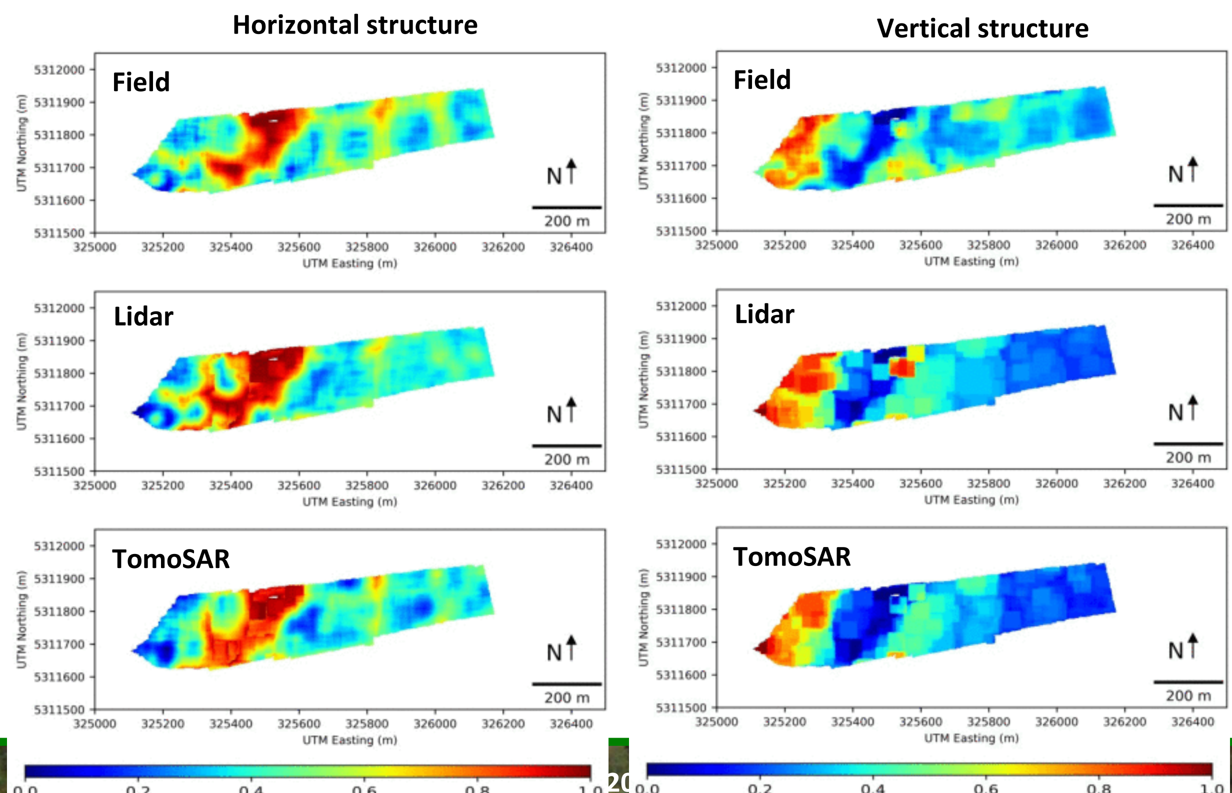


TomoSAR & Forested areas

Classification of forest structure







Site: Traunstein, Germany

Frequency: L-Band

Data-set by DLR

Tello et al., Journal of Selected Topics in Applied Earth Observations and Remote Sensing, 2018

TomoSAR & Forested areas

Correlation between Radar intensity and Above Ground Biomass (AGB)

- 2D SAR intensity is poorly correlated to AGB
- TomoSAR intensity at 0 m is poorly and negatively correlated to AGB
- TomoSAR intensity at main canopy height is highly correlated to AGB (≈ 50 Mg/ha per dB)

Sites: Paracou, Nourages (French Guiana)

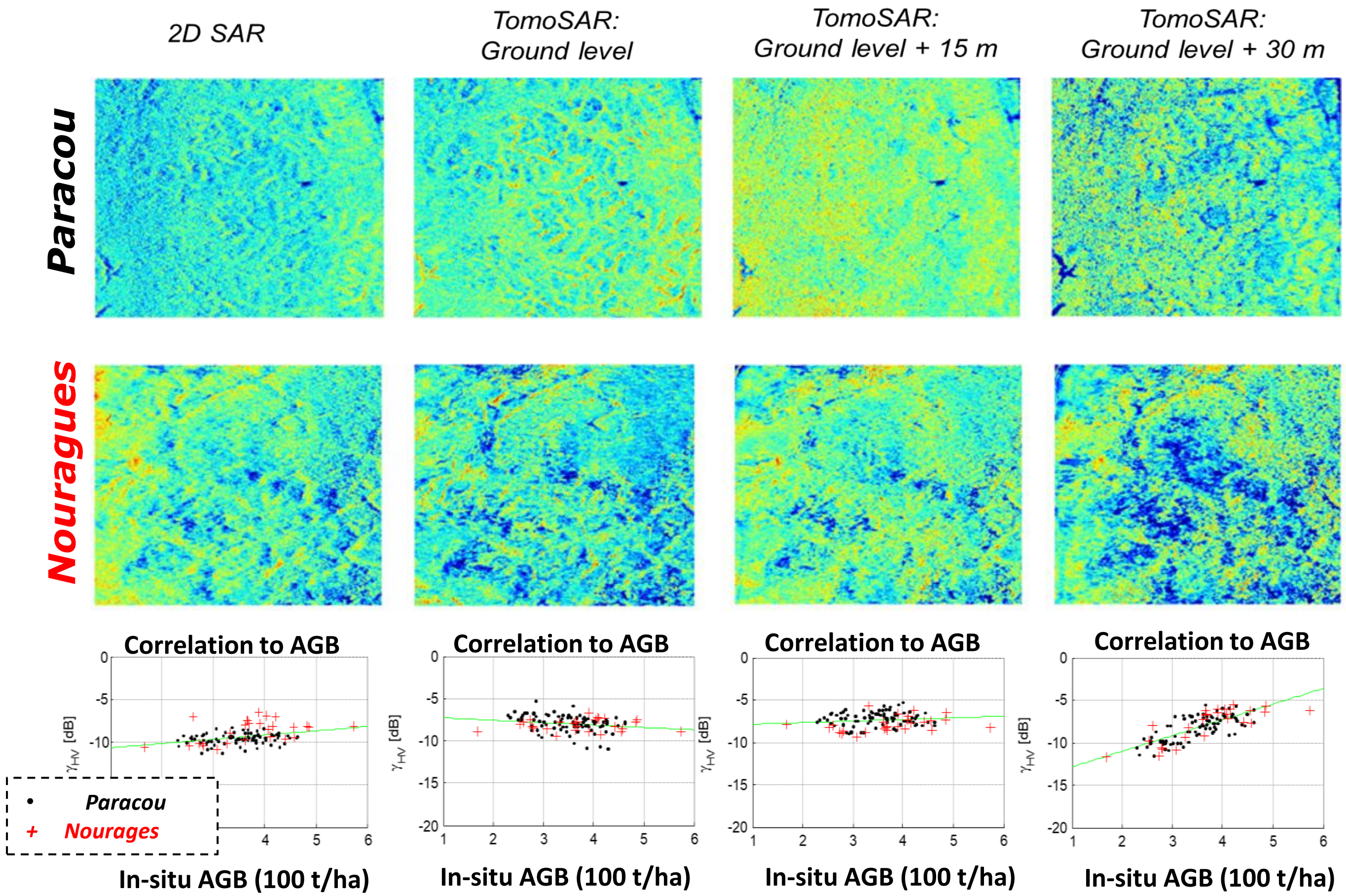
Frequency: P-Band

Data-set: TropiSAR (ESA)

Data-set by ONERA

Ho Tong Minh et al., TGRS, 2014

Ho Tong Minh et al., Remote Sensing of Environment, 2016



TomoSAR & Forested areas

Results were confirmed for three African forest sites…..

Sites: Paracou, Nourages (French Guiana)

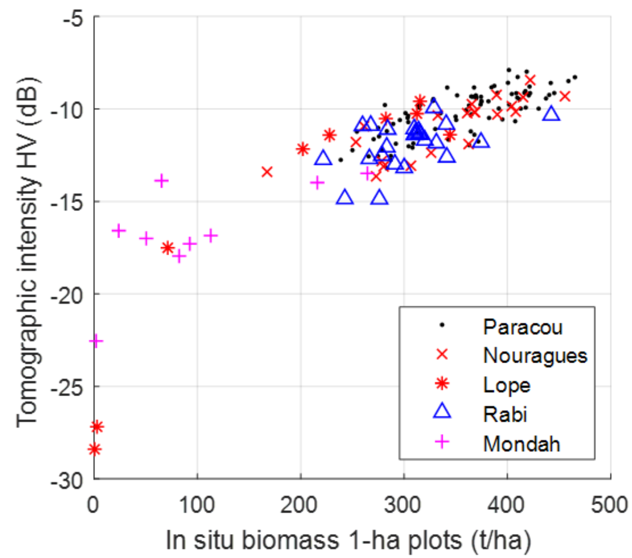
Lopé, Rabi, Mondah (Gabon)

Frequency: P-Band

Data-sets: TropiSAR and AfriSAR (ESA)

Data-set by ONERA

Tebaldini et al., Geophysical Surveys, 2019



…. and for a boreal site at L-Band

Site: Krycklan (Sweden)

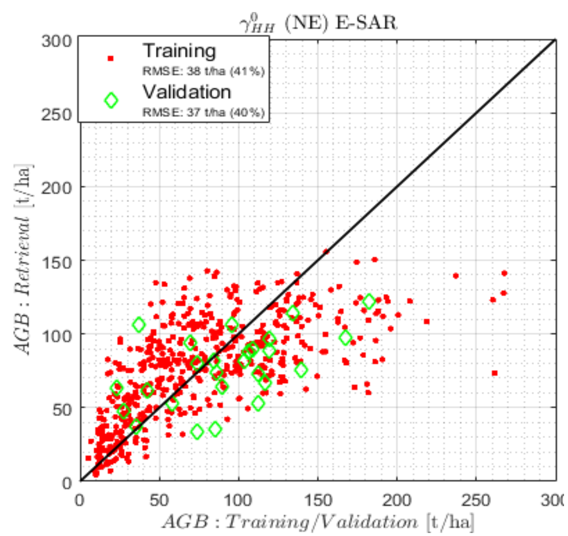
Frequency: L-Band

Data-set: BioSAR 2 (ESA)

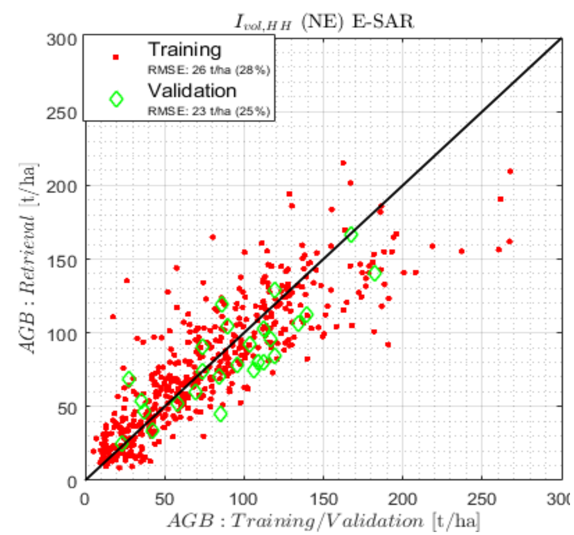
Data-set by DLR

Blomberg et al., GRSL, 2018

2D SAR



TomoSAR



TomoSAR & Glaciers/ Ice sheets

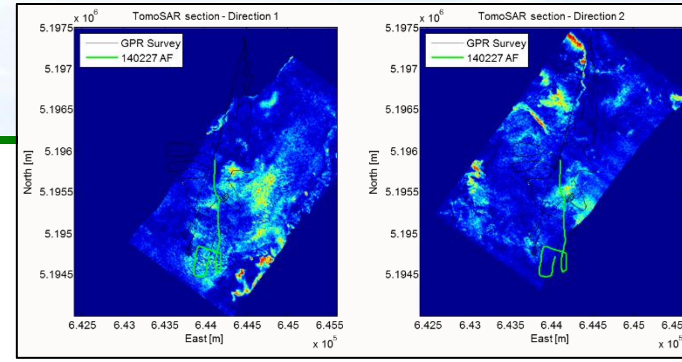
Glaciers: inside view of the ice body

⇒ ***Bedrock detection below the ice surface***

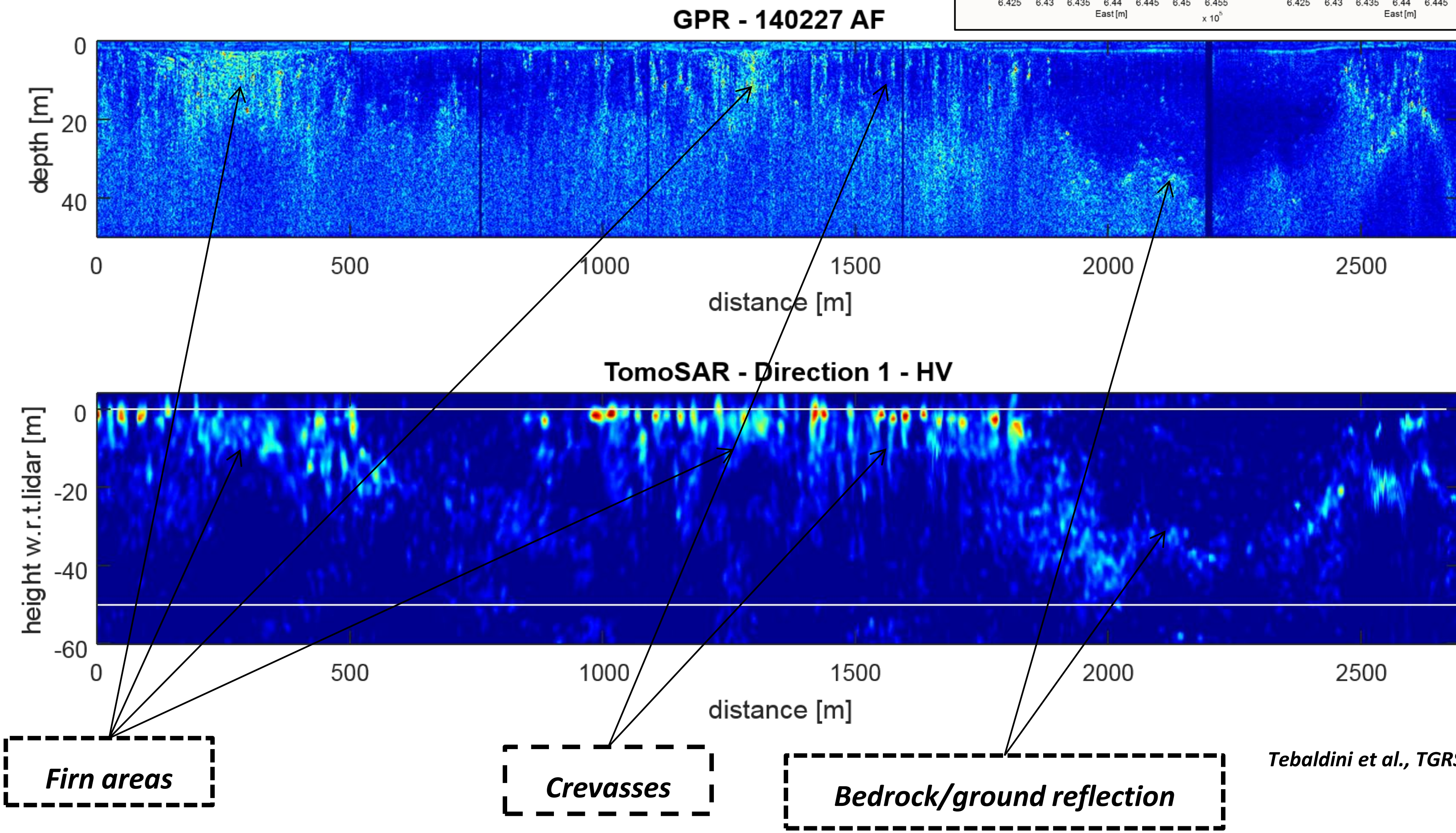
⇒ ***Imaging of internal structures***

Mittelbergferner,
Austrian Alps, March 2014

TomoSAR & Glaciers/ Ice sheets

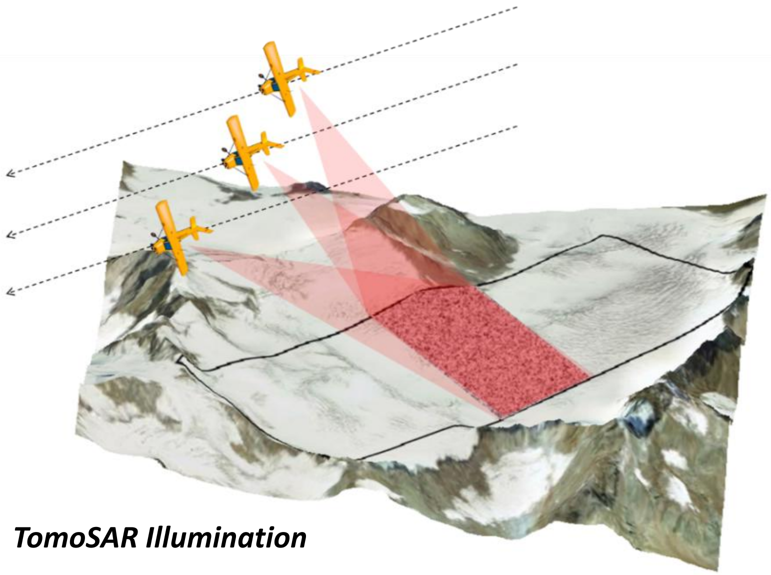


Comparison between 200 MHz Ground Penetrating Radar and L-Band TomoSAR

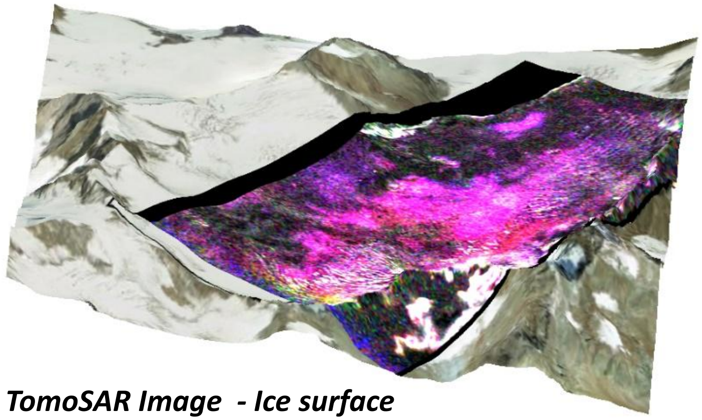


Tebaldini et al., TGRS, 2016

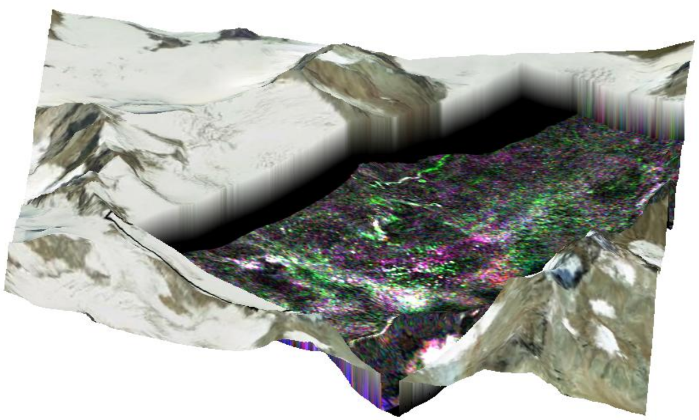
TomoSAR & Glaciers/ Ice sheets



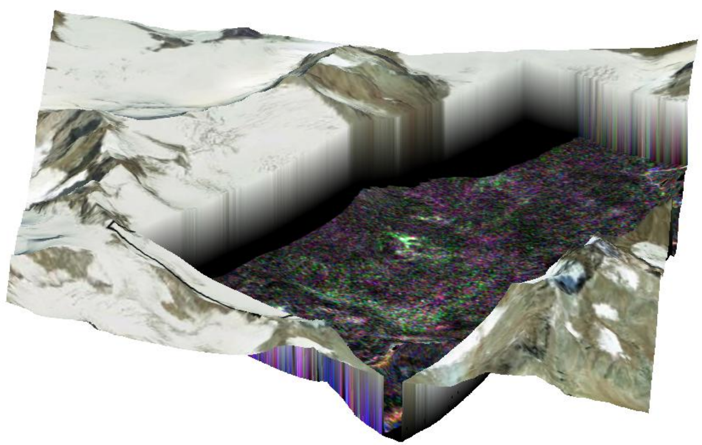
TomoSAR Illumination



TomoSAR Image - Ice surface



TomoSAR Image - 25 m below the Ice surface



TomoSAR Image - 50 m below the Ice surface

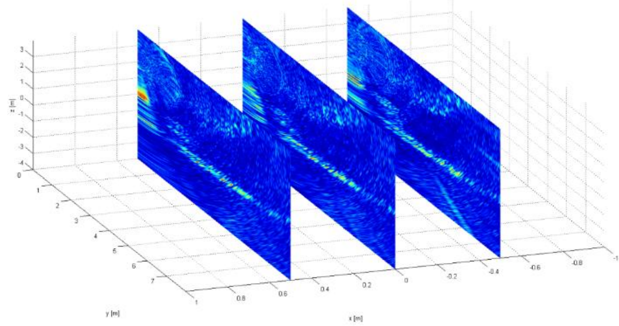
TomoSAR & Snow

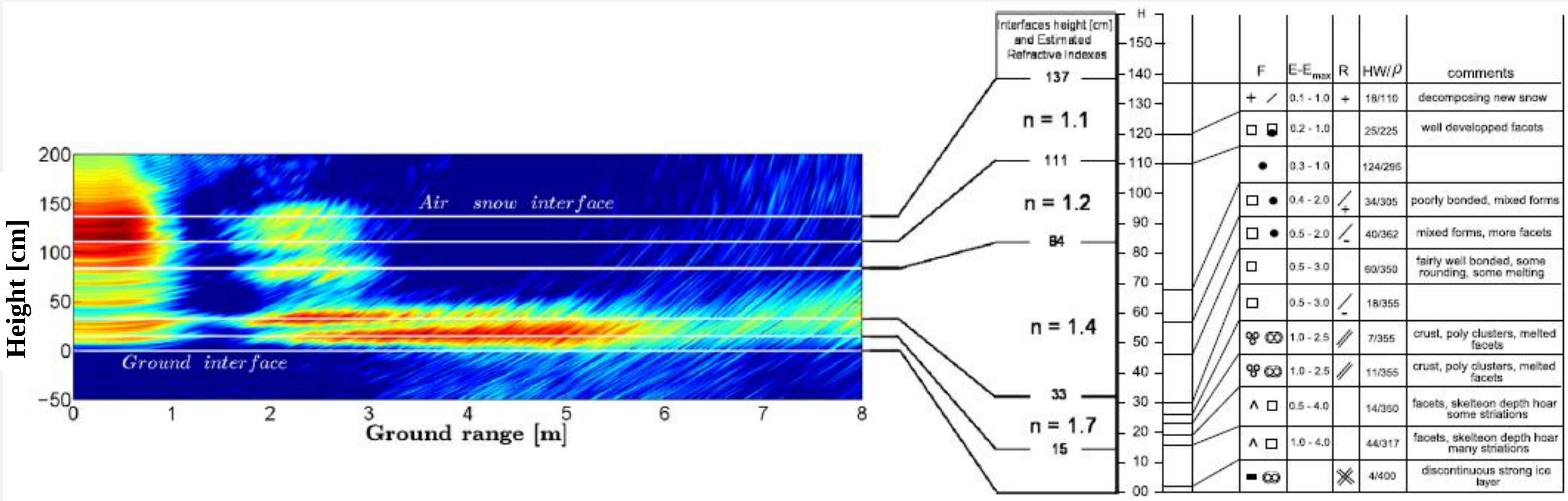
***Snow**: fine structure of snowpack layering*

⇒ *Total Snow depth*

⇒ *Refractive index*

⇒ *Internal layering*





Data from AlpSAR 2013 (Rennes 1, ESA)

Rekioua et al., Comptes Rendus Physique , 2017

A step back…

RADAR (***Radio Detection And Ranging***) is a technology to detect and study far off targets by transmitting EM pulses at radiofrequency and observing the backscattered echoes

Some relevant features:

1. **Active instrument:** ⇔ ***no need for external illumination source***
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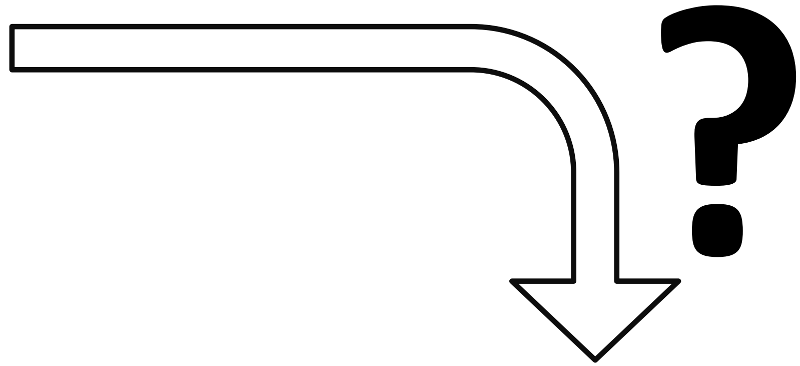

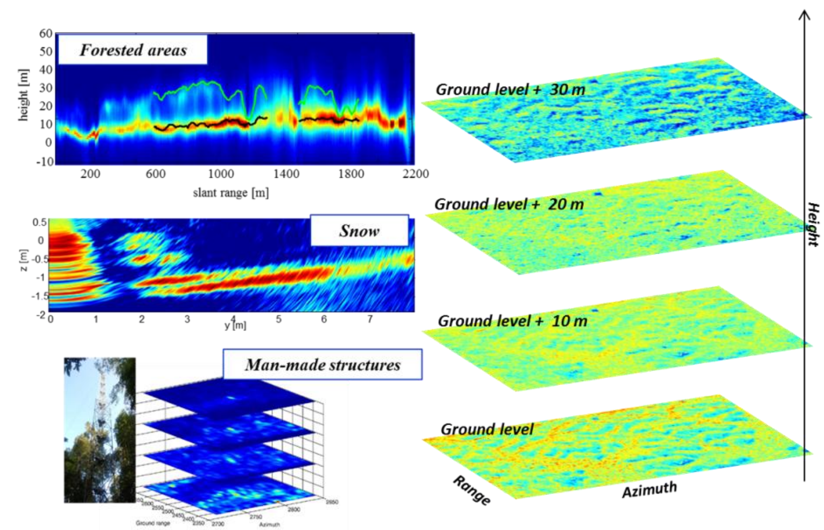

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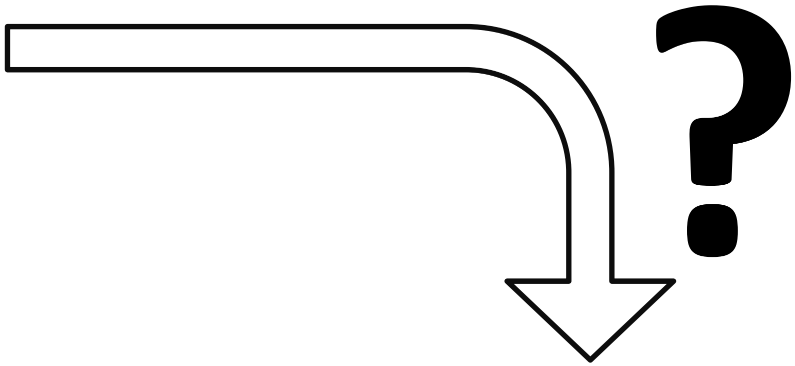
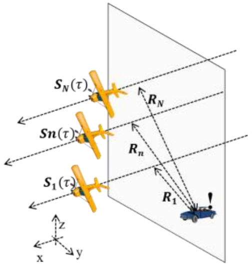
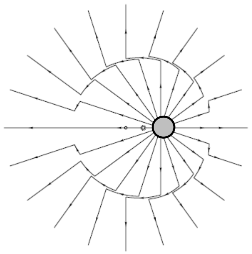


Answer:
Yes (☺), with some efforts concerning

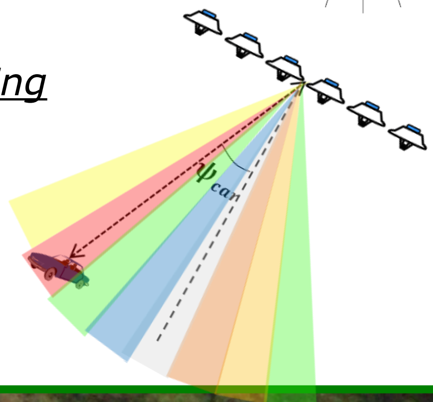


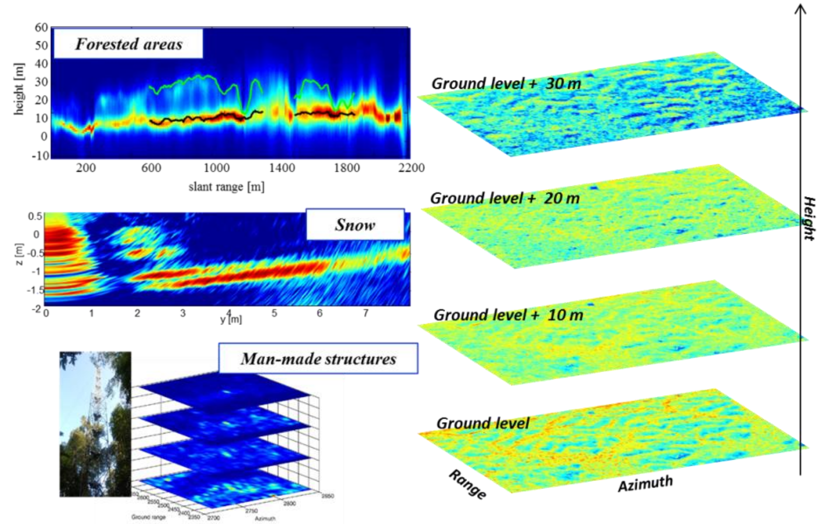
Geometry

Waves



Radar Processing



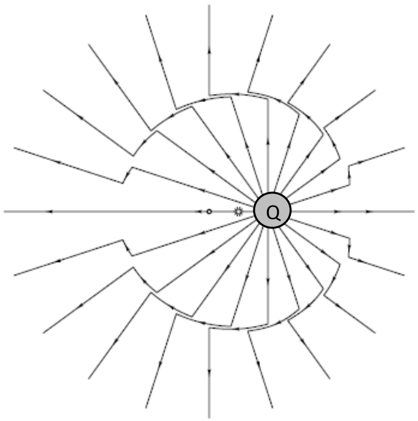


Waves

Must knows about waves (at least for today)

Electromagnetic waves

- EM wave = perturbation in the intensity of the static EM field
- Effect of the universal speed limit
- Triggered by charges in non-uniform motion
- Propagation velocity in free space: $c \approx 3 \cdot 10^8$ m/s



Representation

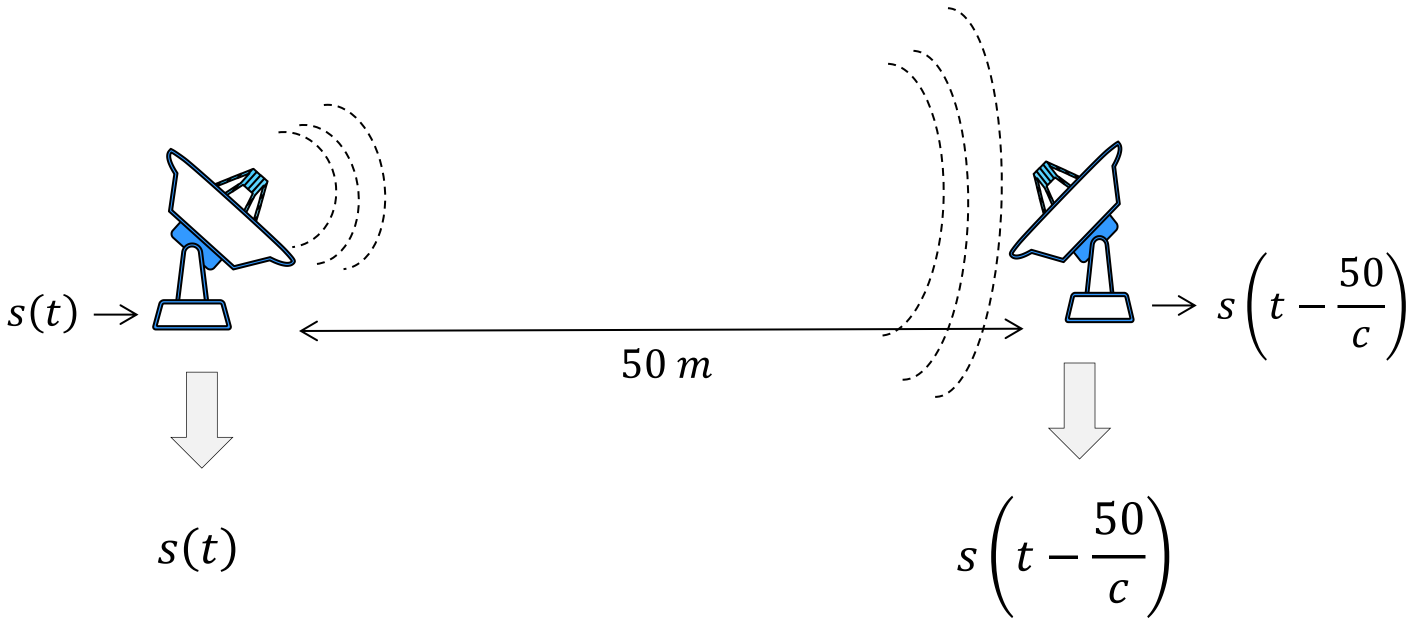
We can describe waves as signals that vary over **time** and **space** as:

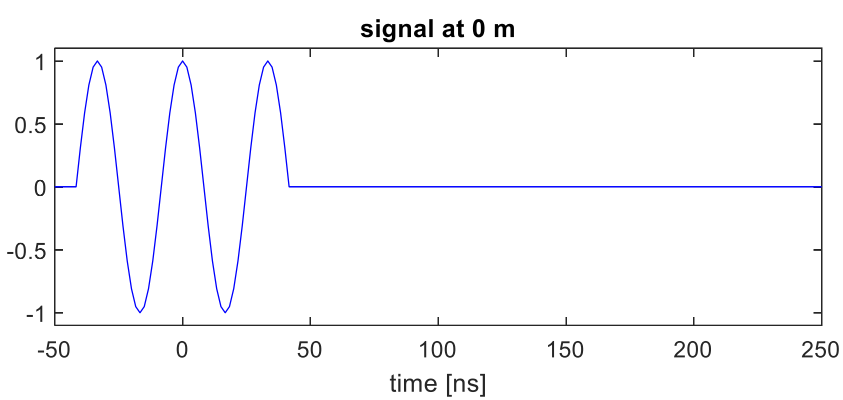
$$s\left(t - \frac{z}{c}\right)$$

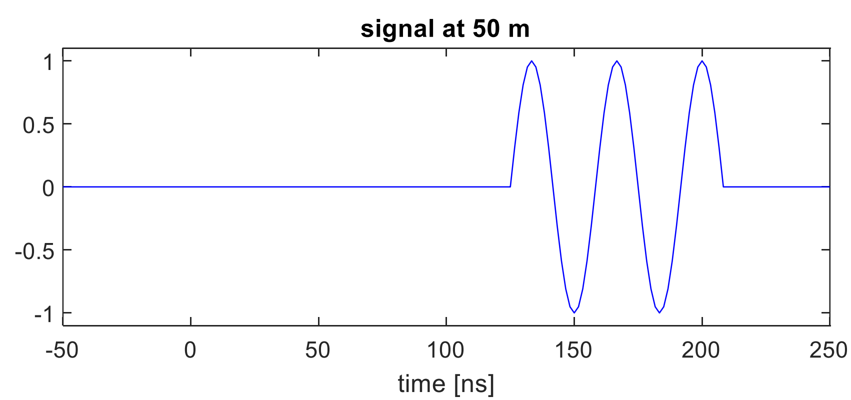
with $s(t)$ some waveform

Must knows about waves (at least for today)

When we express a signal as a function of time, we are implicitly assuming that the signal is measured at a **fixed point in space**

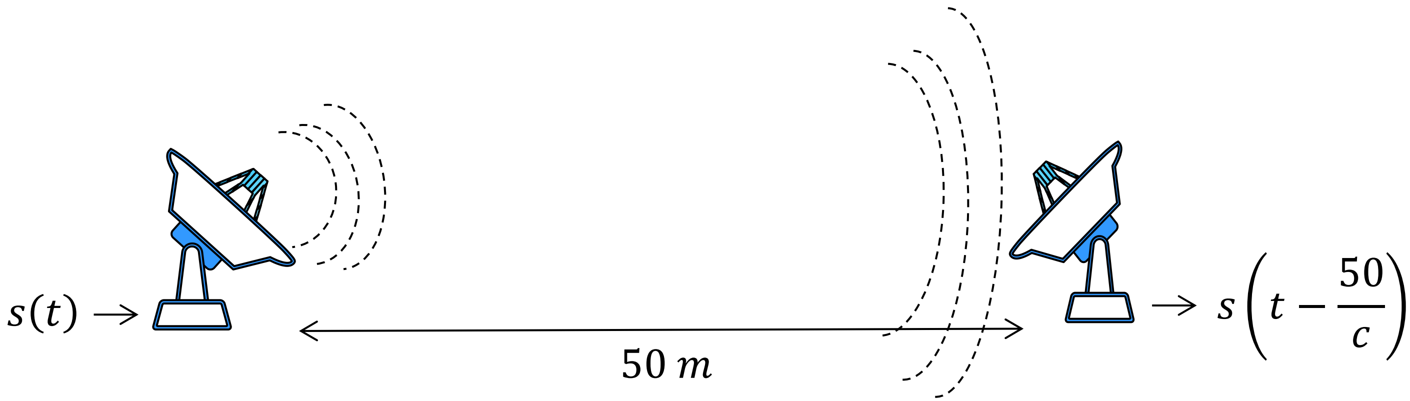


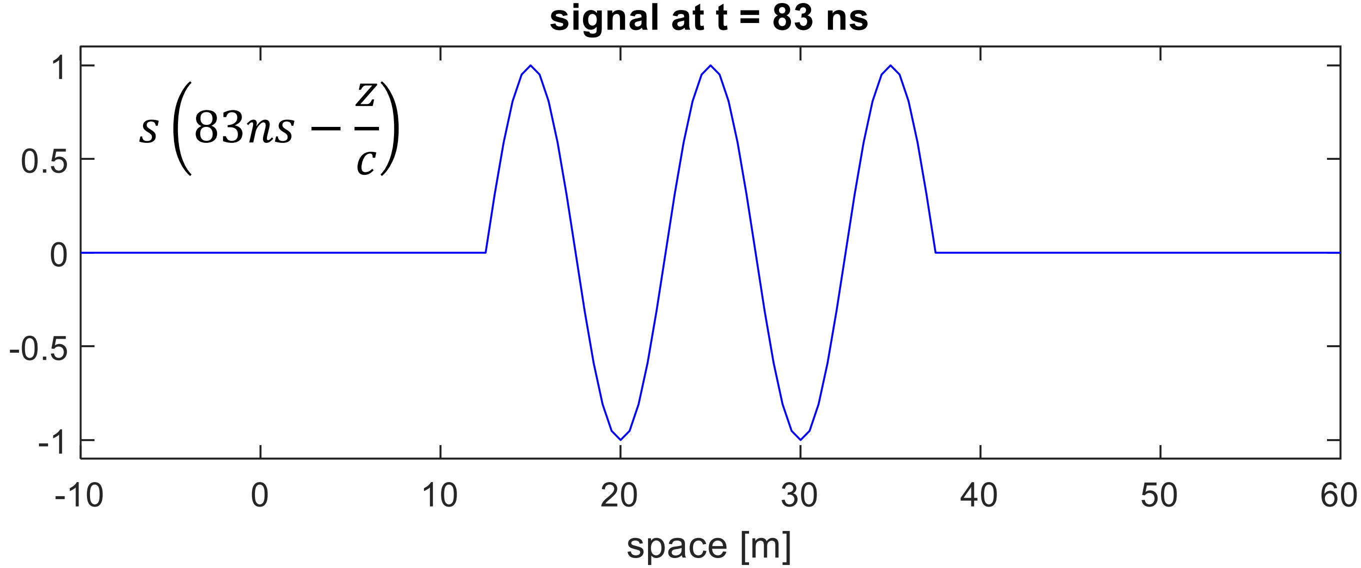




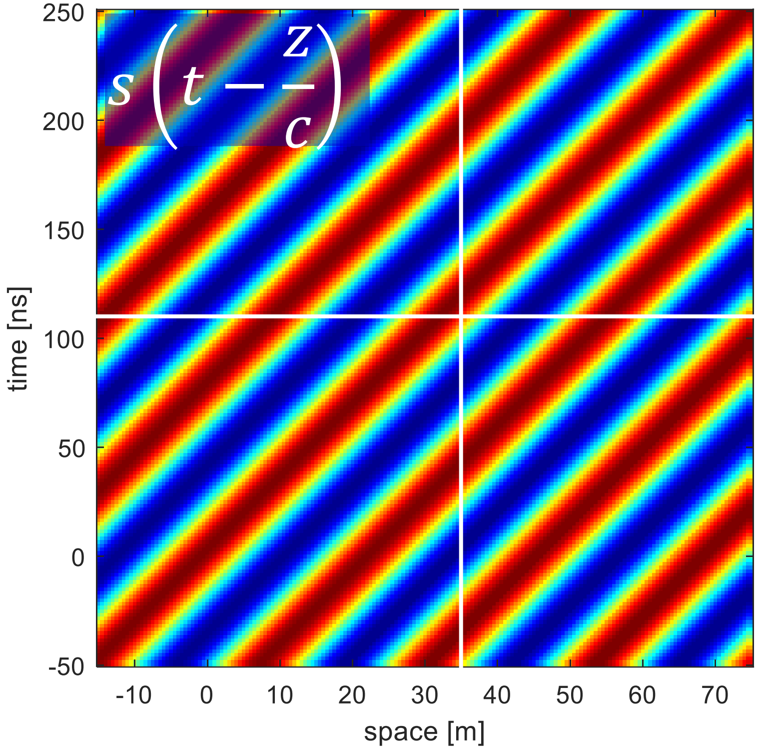
Must knows about waves (at least for today)

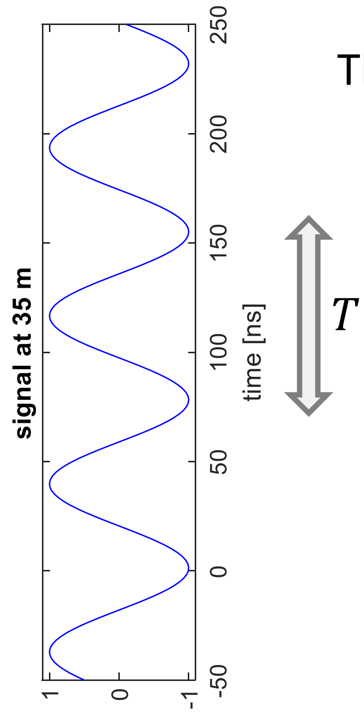
Equivalently, we can express a signal as a function of space by imaging that we can freeze the time at a **fixed instant** and take a snapshot of the signal distribution over space





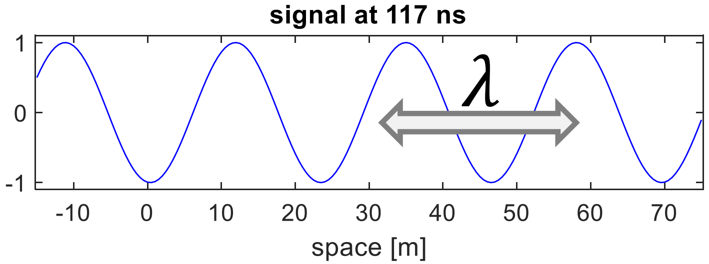
For the case of a monochromatic wave we have $s(t) = cos(2\pi f_o t)$





The temporal period is obtained as

$$T = \frac{1}{f_o}$$



The spatial period, a.k.a. ***the wavelength***, is obtained as

$$\lambda = \frac{c}{f_o}$$

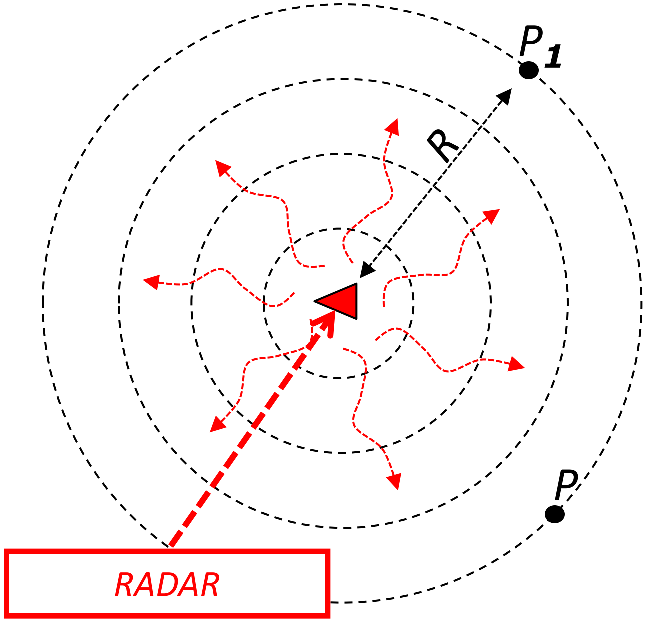
Geometric principles of target localization

<u>*Simplified* </u>*description*

I) *The transmitted signal propagates away from the RADAR sensors in **all directions**[(*)] in the form of a **spherical wave***


Circular wave on the water surface

[(*)]Note: real antennas actually radiate over an angular sector, depending on size and wavelength



Letting $s(t)$ denote the transmitted signal, the signal at point $\boldsymbol{P_1}$ is obtained as

$$s_1(t) = s\left(t - \frac{R}{c}\right)$$

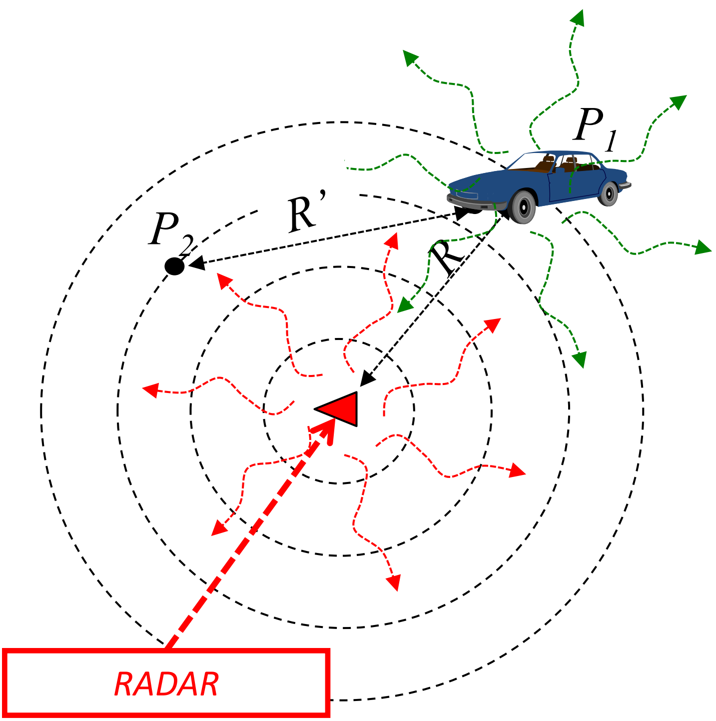
R = distance between P_1 and the RADAR

c = speed of light

Simplified description

II) The signal interacts with surrounding objects (targets) ⇔ backscattered echoes

As a first approximation the backscattered echo can be represented by imaging the target as a new source of spherical waves triggered by the impinging signal $\boldsymbol{s_1(t)}$



$s_1(t)$ = signal at point P_1

$$s_1(t) = s\left(t - \frac{R}{c}\right)$$

R = distance between P_1 and the RADAR

c = speed of light

$s_2(t)$ = backscattered signal at point P_2

$$s_2(t) = A \cdot s_1\left(t - \frac{R'}{c}\right) = A \cdot s\left(t - \frac{R' + R}{c}\right)$$

R' = distance from P_1 to P_2

A = constant accounting for the interaction of the impinging signal with the target

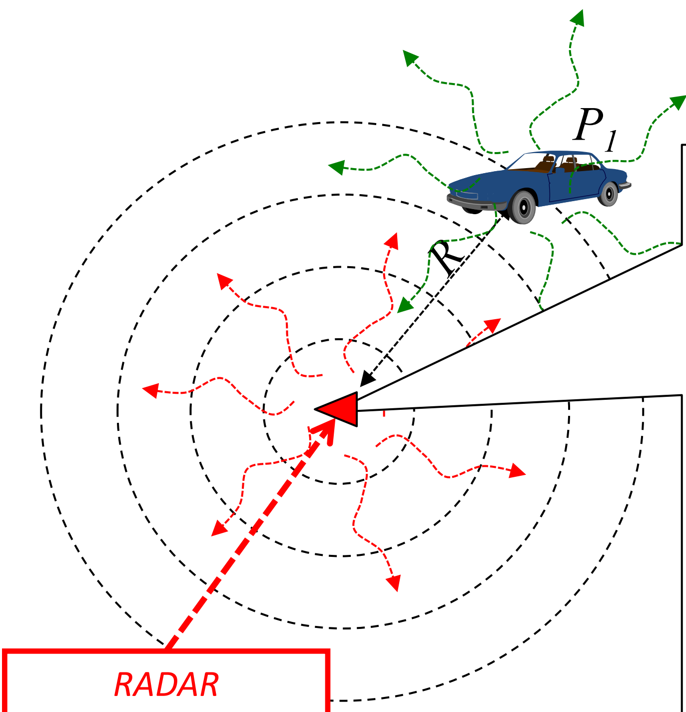
Simplified description:

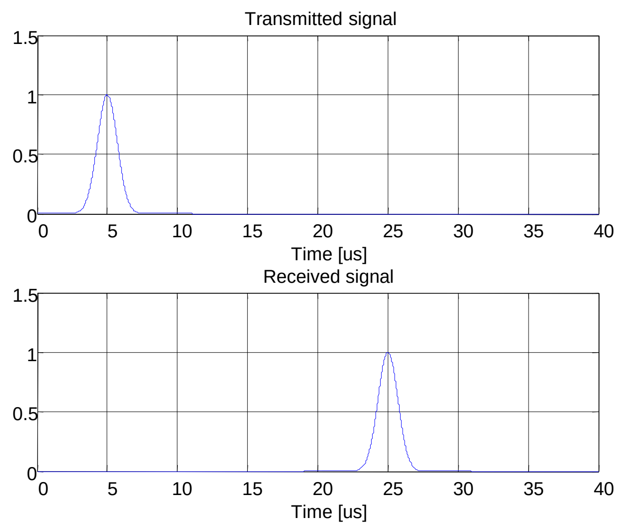
III) The backscattered echo is received by the RADAR sensor

$s_{Rx}(t)$ = backscattered signal received by the RADAR

$$s_{Rx}(t) = A \cdot s_1\left(t - \frac{R}{c}\right) = A \cdot s\left(t - \frac{2R}{c}\right)$$

⇒ *The distance of a target from the RADAR is known by measuring the pulse round-trip time*





Example

Measured delay: $\tau = 20$ *microseconds*

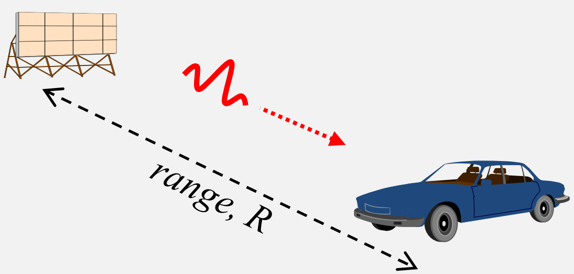
Propagation velocity: c = 300000 Km/s

⇒ **Range: R = $c \cdot \tau/2$ = 3 Km**

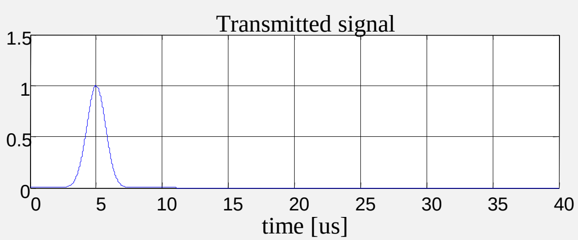
Localization in 1D

Delay measurement

Transmission

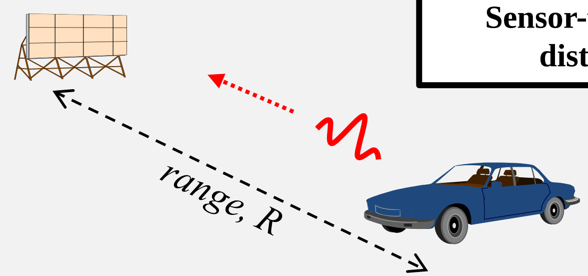


$$s_{Tx}(t) = s(t)$$



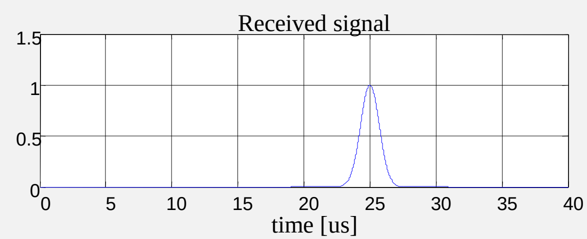
Reception

Signal delay
⇔
Sensor-to-target distance



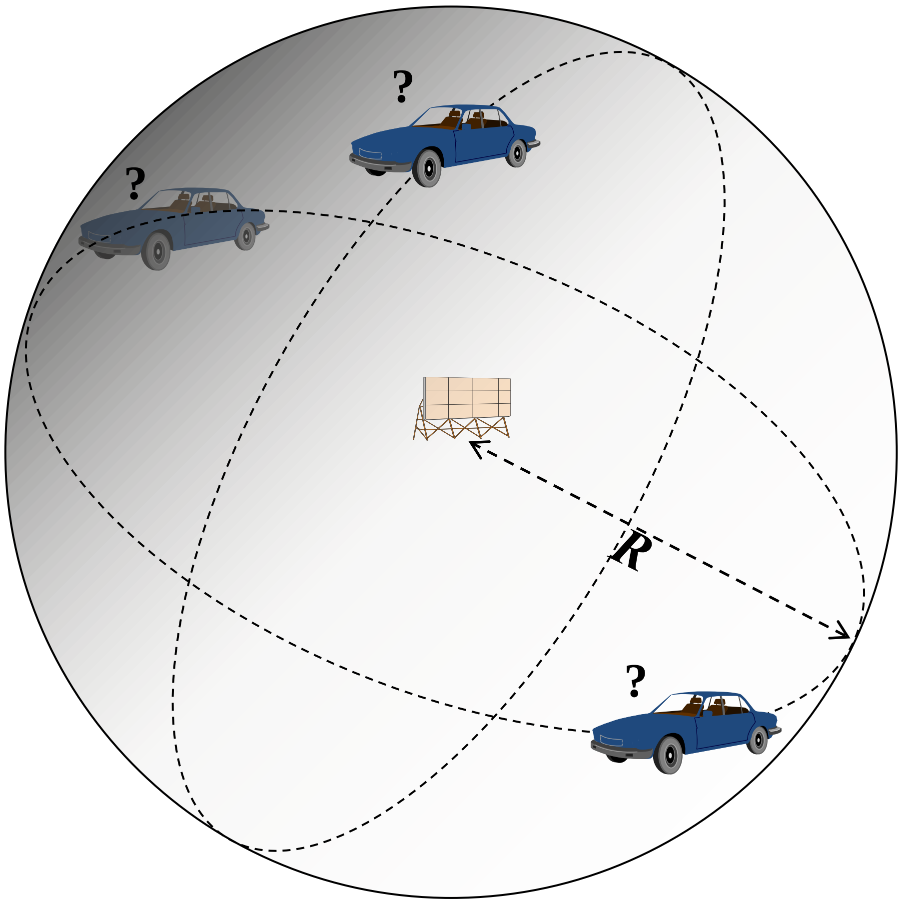
$$s_{Rx}(t) = s\left(t - \frac{2R}{c}\right)$$

r : sensor-to-target distance
c : speed of light



Localization in 1D

Delay measurement ⇔ Localization on the surface of a sphere



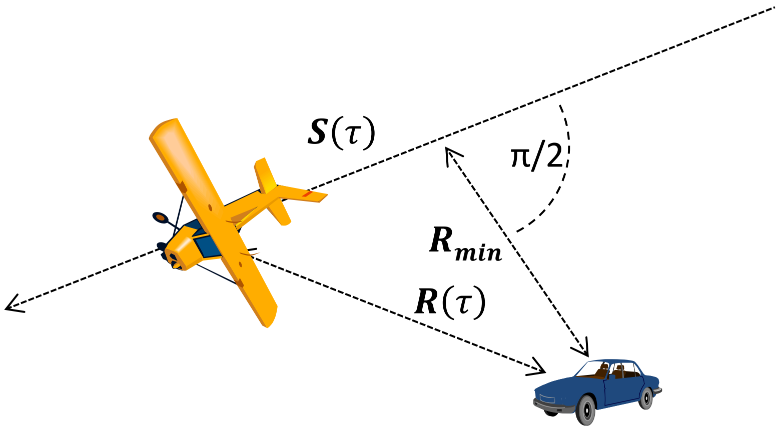
The target is bound to lie on a sphere

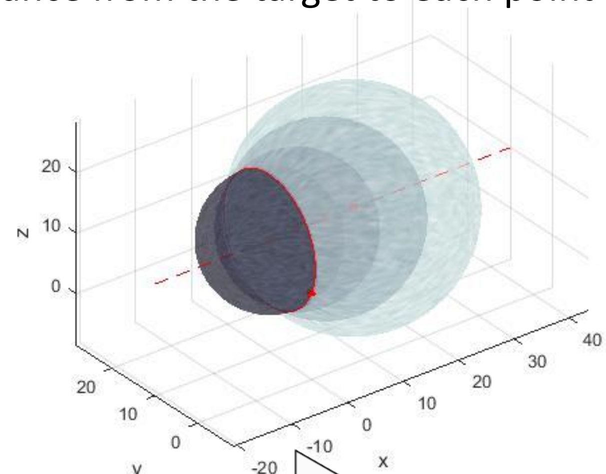
- Centered on the RADAR
- Of radius R

⇒ ***1D Localization***

Localization in 2D (SAR)

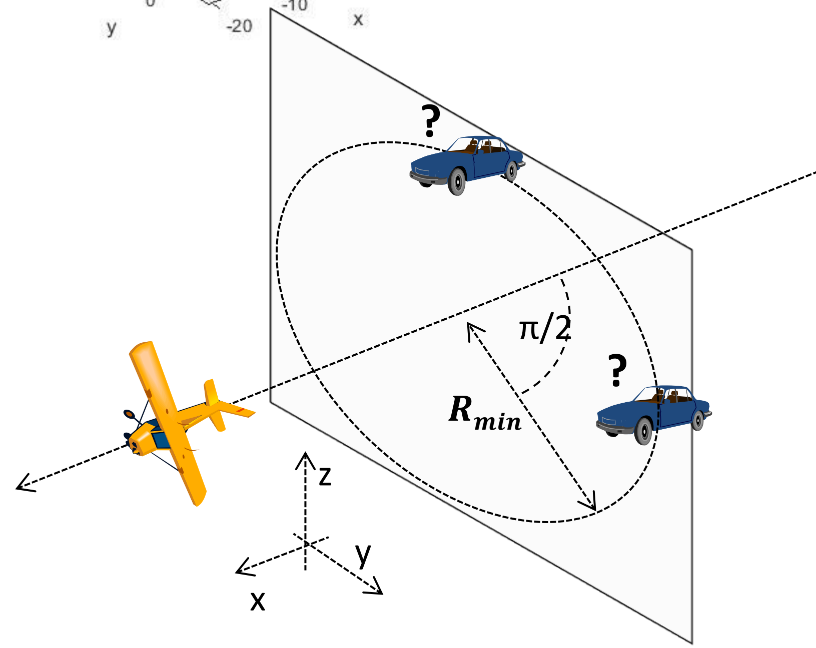
Flying a RADAR along a straight line = measuring the distance from the target to each point on the line





The target is bound to lie on the intersection of all the spheres:

- Centered in $S(\tau)$
- Of radius $R(\tau)$



$\Rightarrow$ The target is bound to lie on the circle:

- Centered on the trajectory
- Perpendicular to the trajectory (*yz* plane)
- Of radius R_{min}

$\Rightarrow$ 2D Localization

Localization in 3D (TomoSAR)

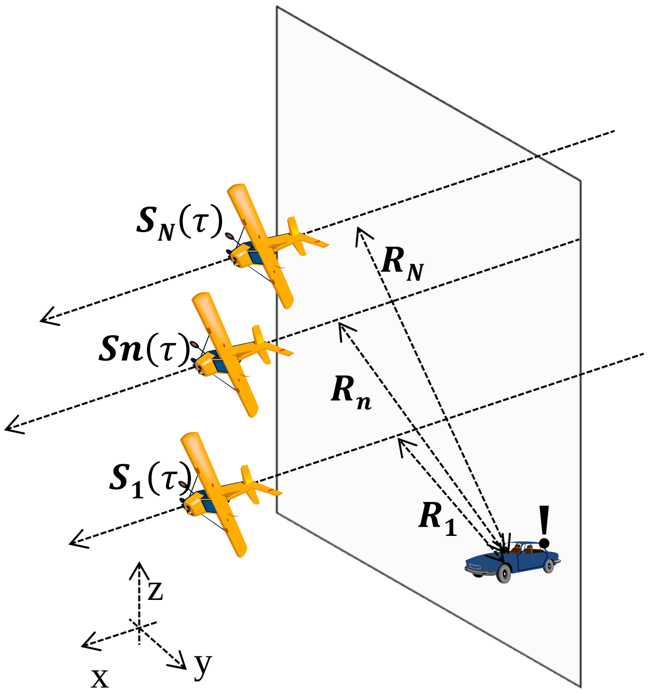
Flying a RADAR along multiple lines = measuring the distance from the target to multiple lines

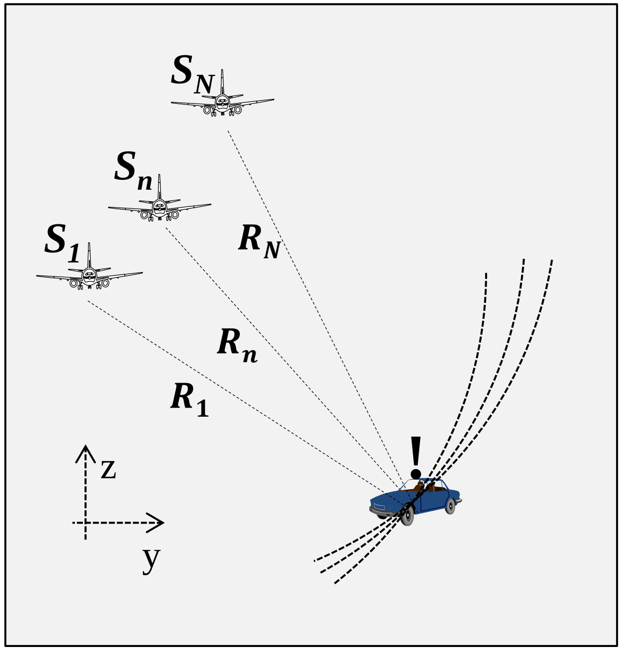
The target is bound to lie on the circles:

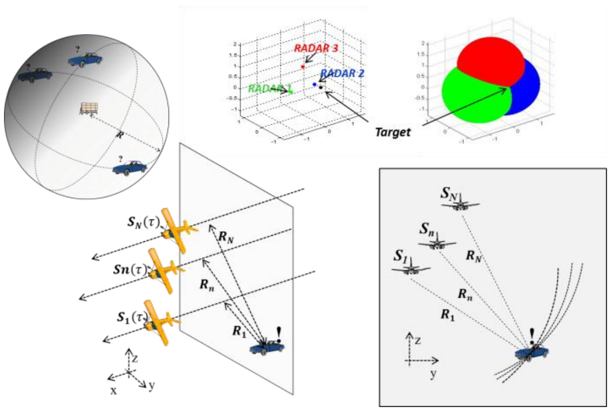
- Centered on each trajectory
- Perpendicular to the trajectory ,
- Of radius R_1 … R_n …. R_N

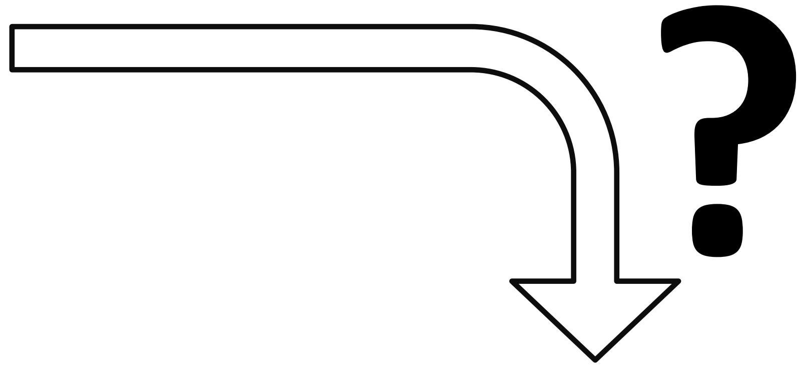
⇒ Only 1 solution in the 3D space !

⇒ 3D localization

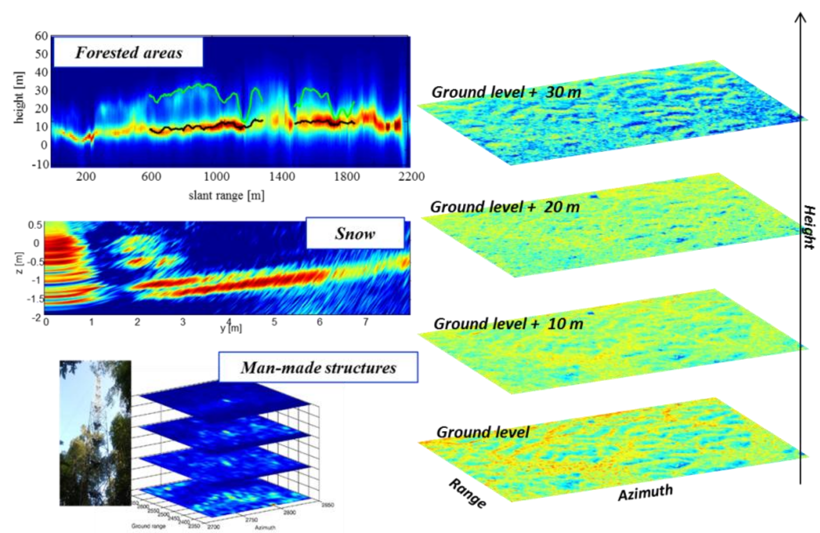


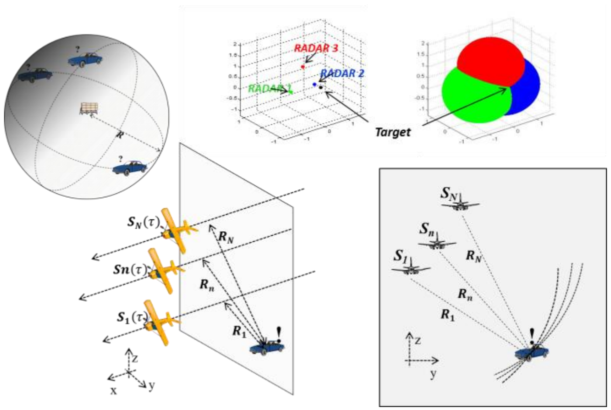


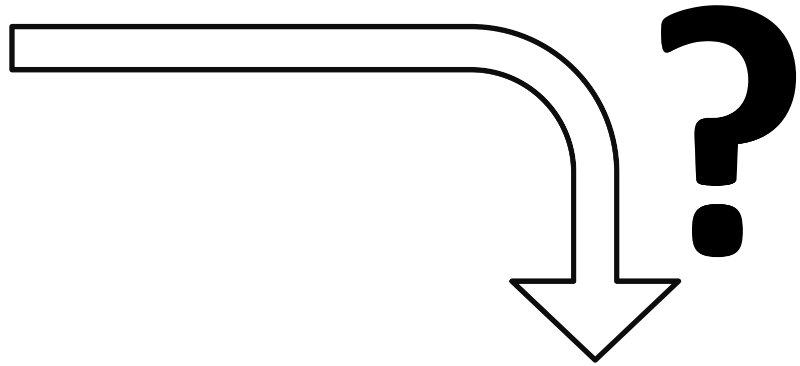




Geometry unveils the principle why flying multiple trajectories results in the capability to localize a target in the 3D space



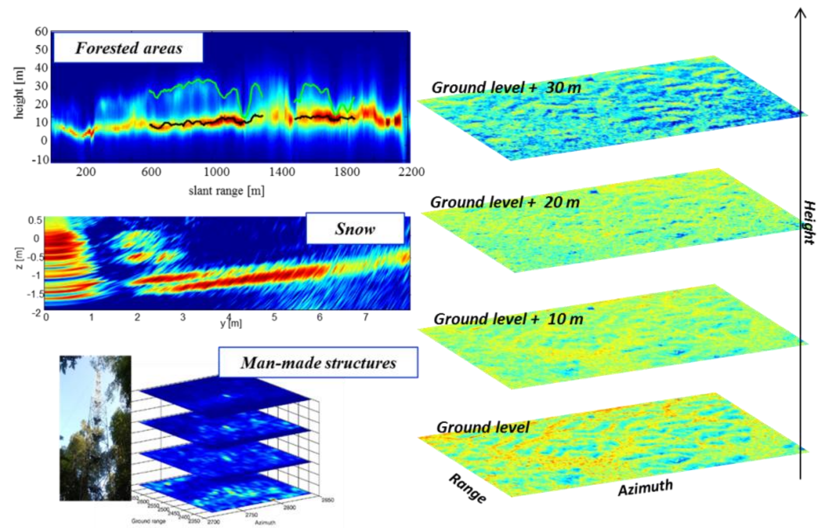




Geometry unveils the principle why flying multiple trajectories results in the capability to localize a target in the 3D space

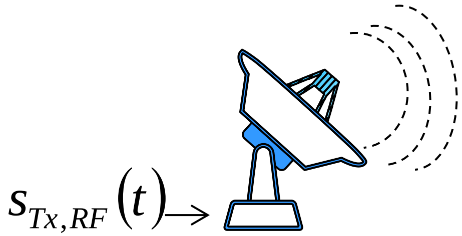
Missing elements:

- ***Resolution***
- ***What if there are many targets ?!!!***



RADAR signals

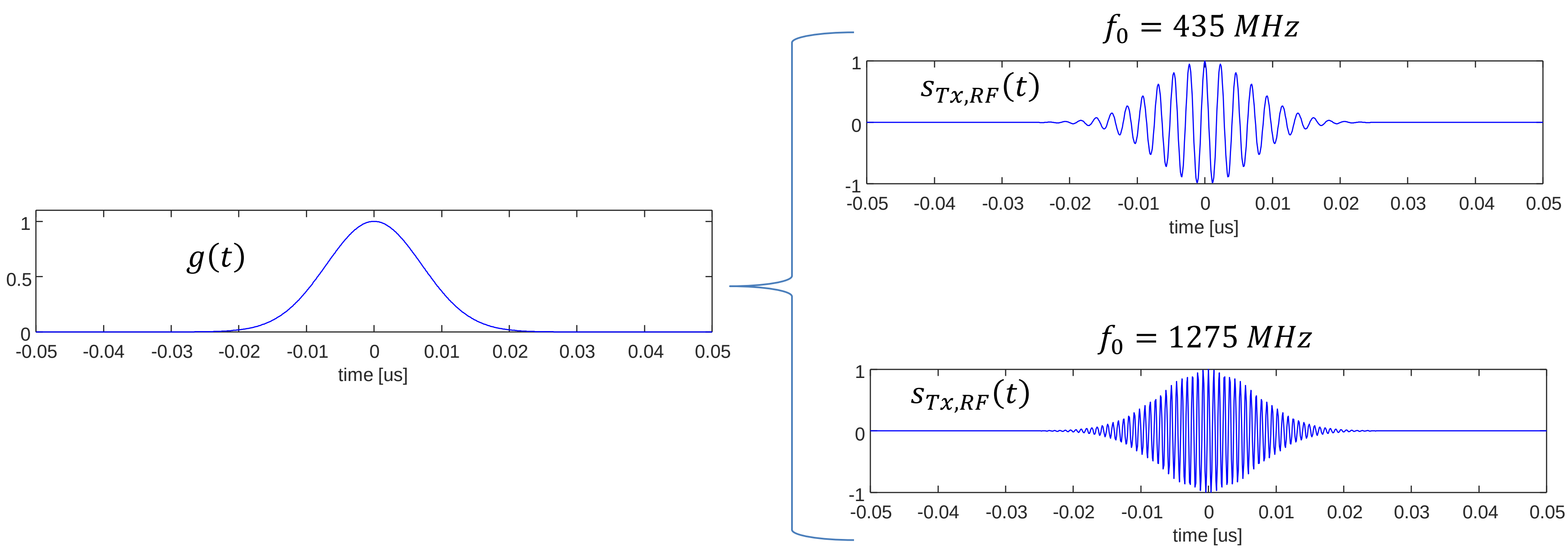
RADARs transmit and receive Radiofrequency (RF) pulses



$$s_{Tx,RF}(t) = g(t) \cdot cos(2\pi f_0 t)$$

g(t) = short EM pulse

f_0 = carrier frequency



Radar signals

RADARs transmit and receive Radiofrequency (RF) pulses

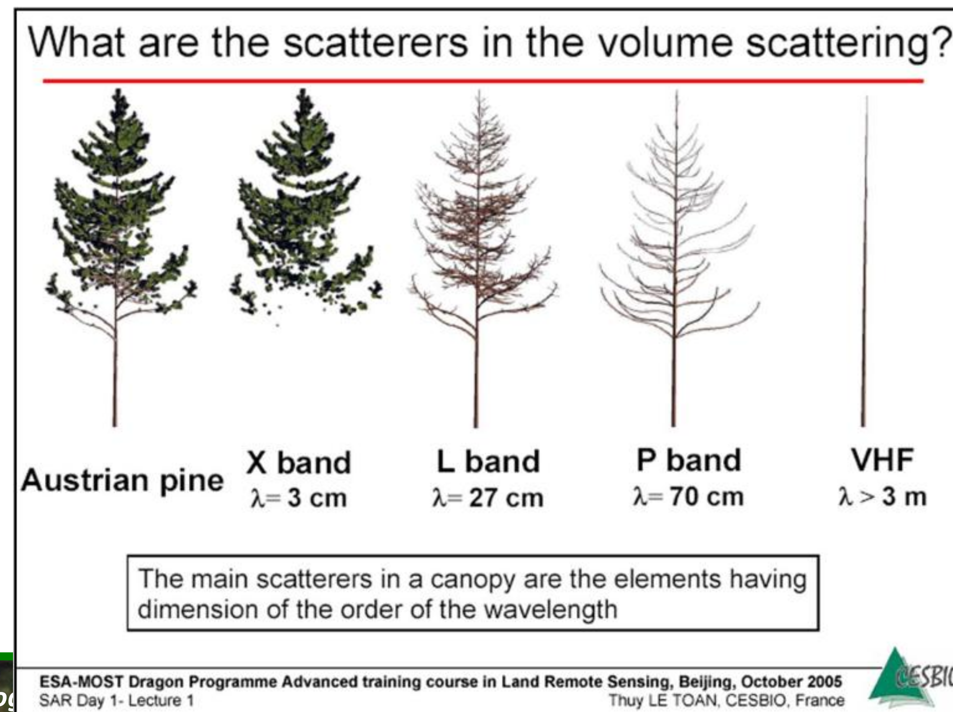
$s_{Tx,RF}(t) \rightarrow$

$$s_{Tx,RF}(t) = g(t) \cdot cos(2\pi f_0 t)$$

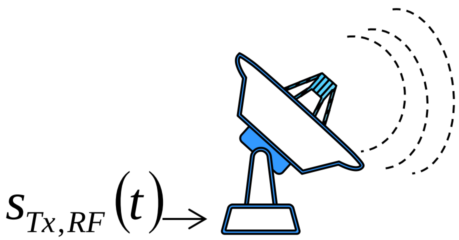
g(t) = short EM pulse
f_0 = carrier frequency

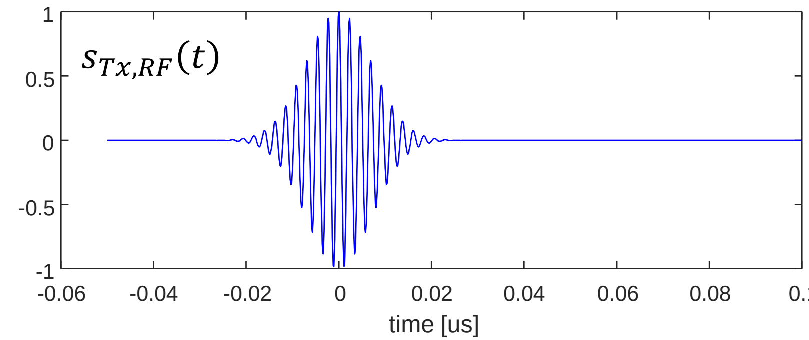
The carrier frequency (or wavelength $\lambda = \frac{c}{f_o}$) is perhaps the most important parameter in the design of a Radar sensor, as it determines:

- The antenna to be used
- The RF hardware to be used
- **The features of the observed targets which the signal is sensitive to**

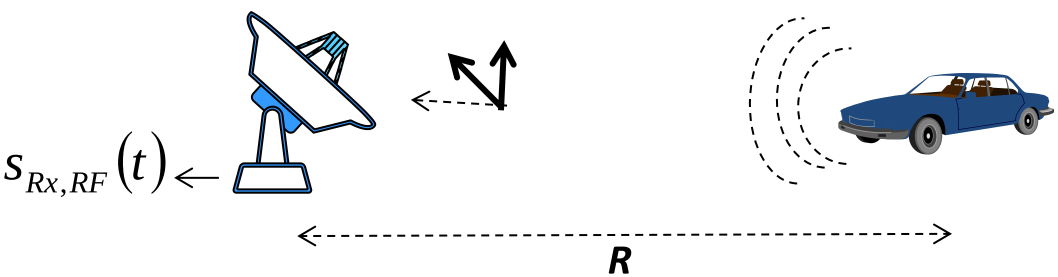


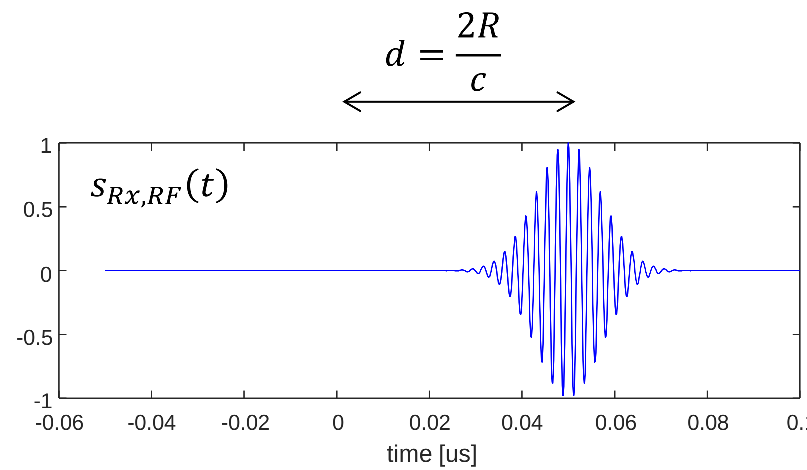
On a mathematical ground, the signal backscattered by a target is represented as a delayed version of the transmitted signal





The signal is received after a **time delay** $\boldsymbol{d = \frac{2R}{c}}$





$$s_{Rx,RF}(t) = A \cdot s_{Tx,RF}(t-d)$$

$$= A \cdot g(t-d) \cdot cos\left(2\pi f_0 (t-d)\right)$$

Radar signals

Following basic trigonometry, the received signal is expressed as:

$$s_{Rx,RF}(t) = \underbrace{A \cdot g(t-d) \cdot cos(2\pi f_0 d)}_{\text{In phase component } I(t)} \cdot cos(2\pi f_0 t) - \underbrace{A \cdot g(t-d) \cdot sin(2\pi f_0 d)}_{\text{Quadrature component } -Q(t)} \cdot sin(2\pi f_0 t)$$

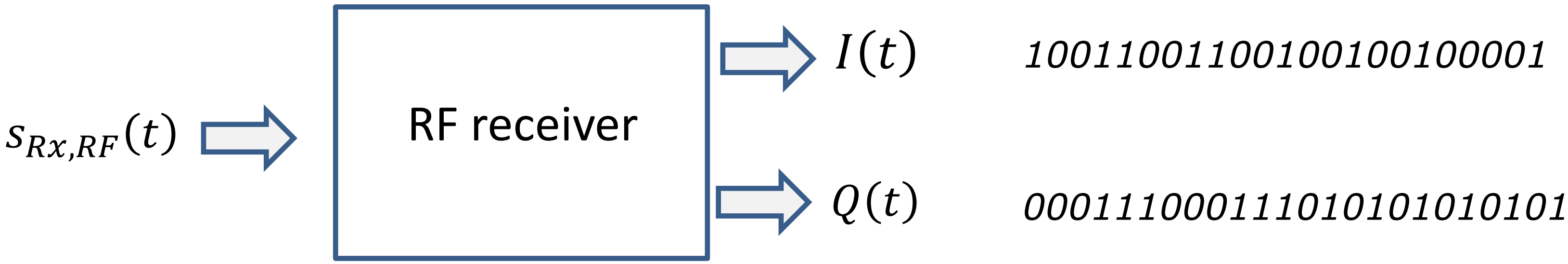
Where we define the in-phase and quadrature signals as:

- $I(t) = A \cdot g(t-d) \cdot cos(2\pi f_0 d)$
- $Q(t) = -A \cdot g(t-d) \cdot cos(2\pi f_0 d)$

The **information** about the target is carried by the amplitude and delay parameters *A* and *d*, which are embedded in the in-phase and quadrature signals $I(t)$ and $Q(t)$
(we already know the value of the carrier, so no new info in it)

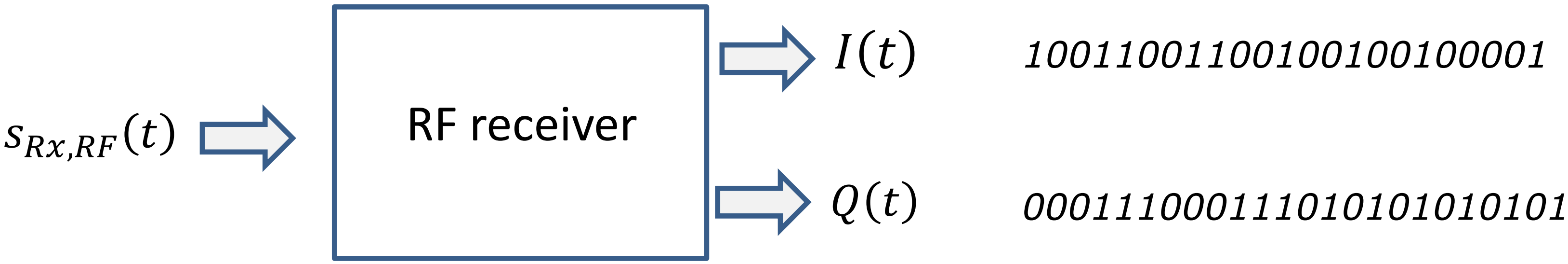
Radar signals

The $I(t)$ and $Q(t)$ of RF signals are extracted by RF circuitry, stored as numerical signals…



Radar signals

The $I(t)$ and $Q(t)$ of RF signals are extracted by RF circuitry, stored as numerical signals…



… and represented as a single **complex** signal, simply referred to as the (complex envelope of the) received signal

$$s_{Rx}(t) = I(t) + jQ(t)$$

j = imaginary unit

The complex representation is ubiquitous in the study of all wave phenomena

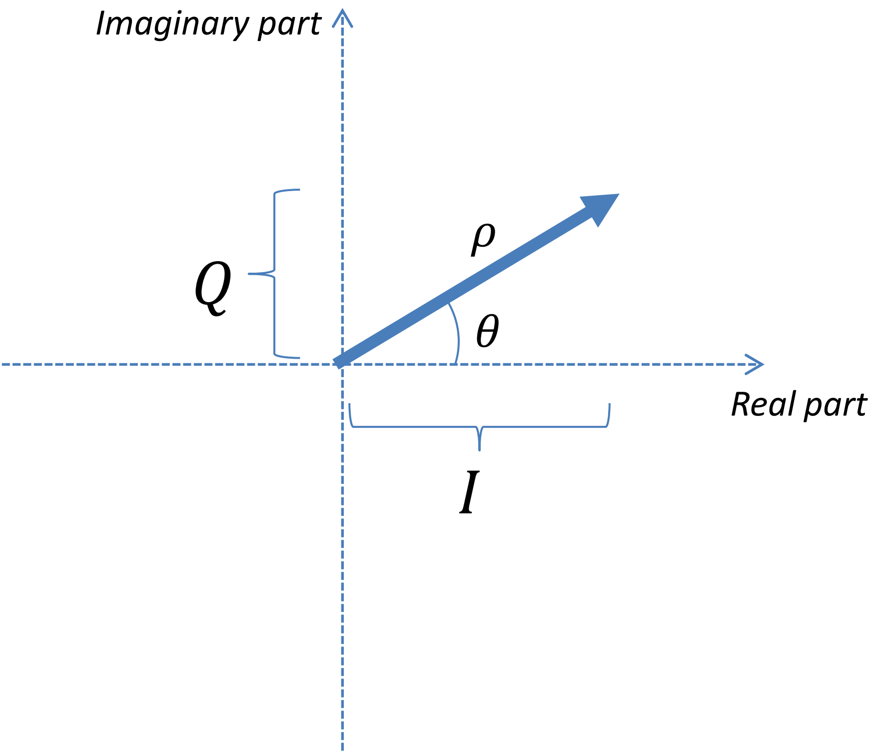
Reason why it is used: it allows to make systematic use of the properties of complex exponentials, which leads to simpler mathematical expressions

Radar signals

$$s_{Rx}(t) = I(t) + jQ(t)$$

j = imaginary unit

For any fixed time, this signal can be represented as a vector in a 2D space



The ***real*** and ***imaginary*** parts (I and Q) represent the coordinates of the vector

The ***phase*** θ is the angle formed with the real axis

The ***magnitude*** ρ is simply the length of the vector

$$\rho = |s_{Rx}(t)| = \sqrt{I^2 + Q^2}$$

Radar signals

Going back to the case of the received signal, we have:

$$s_{Rx,RF}(t) = A \cdot g(t-d) \cdot cos(2\pi f_0 d) \cdot cos(2\pi f_0 t) - A \cdot g(t-d) \cdot sin(2\pi f_0 d) \cdot sin(2\pi f_0 t)$$

In phase component $I(t)$ — Quadrature component $-Q(t)$

$$\Downarrow$$

$$s_{Rx}(t) = I(t) + jQ(t) = A \cdot g(t-d) \cdot e^{-j2\pi f_0 d}$$

Radar signals

Finally, recalling that:

- The delay is obtained as $d = \frac{2R}{c}$
- The wavelength is obtained is $\lambda = \frac{c}{f_0}$

we obtain the standard expression of the received signal (used everywhere in wave literature)

$$s_{Rx}(t) = A \cdot g\left(t - \frac{2R}{c}\right) \cdot e^{-j\frac{4\pi}{\lambda}R}$$

Radar signals

Finally, recalling that:

- The delay is obtained as $d = \frac{2R}{c}$
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we obtain the standard expression of the received signal (used everywhere in wave literature)

$$s_{Rx}(t) = A \cdot g\left(t - \frac{2R}{c}\right) \cdot e^{-j\frac{4\pi}{\lambda}R}$$

Amplitude:
this term is related to the strength of the wave backscattered by the target

Delayed pulse:
this term allows for the determination of a target's distance from the Radar

Phase:
this is where the magic starts…

The frequency domain

Frequency domain

The signals presented in the last section were represented by drawing their variation over time, or by writing equations where the signal amplitude depends on time, like $g(t)$

This particular representation is referred to as ***time domain***

Frequency domain

The signals presented in the last section were represented by drawing their variation over time, or by writing equations where the signal amplitude depends on time, like $g(t)$

This particular representation is referred to as ***time domain***

An alternative representation is built by representing a signal as a collection of **sinusoids** (either real or complex), i.e.:

$$g(t) = G_1 e^{j2\pi f_1 t} + G_2 e^{j2\pi f_2 t} + G_3 e^{j2\pi f_3 t} + \cdots$$

we say that the signal $g(t)$ ”contains” the sinusoids at frequency $f_1,\ f_2,\ f_3$… and the amplitudes G_1,G_2, G_3 represent the “strength” of each of those sinusoids.

Note: complex sinusoids are defined as:

$$e^{j2\pi ft} = cos(2\pi ft) + j \cdot sin(2\pi ft)$$

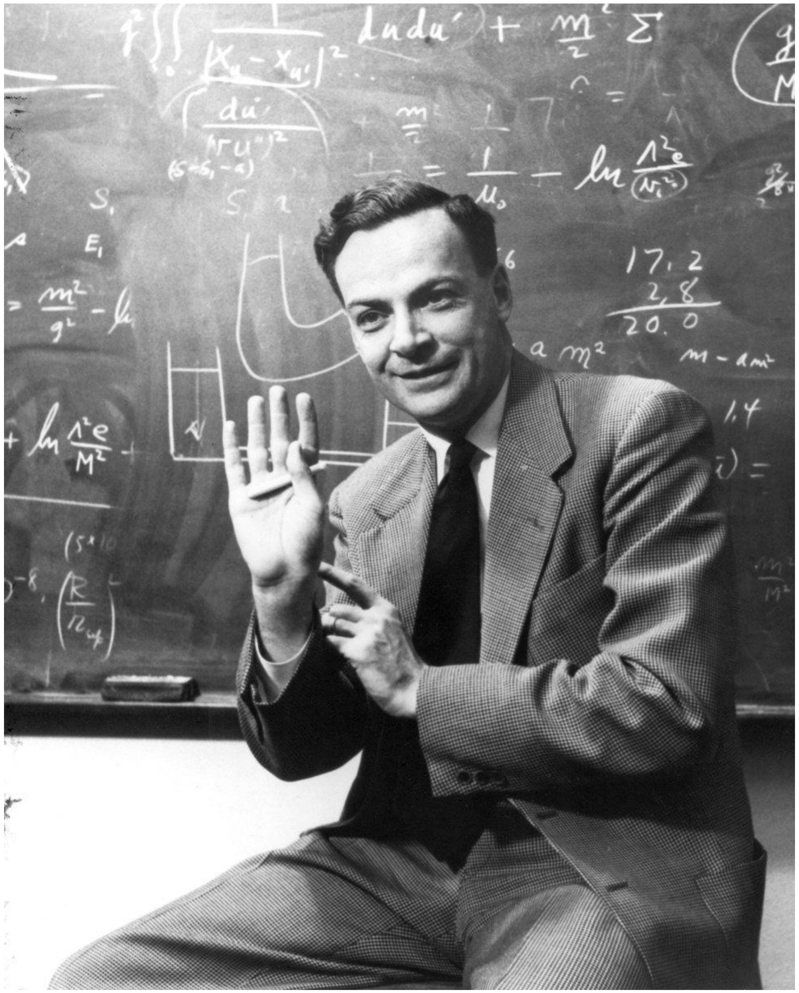
Again, we use complex sinusoids since this leads to simpler mathematical expression

Question: can we represent any signal in the frequency domain?

In other terms, can we always represent a signal as a collection of sinusoids?

Frequency domain

We answer with the help of Richard Feynman:



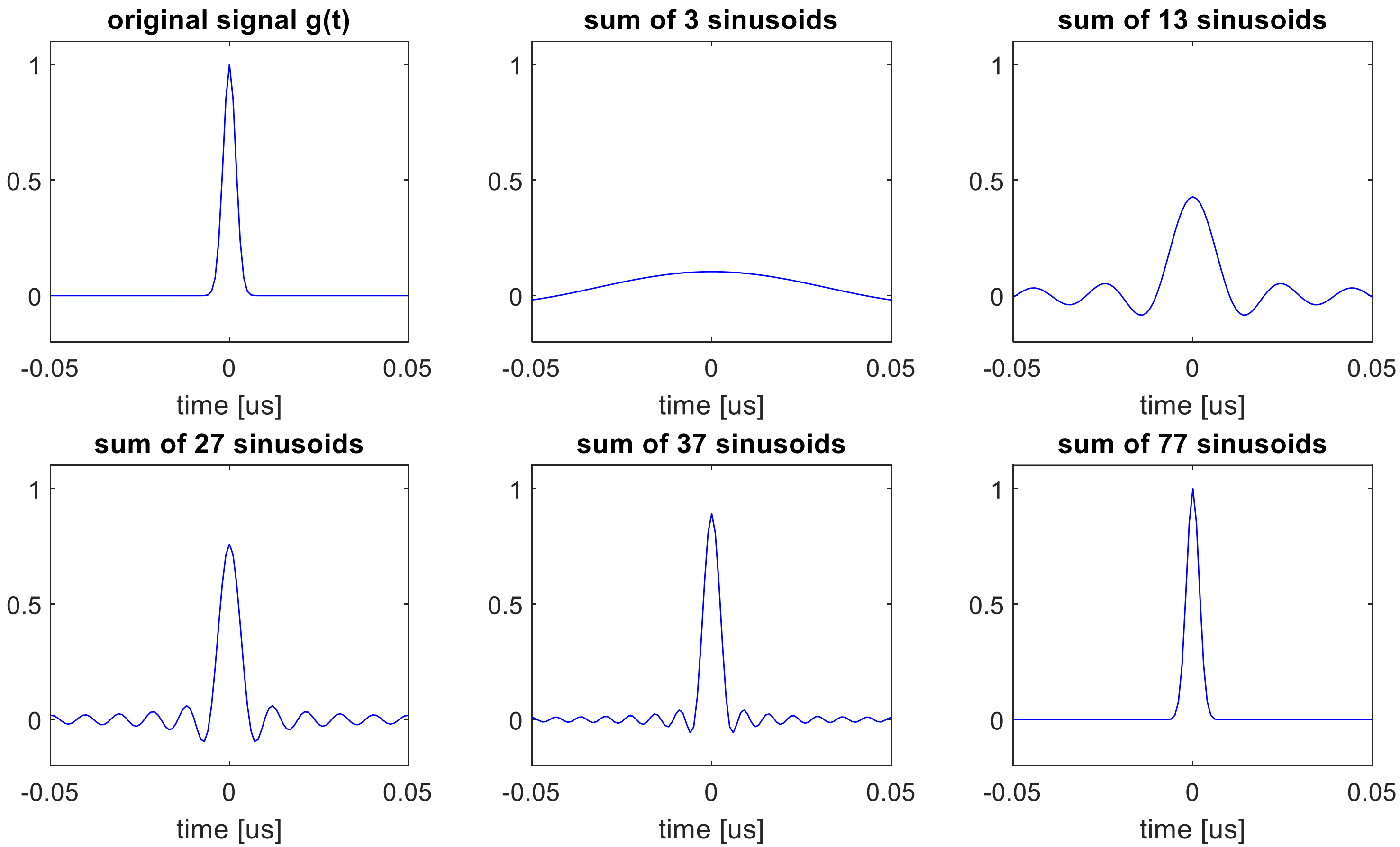
In what circumstances can a curve be represented as a sum of a lot of cosines?

Answer:

In all ordinary circumstances, except for certain cases the mathematicians can dream up. Of course, the curve must have only one value at a given point, and it must not be a crazy curve which jumps an infinite number of times in an infinitesimal distance, or something like that. But aside from such restrictions any reasonable curve (one that a singer is going to be able to make by shaking her vocal cords) can always be compounded by adding cosine waves together.

Frequency domain

Practically, this means we can **always** represent a signal in terms of a sum of sinusoids, as long as we consider a sufficient number of sinusoids



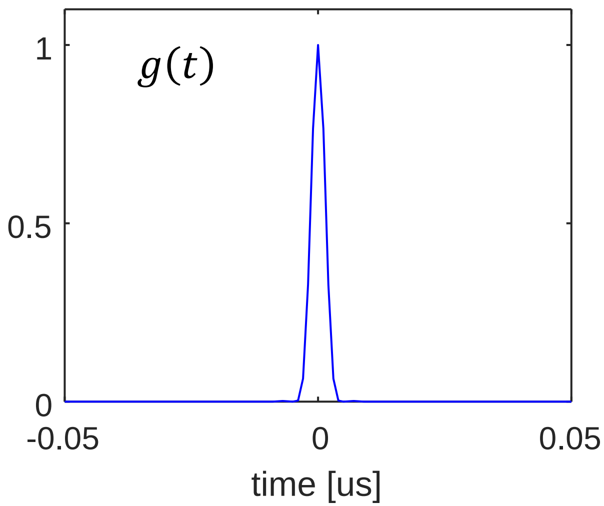
Frequency domain

Practically, this means we can **always** represent a signal in terms of a sum of sinusoids, as long as we consider a sufficient number of sinusoids

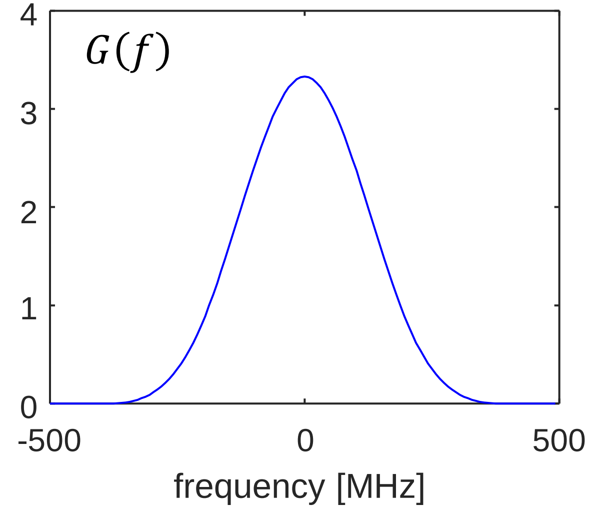
In this way, we can represent the signal g by drawing (or writing) the series of the amplitudes as a function of the frequency of the associated sinusoid, i.e.: $G(f)$

$$g(t) = G_1 e^{j2\pi f_1 t} + G_2 e^{j2\pi f_2 t} + G_3 e^{j2\pi f_3 t} + \cdots$$

This particular representation is referred to as ***frequency domain***



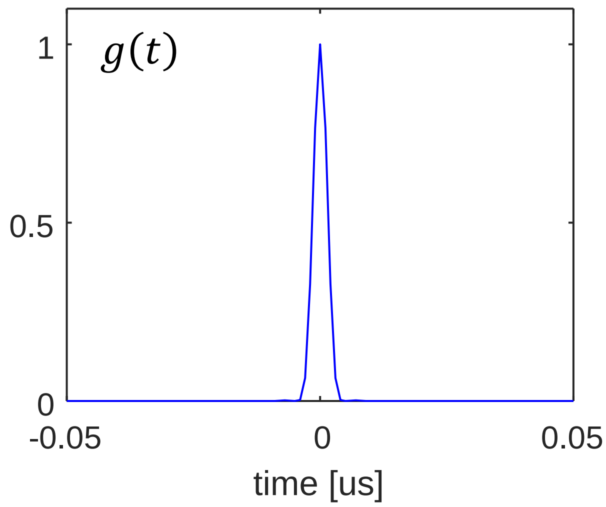
Time domain representation

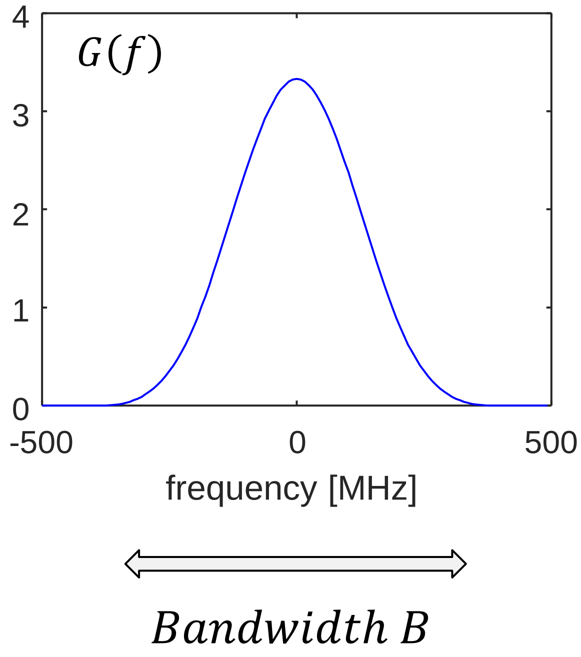


Frequency domain representation

Bandwidth

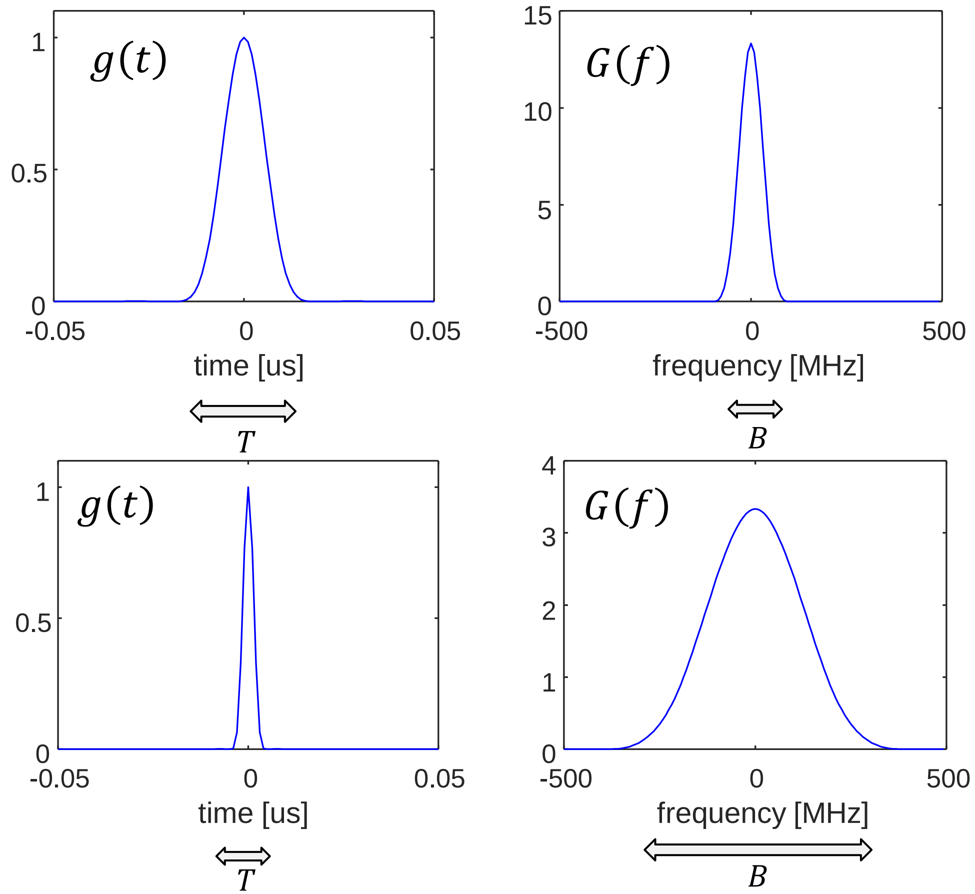
The **bandwidth** of a signal is defined as the "length" of the interval where we find the frequencies that are contained in it





For Radar pulses we can state the rule that (with some exceptions we will not discuss):

signal bandwidth is inversely proportional to signal duration



$$T \approx \frac{1}{B}$$

$$T \approx \frac{1}{B}$$

Question: how do we get to know which frequencies contribute to a signal?

How do we compute their amplitudes $G(f)$?

The Fourier Transform

Answer: we calculate the ***Fourier Transform*** of the signal

For a signal represented as a sequence of time samples in our computer, the Fourier Transform is expressed as:

$$G(f) = \sum_n g(t_n) \cdot e^{-j2\pi f t_n}$$

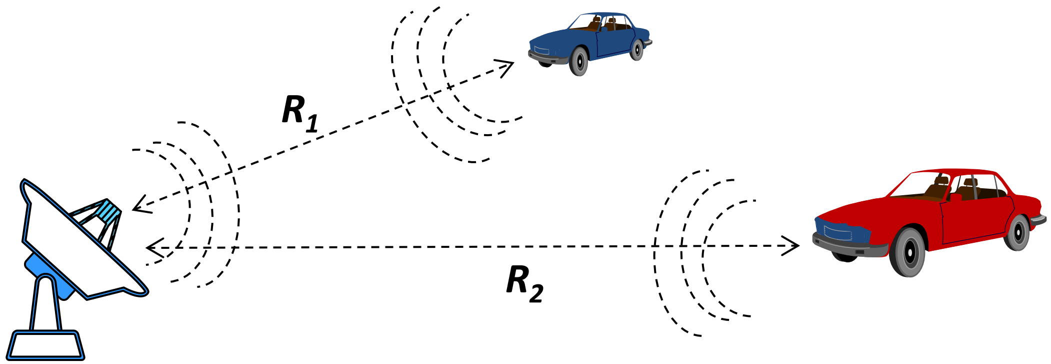
Which states a simple recipe:

- Choose (at will) a particular frequency f
- Take the original signal $g(t_n)$ and multiply its time samples times $e^{-j2\pi f t_n}$
- Sum over all samples
- Repeat for any frequency f we want to evaluate

Range resolution

Range resolution

The concept of bandwidth leads us to directly to the important concept of ***range resolution***, intended as the capability to distinguish (resolve) two targets found at slightly different distances from the Radar

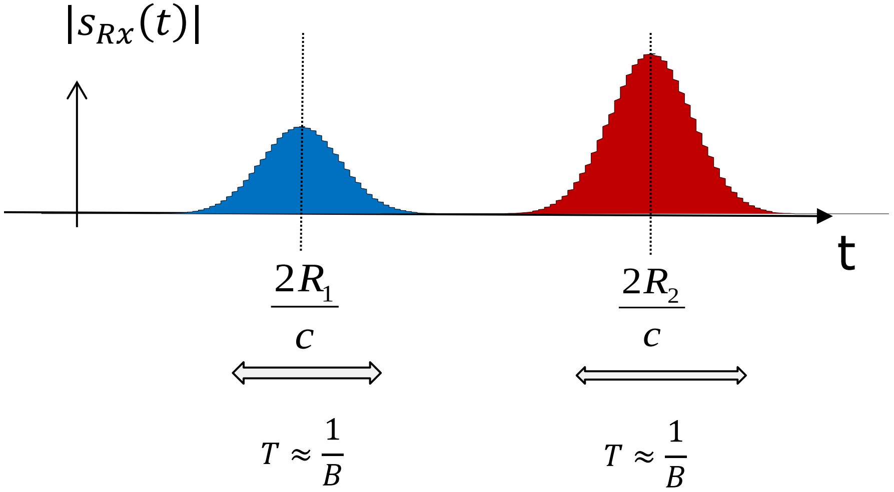


The received signal is now expressed as the sum of two signals associated with the two targets

$$s_{Rx}(t) = A_1 \cdot g\left(t - \frac{2R_1}{c}\right) \cdot e^{-j\frac{4\pi}{\lambda}R_1} + A_2 \cdot g\left(t - \frac{2R_2}{c}\right) \cdot e^{-j\frac{4\pi}{\lambda}R_2}$$

Range resolution

If we plot signal magnitude, we obtain the following graph



$\Rightarrow$ We can tell there are two targets as long as the received signal exhibits **two distinct peaks**

This occurs upon the condition that:

$$\frac{2|R_2 - R_1|}{c} \geq T \quad \Longrightarrow \quad |R_2 - R_1| \geq \frac{c}{2B} = \Delta R$$

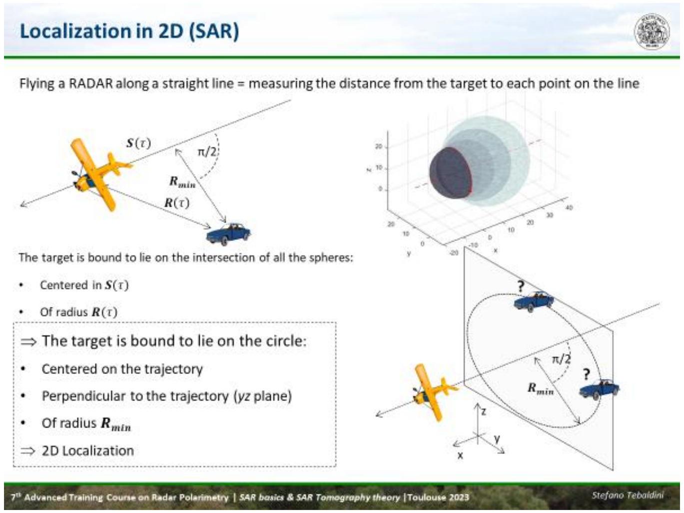
Where $\Delta R = \frac{c}{2B}$ is referred to as the **range resolution** of the Radar

Range resolution

B	ΔR	Typical SAR case
6 MHz	25 m	P-Band (≈ 400 MHz carrier) spaceborne SAR (due to ITU regulations)
40 MHz	3.75 m	L-Band (≈ 1300 MHz carrier) spaceborne SAR
150 MHz – 500 MHz	1 m – 0.3 m	Low frequency airborne SAR (hundreds of MHz to few GHz) X-band (≈ 10 GHz carrier) spaceborne SAR
1 GHz – 5 GHz	0.15 m – 0.03 m	Higher frequency (≥ 10 GHz carrier) airborne SAR Higher frequency (≥ 70 GHz carrier) automotive Radar

Angular resolution

Angular resolution



Moving a RADAR along a straight line = measuring the distance from the target to each point on the line

⇔ 2D Localization

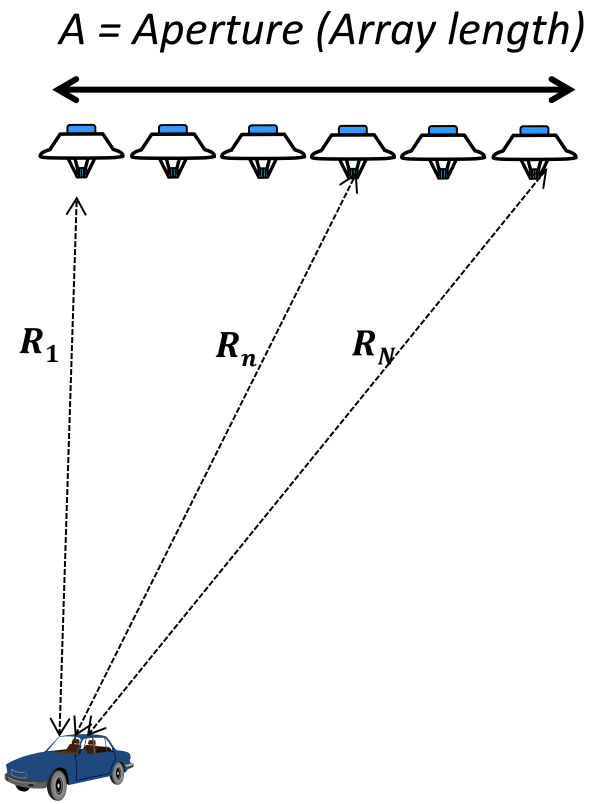
How ?

Let's take a step back….

Angular resolution

Consider an array of N antennas, sequentially emitting a ***monochromatic wave***

Note: Monochromatic wave ⬄ 0 bandwidth ⬄ $g(t) = 1$ ⬄ no range resolution



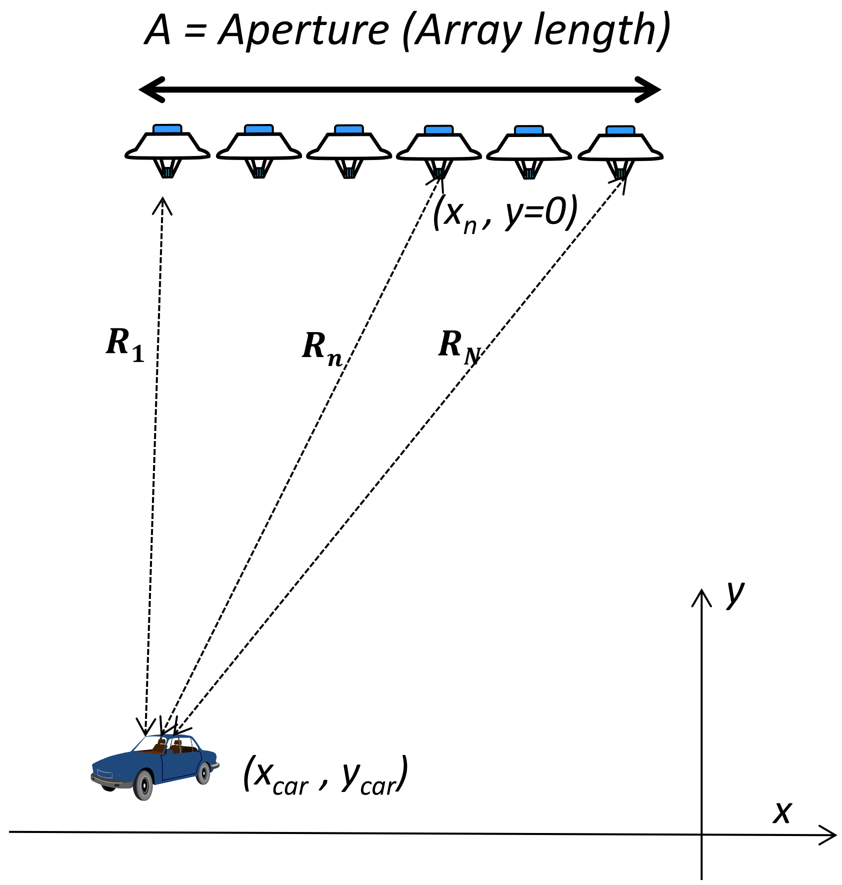
Received signal at the n-th antenna

$$s_{Rx}(n) = A_{car} \cdot e^{-j\frac{4\pi}{\lambda}R_n}$$

Angular resolution

Consider an array of *N* antennas, sequentially emitting a ***monochromatic wave***

Note: Monochromatic wave ⇔ 0 bandwidth ⇔ $g(t) = 1$ ⇔ no range resolution



Received signal at the n-th antenna

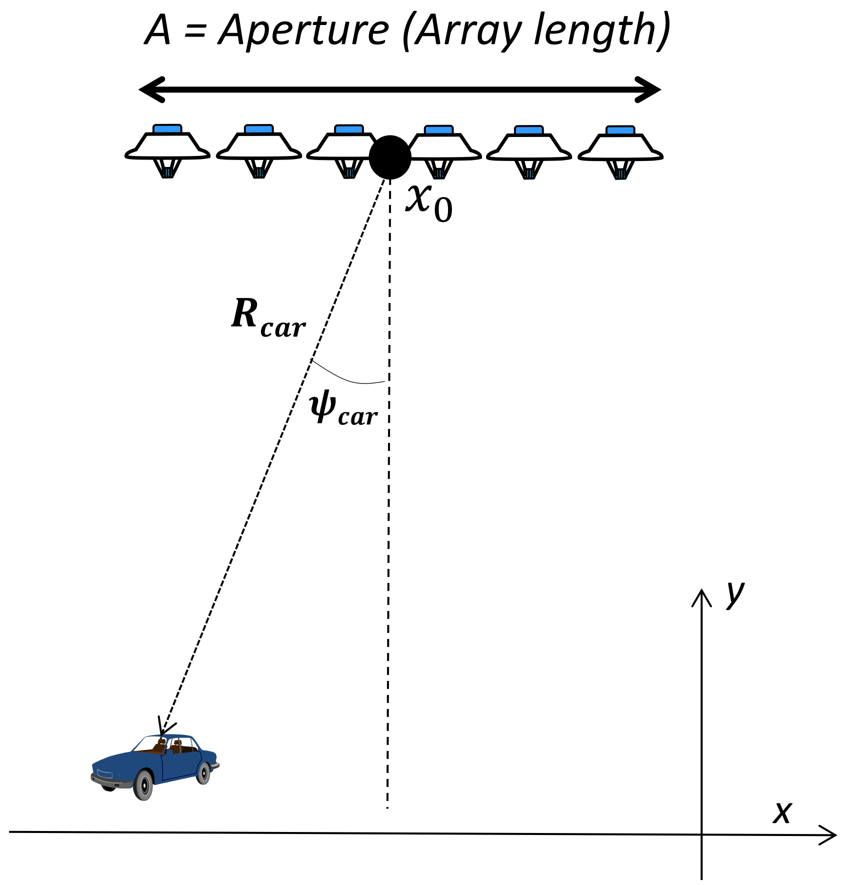
$$s_{Rx}(n) = A_{car} \cdot e^{-j\frac{4\pi}{\lambda}R_n}$$

$$R_n = \sqrt{(x_n - x_{car})^2 + (y_{car})^2}$$

Angular resolution

Consider an array of *N* antennas, sequentially emitting a ***monochromatic wave***

Note: Monochromatic wave ⇔ 0 bandwidth ⇔ $g(t) = 1$ *⇔ no range resolution*



Received signal at the n-th antenna

$$s_{Rx}(n) = A_{car} \cdot e^{-j\frac{4\pi}{\lambda}R_n}$$

$$R_n \cong R_{car} + sin(\psi_{car}) \cdot (x_n - x_0)$$

Valid for $R_{car} >> A$

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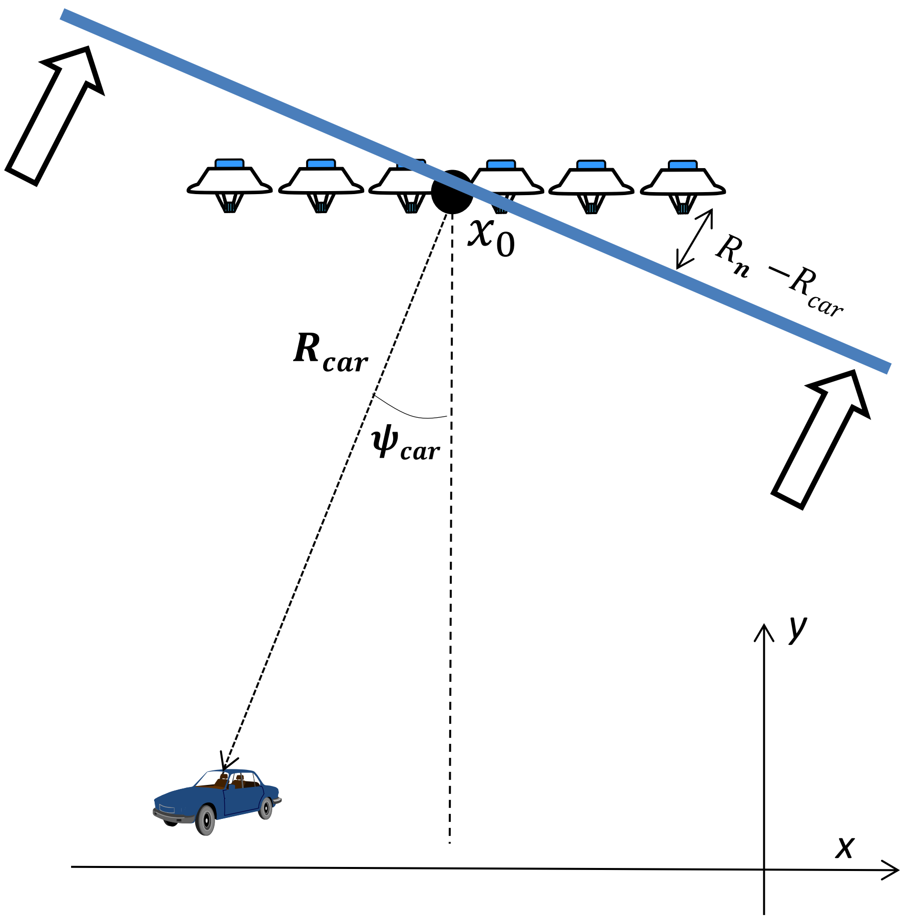


$$R_n \cong R_{car} + sin(\psi_{car}) \cdot (x_n - x_0)$$

Valid for $R_{car} >> A$

Equivalent to a planar wavefront from the car to the antenna array

Angular resolution

Plane wavefront approximation

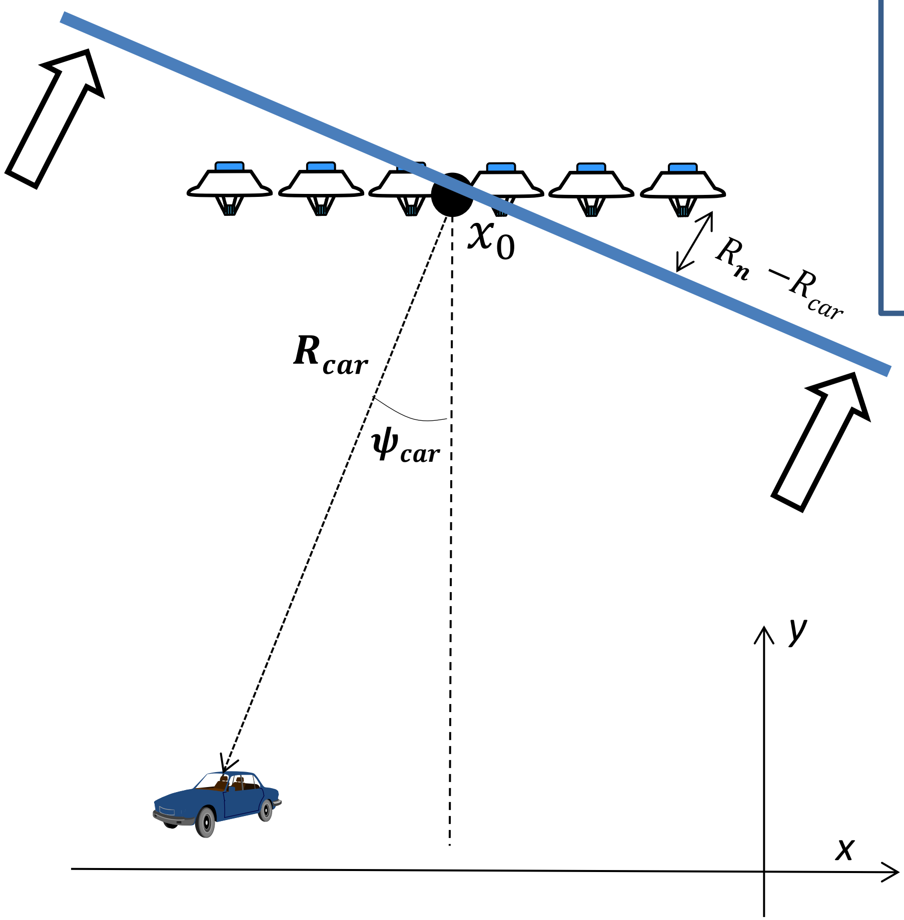
$$R_{\boldsymbol{n}} \cong R_{car} + sin(\psi_{car}) \cdot (x_n - x_0)$$

Received signal

$$s_{Rx}(n) \cong A_{car} e^{-j\frac{4\pi}{\lambda}R_{car}} \cdot e^{-j\frac{4\pi}{\lambda}sin(\psi_{car})\cdot x_n}$$

Complex sinusoid with spatial frequency

$$f_{car} = \frac{2}{\lambda} sin(\psi_{car})$$



The Direction of Arrival (DoA) of the wavefront impinging on the array can be found by measuring the spatial frequency along the array

Angular resolution

Plane wavefront approximation

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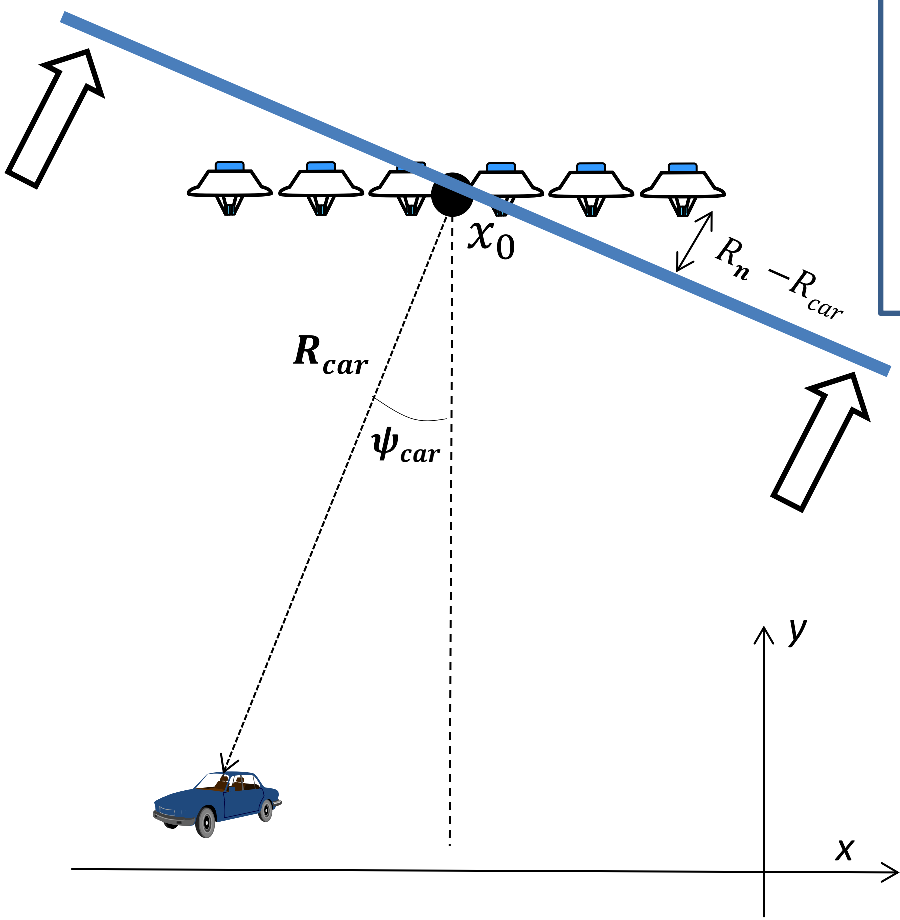


The Direction of Arrival (DoA) of the wavefront impinging on the array can be found by measuring the spatial frequency along the array

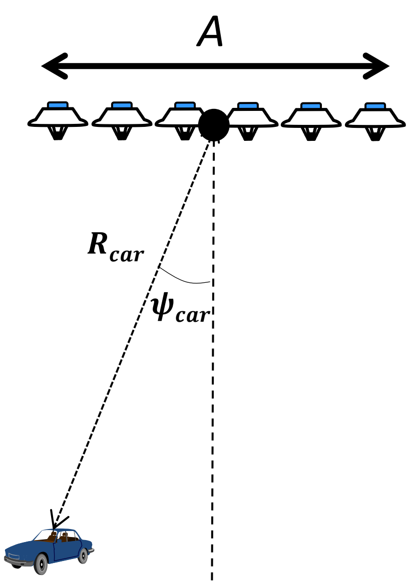
We need a Fourier Transform!

Angular resolution

Signal along the array

$$s_{Rx}(n) \cong A_{car} e^{-j\frac{4\pi}{\lambda}R_{car}} \cdot e^{-j2\pi f_{car} \cdot x_n}$$

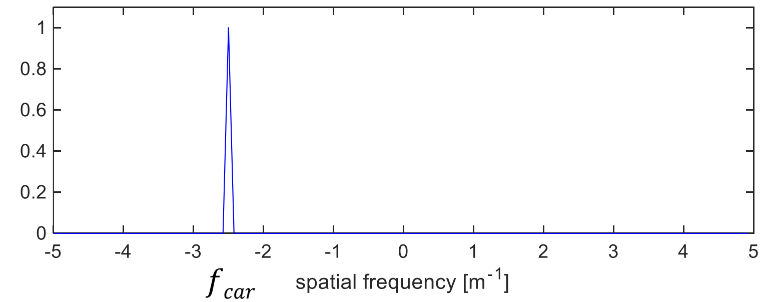
$$f_{car} = \frac{2}{\lambda} sin(\psi_{car})$$



Fourier Transform

The signal to be transformed contains a single sinusoid at frequency f_{car}

$\Rightarrow$ We would expect its Fourier Transform to show a single peak at frequency $f_x = f_{car}$



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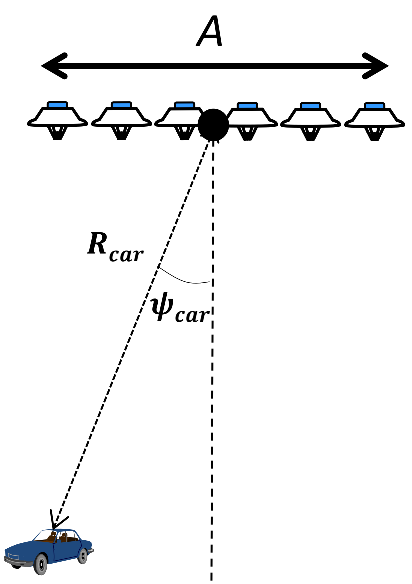
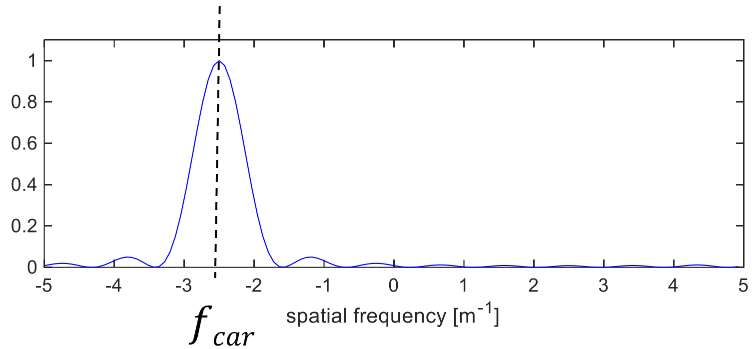


Fourier Transform

The signal to be transformed contains a single sinusoid at frequency f_{car}

However, we find something *quite different*

- A peak is present at the right position ($f_x = f_{car}$)…
- … but is spread across an interval of frequencies



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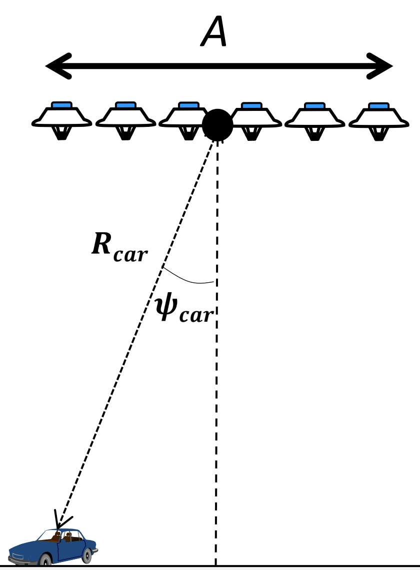

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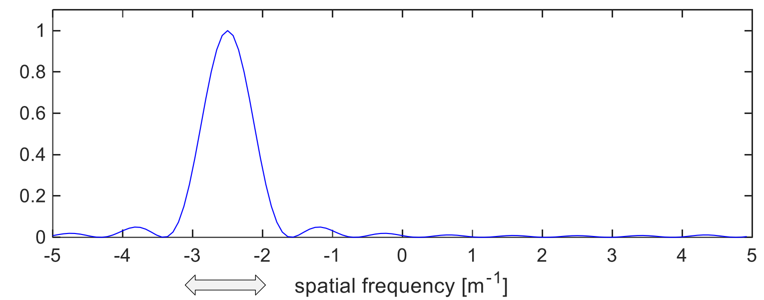


$$\Delta f \approx \frac{1}{A}$$

The reason for the spread is the inverse proportionality between signal duration and bandwidth

The signal along the array has a "*duration*" of *A* meters hence its FT has a bandwidth $\Delta f \approx \frac{1}{A}$

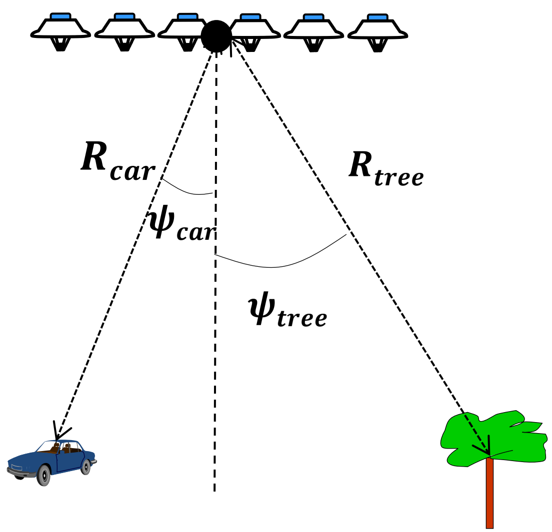
Angular resolution

The link between array aperture and spatial bandwidth leads us directly to the important concept of ***angular resolution***, intended as the capability to distinguish (resolve) two targets found at slightly different angles w.r.t. the Radar

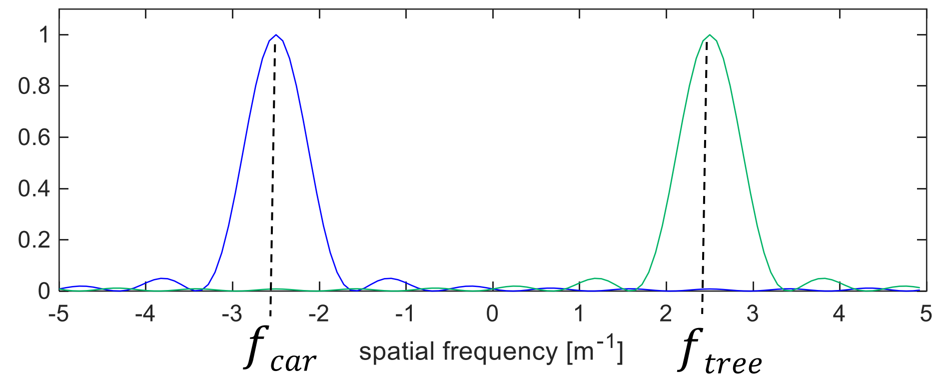
Signal along the array

$$s_{Rx}(n) \cong A_{car} e^{-j\frac{4\pi}{\lambda}R_{car}} \cdot e^{-j2\pi f_{car} \cdot x_n} + A_{tree} e^{-j\frac{4\pi}{\lambda}R_{tree}} \cdot e^{-j2\pi f_{tree} \cdot x_n}$$

$$f_{car} = \frac{2}{\lambda} sin(\psi_{car}) \qquad f_{tree} = \frac{2}{\lambda} sin(\psi_{tree})$$



Fourier Transform



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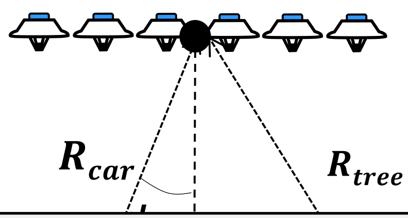
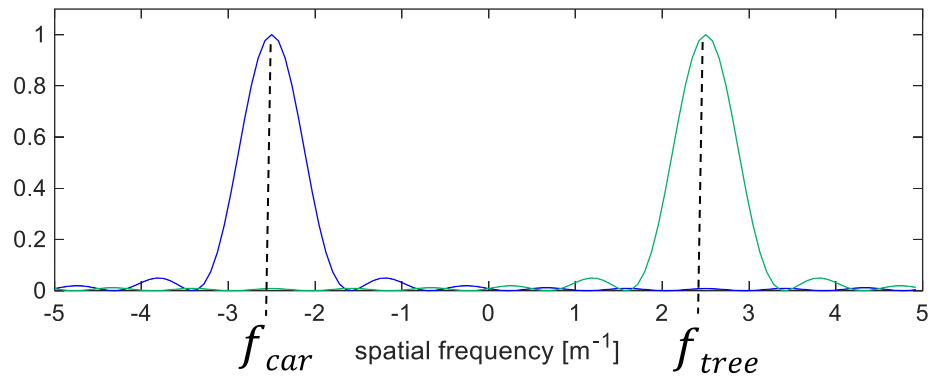


Fourier Transform



$\Rightarrow$ We can tell there are two targets as long as the received signal exhibits **two distinct peaks**

This occurs upon the condition that:

$$|f_{car} - f_{tree}| \geq \Delta f \approx \frac{1}{A} \Longrightarrow |\psi_{car} - \psi_{tree}| \geq \Delta\psi \approx \frac{\lambda}{2A}$$

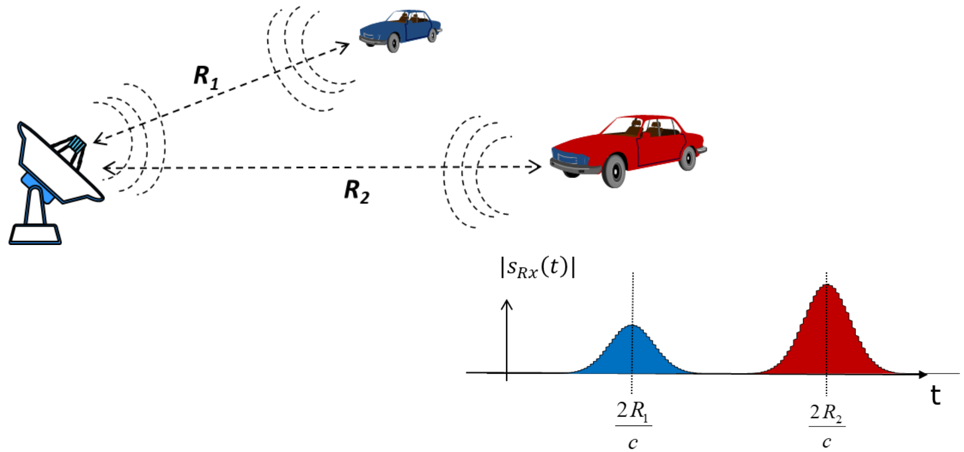
Where $\Delta\psi = \frac{\lambda}{2A}$ is referred to as the **angular resolution** of the array

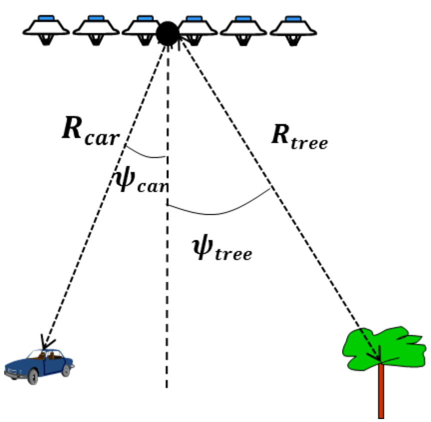
2D resolution

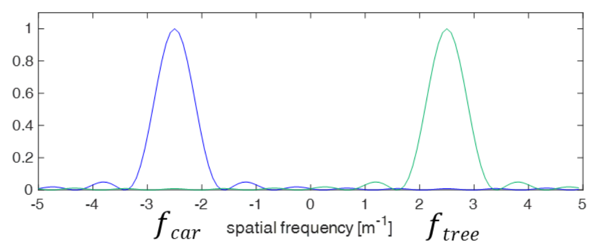
Bandwidth

⇔

Range resolution







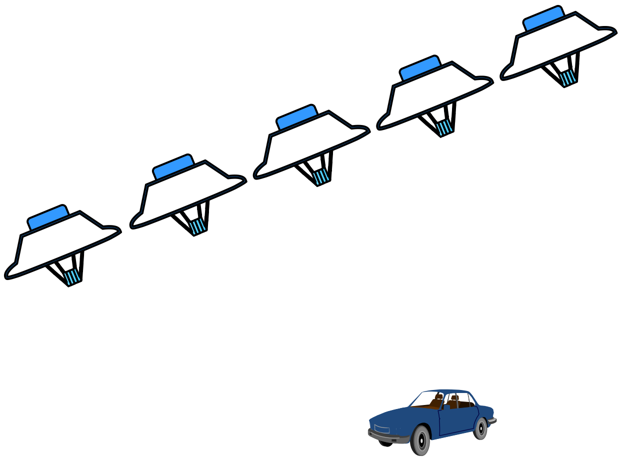
Antenna array emitting a monochromatic wave

⇔

Angular resolution

2D resolution

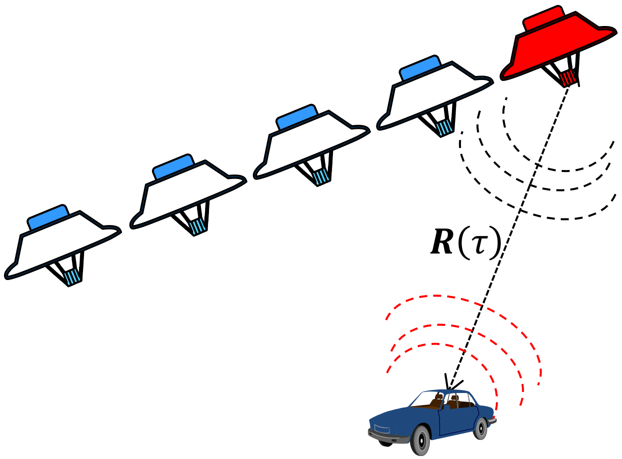
Antenna array emitting RF pulses

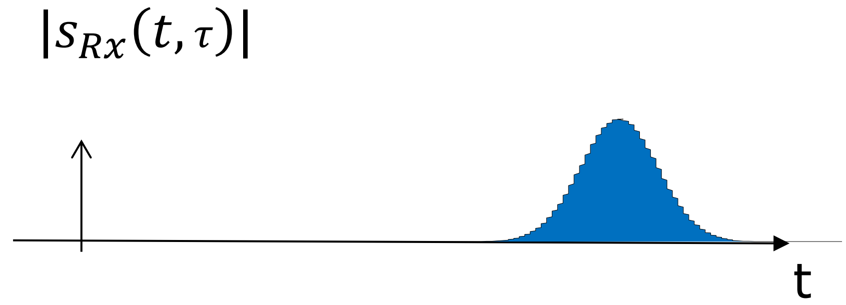


2D resolution

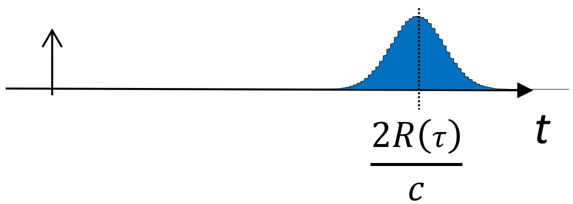
τ = flight time (or *slow time*, in jargon)

t = time w.r.t. transmission (or *fast time*, in jargon)





$$s_{Rx}(t,\tau) = A_{car}\, g\left(t - \frac{2R(\tau)}{c}\right) \cdot e^{-j\frac{4\pi}{\lambda}R(\tau)}$$



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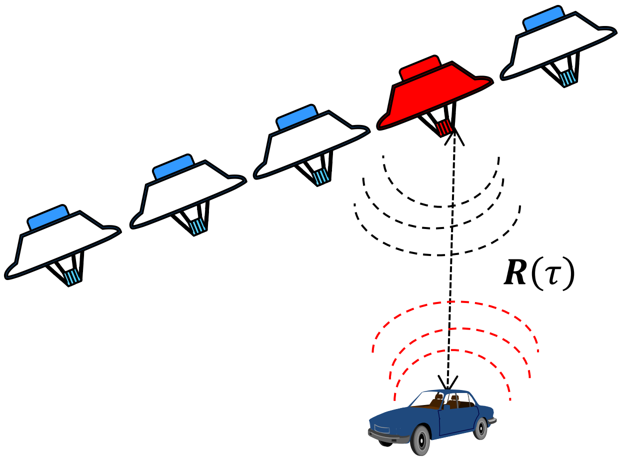

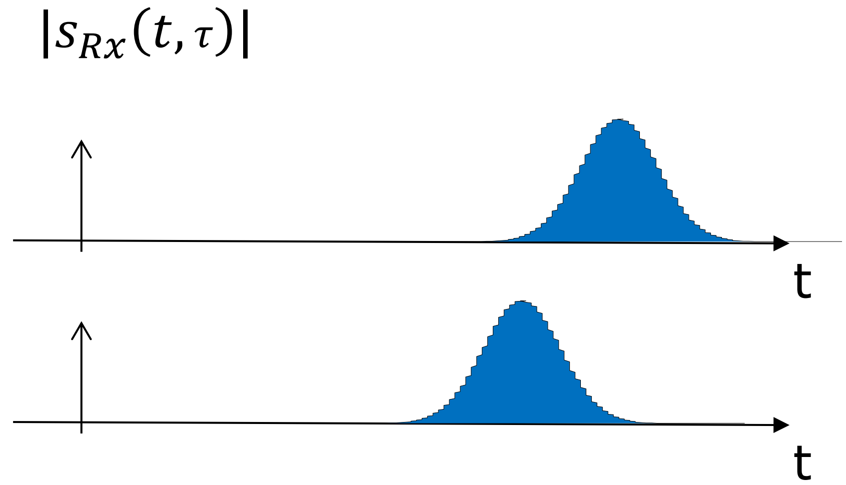

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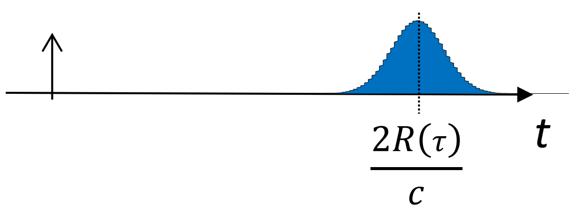

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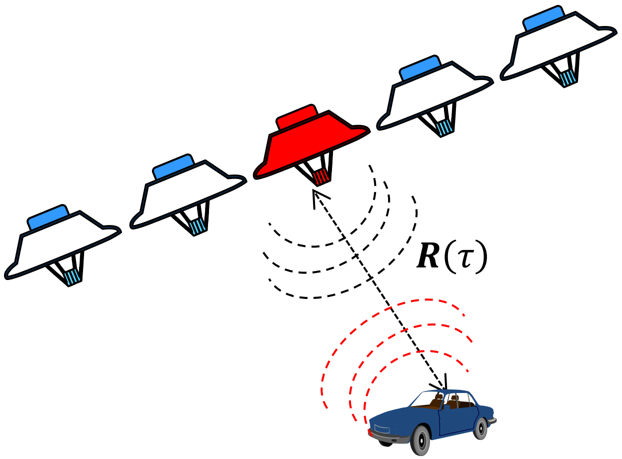
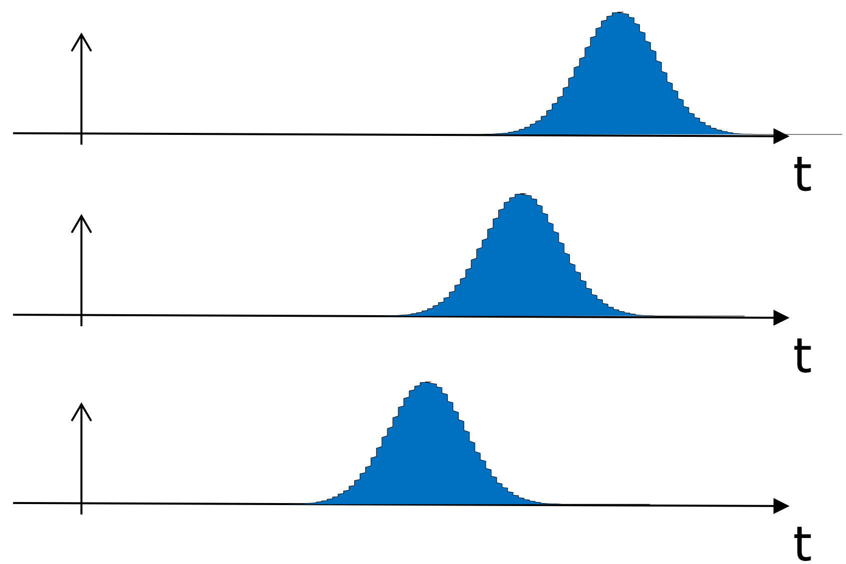


$|s_{Rx}(t,\tau)|$



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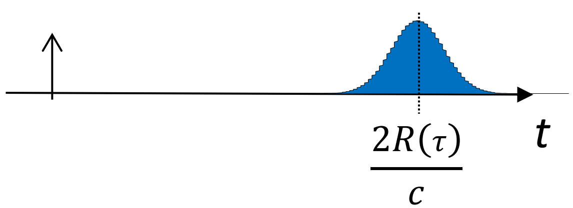

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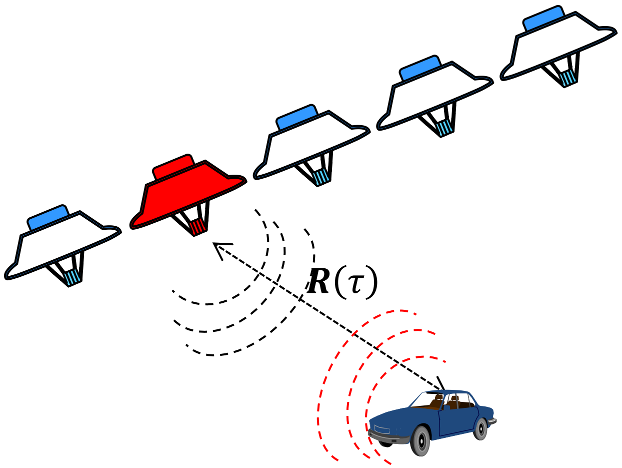

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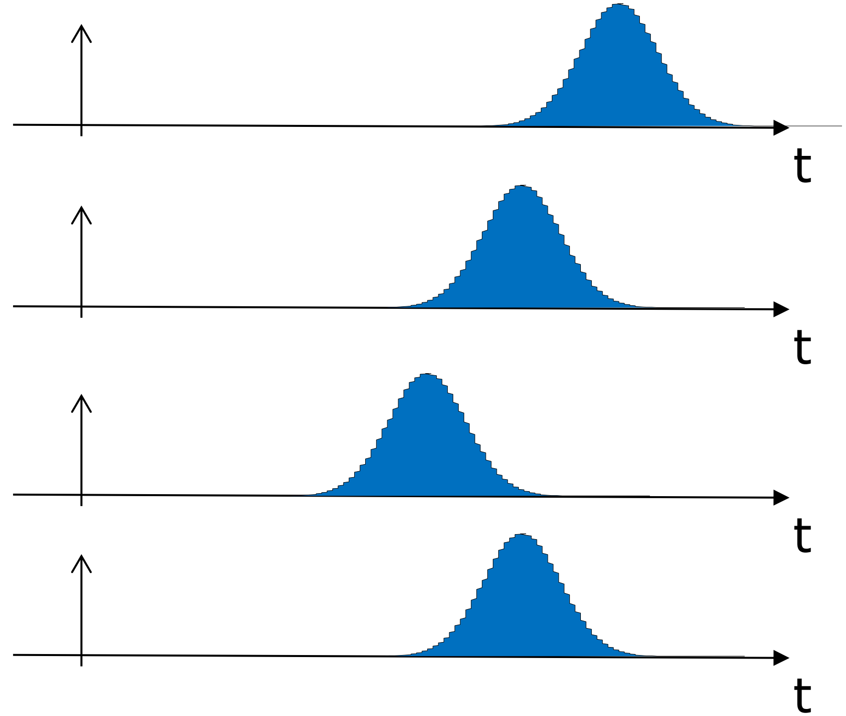

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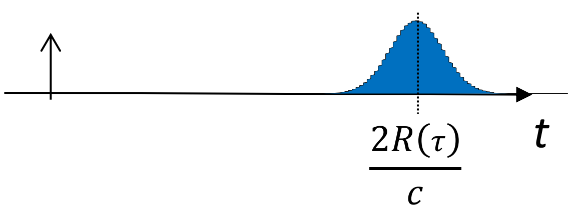

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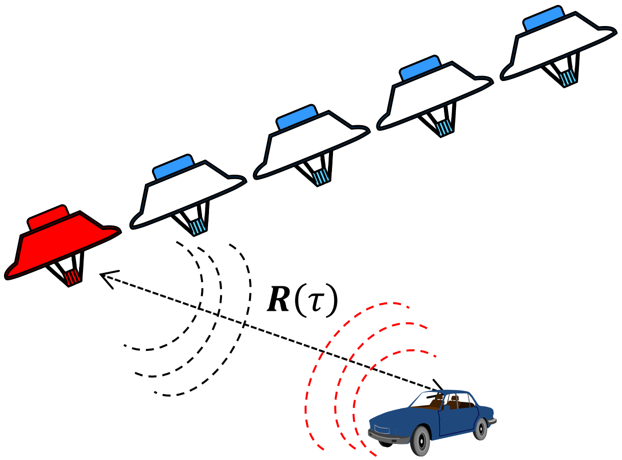

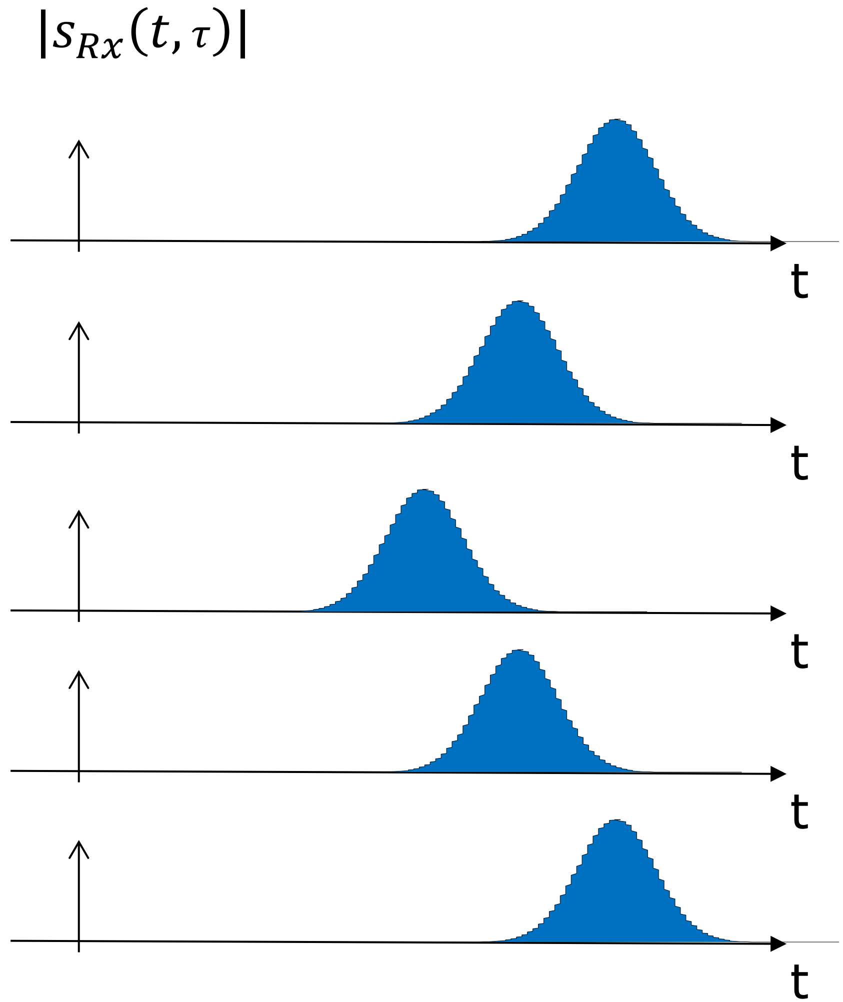

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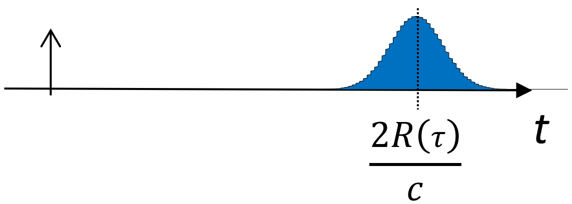

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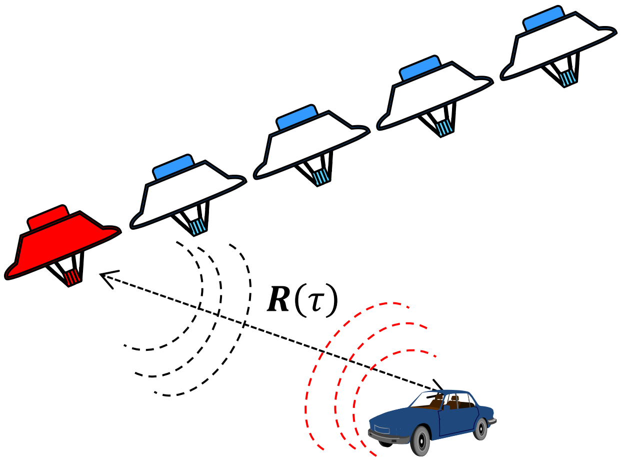
*The delay variation is referred to as **range migration***



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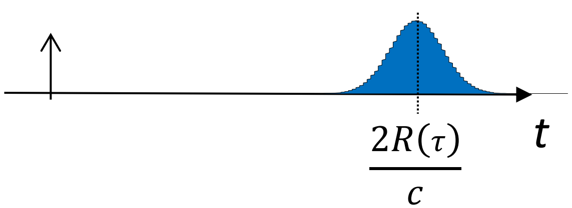
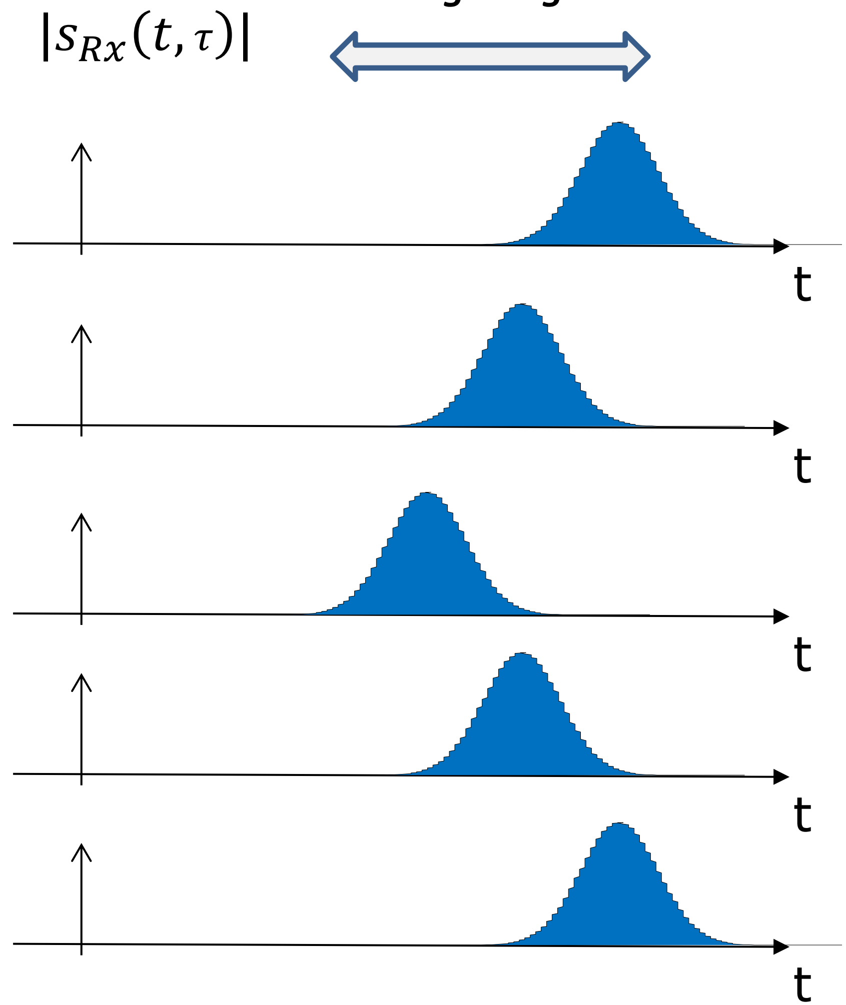
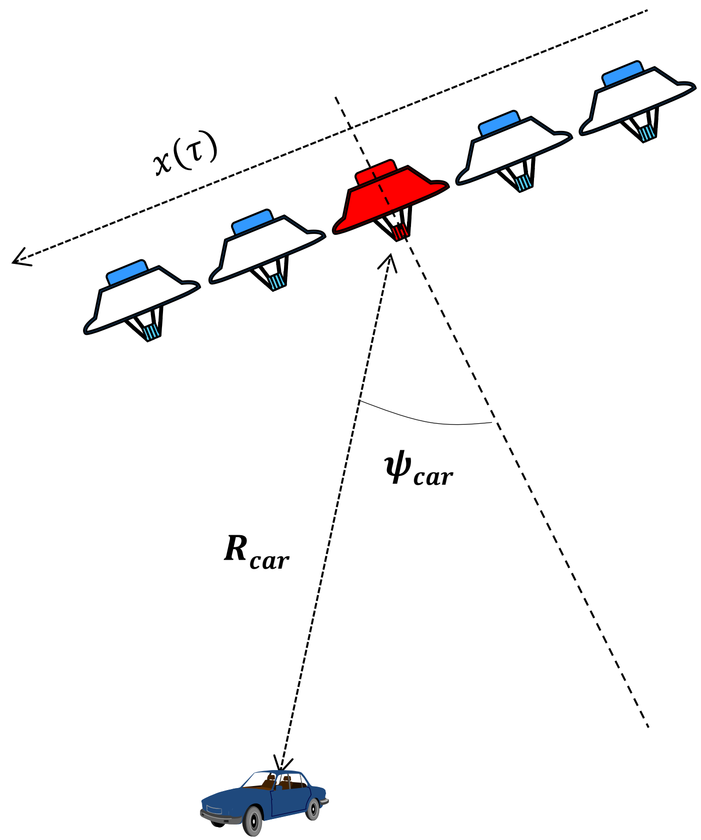


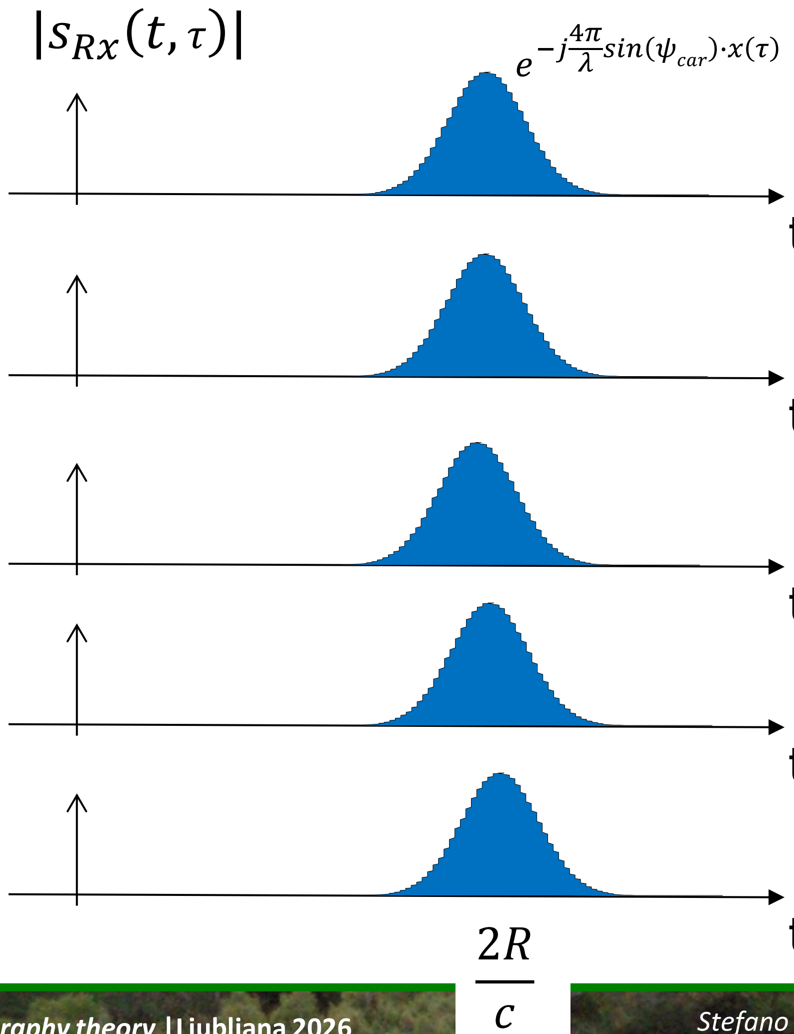


2D resolution

Hp1: range migration is negligible ⇔ we can tell the range from the delay (along t)

Hp2: plane wavefront approximation ⇔ we can tell the angular position from the frequency (along τ)





2D resolution

Practically, we compute a FT for any value of the fast time

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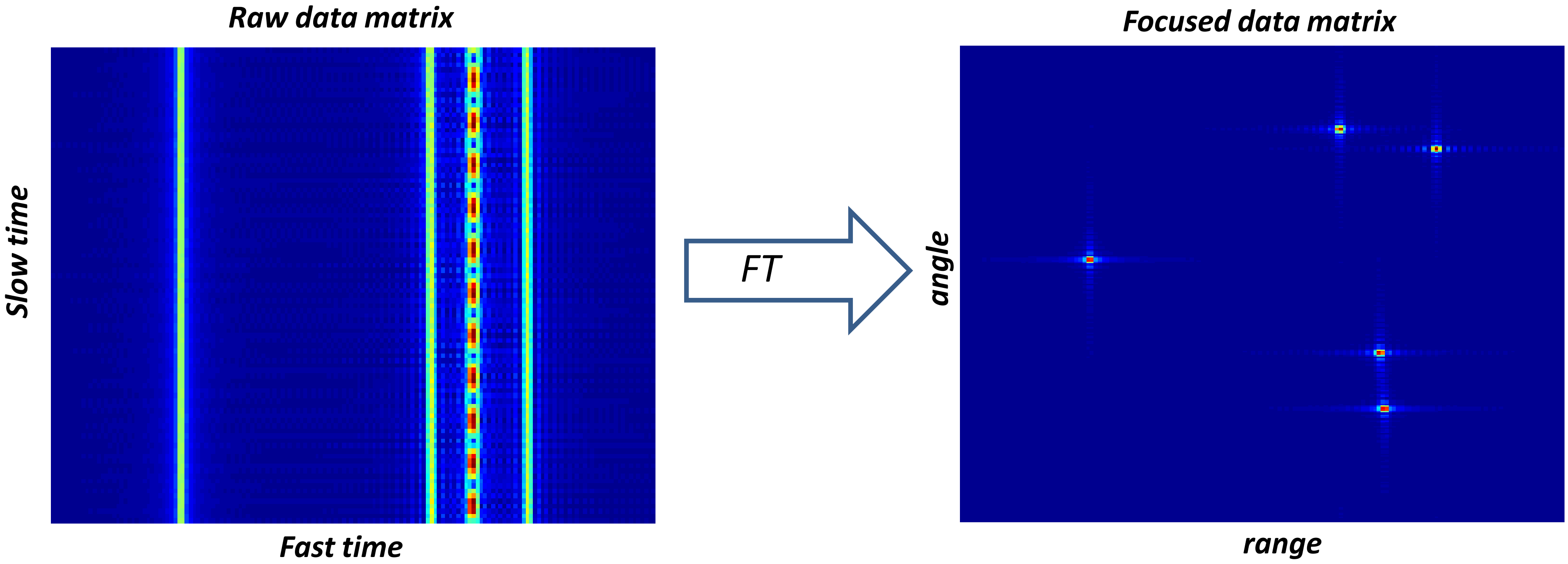

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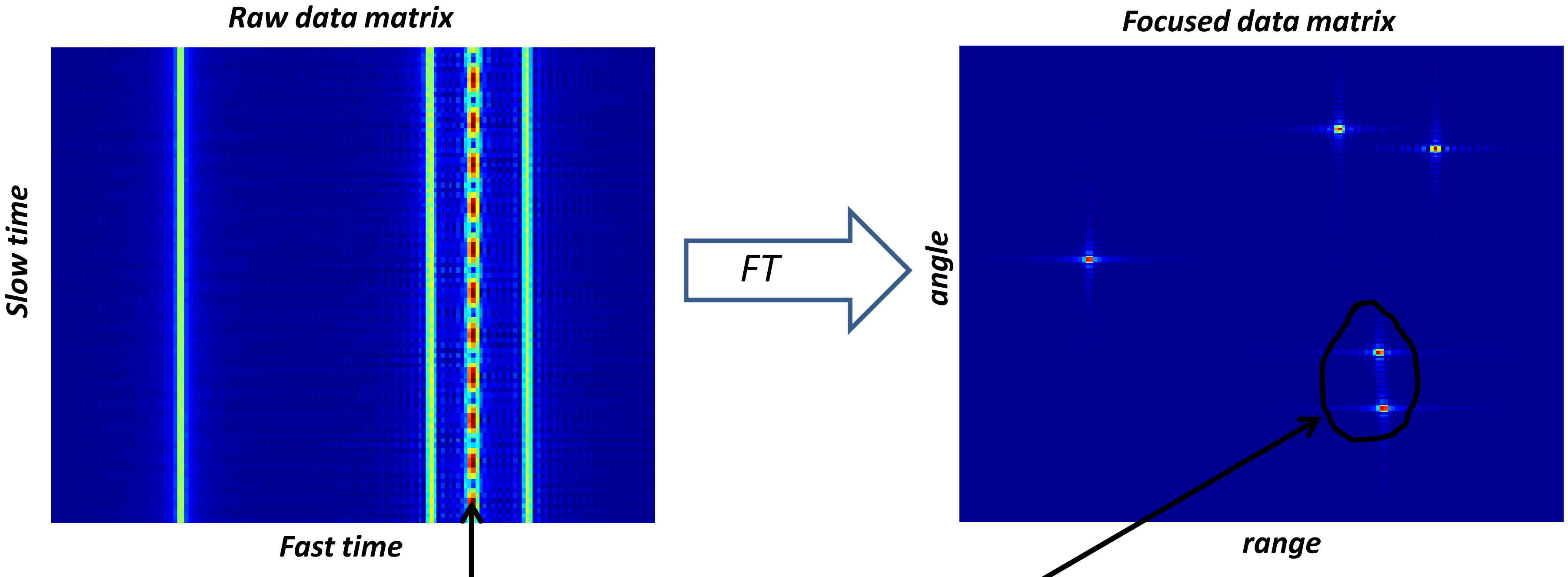


Interference of two targets at the same distance

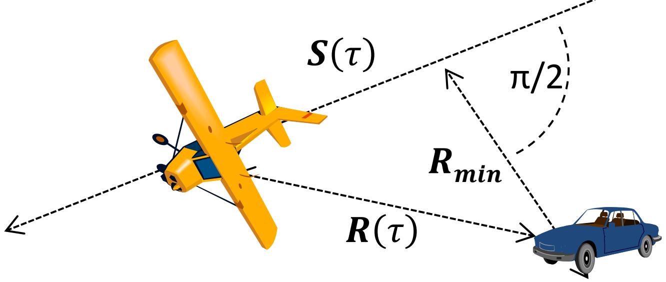
Targets resolved in the angular domain

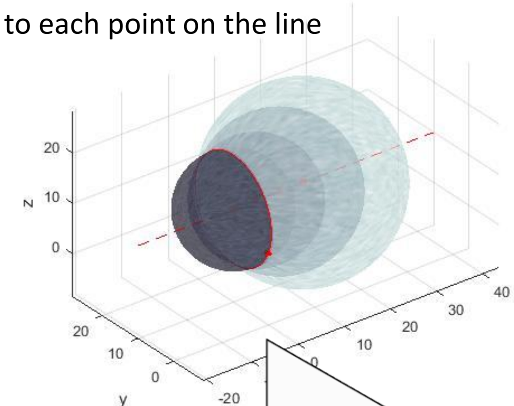
SAR Imaging

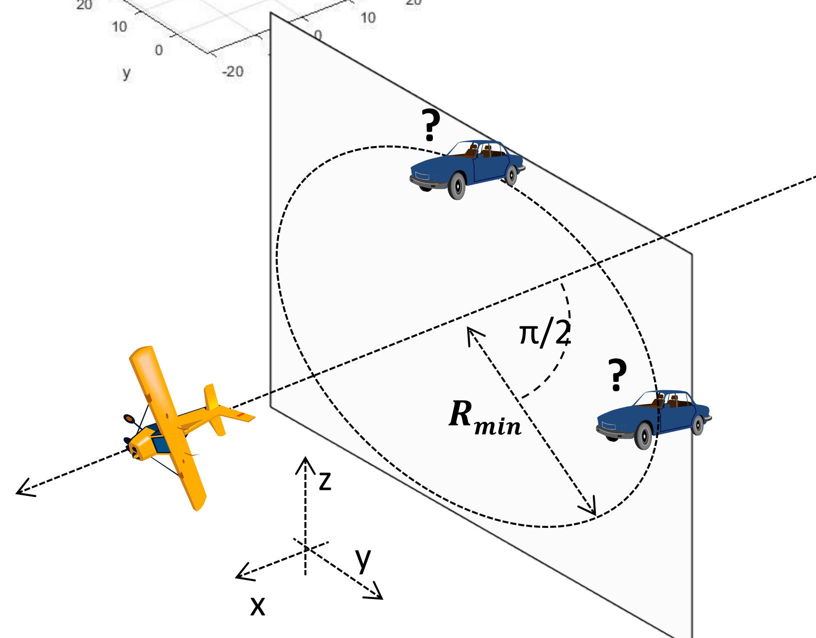
SAR imaging – geometrical interpretation

Synthetic Aperture Radars (SAR) employ a moving RADAR sensor, flown onboard a satellite or an aircraft, in order to synthesize an antenna as long as several kilometers

Flying a RADAR along a straight line = measuring the distance from the target to each point on the line







The target is bound to lie on the intersection of all the spheres:

- Centered in $\boldsymbol{S}(\tau)$
- Of radius $\boldsymbol{R}(\tau)$

⇒ The target is bound to lie on the circle:

- Centered on the trajectory
- Perpendicular to the trajectory (*yz* plane)
- Of radius $\boldsymbol{R}_{min}$

⇒ 2D Localization

SAR imaging

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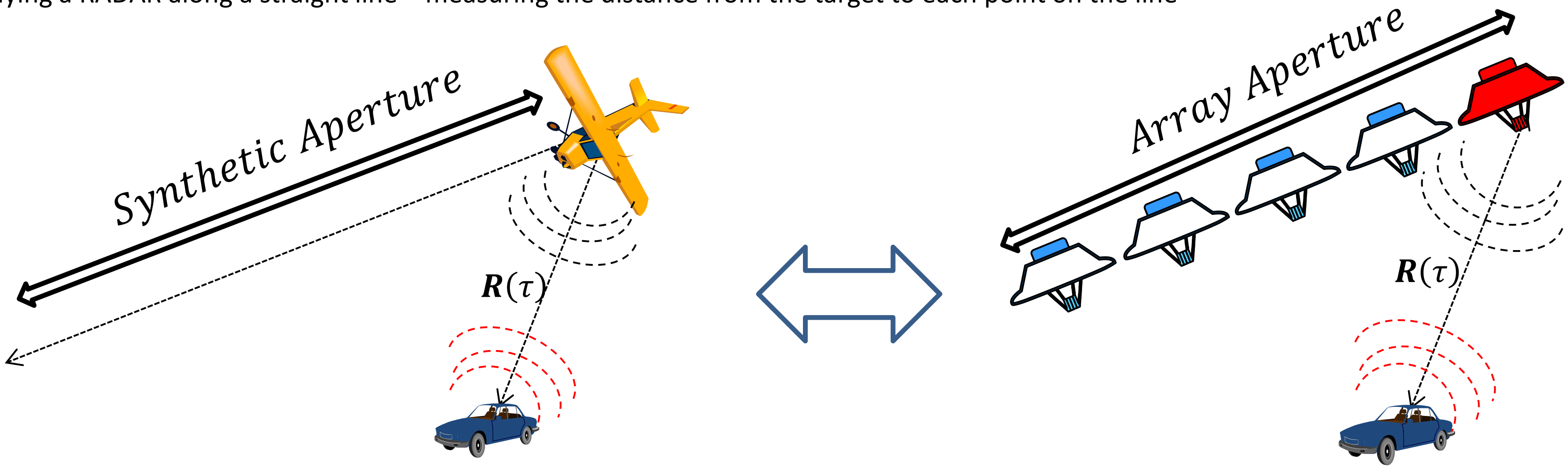
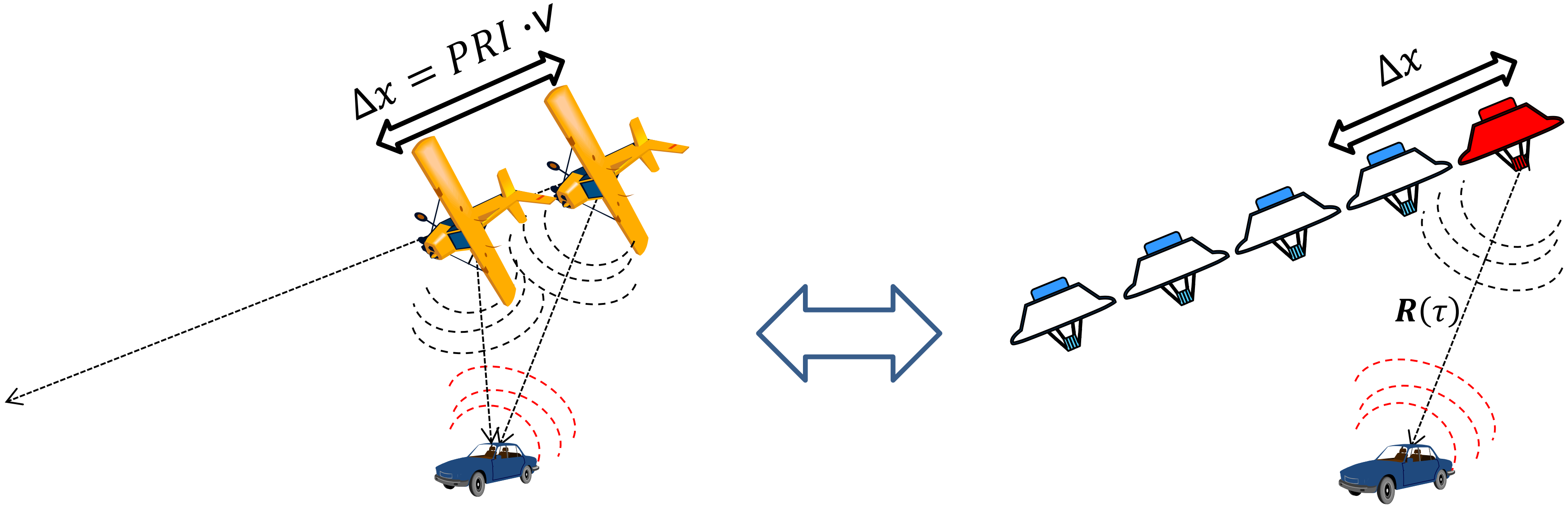


Start-stop approximation:

the platform is assumed to be completely still in air (or in space) during pulse transmission and reception



$\Rightarrow$ ***Equivalent to an antenna array !***

PRI = Pulse Repetition Interval [s]

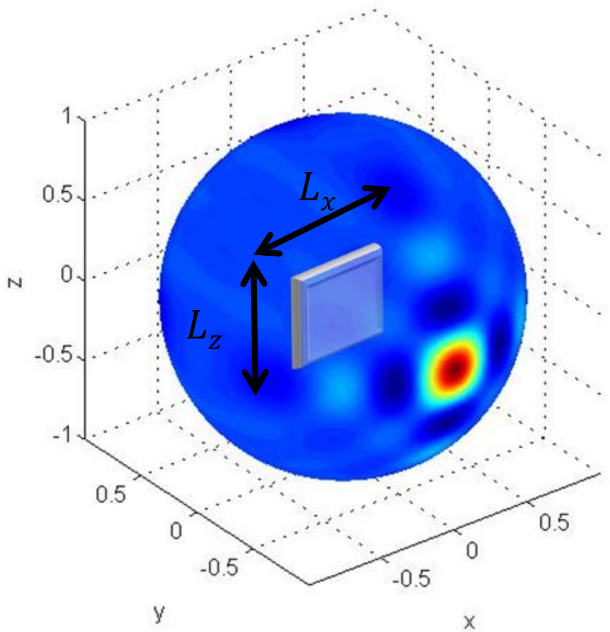
PRF = Pulse Repetition Frequency [Hz]

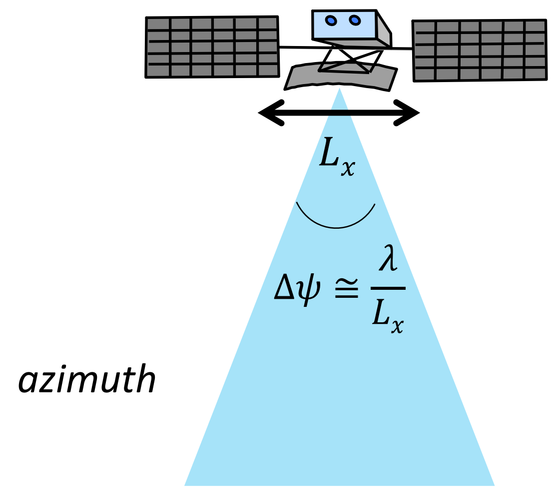
v = Platform speed [m/s]

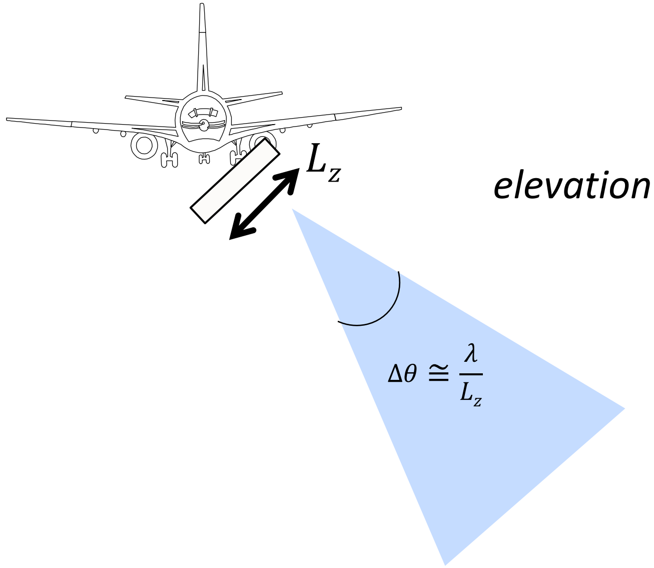
How long is the synthetic aperture ?

Target illumination is limited to an angular sector, depending on wavelength and antenna size

$$\Delta\psi \cong \frac{\lambda}{L_x} \qquad \Delta\theta \cong \frac{\lambda}{L_z}$$







SAR imaging

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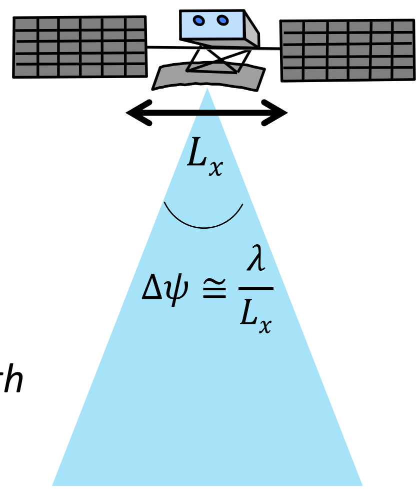
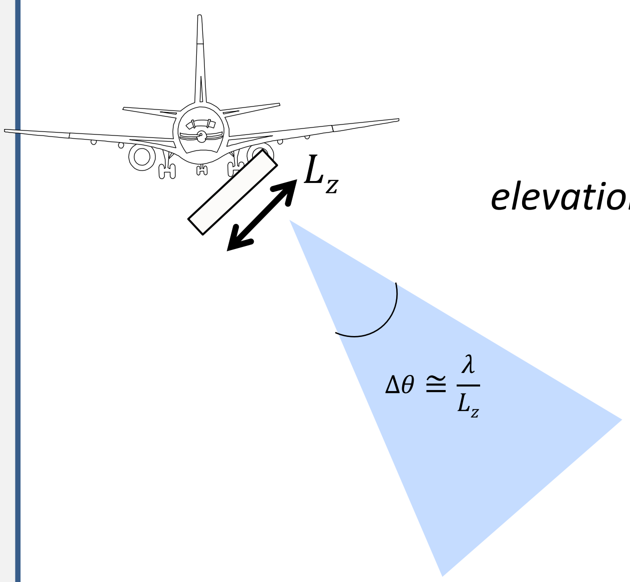


azimuth



elevation

Spaceborne SAR at C-Band

Lx = 10-15 m

λ = 5.6 cm

$\Delta\psi \cong 0.5\,°$

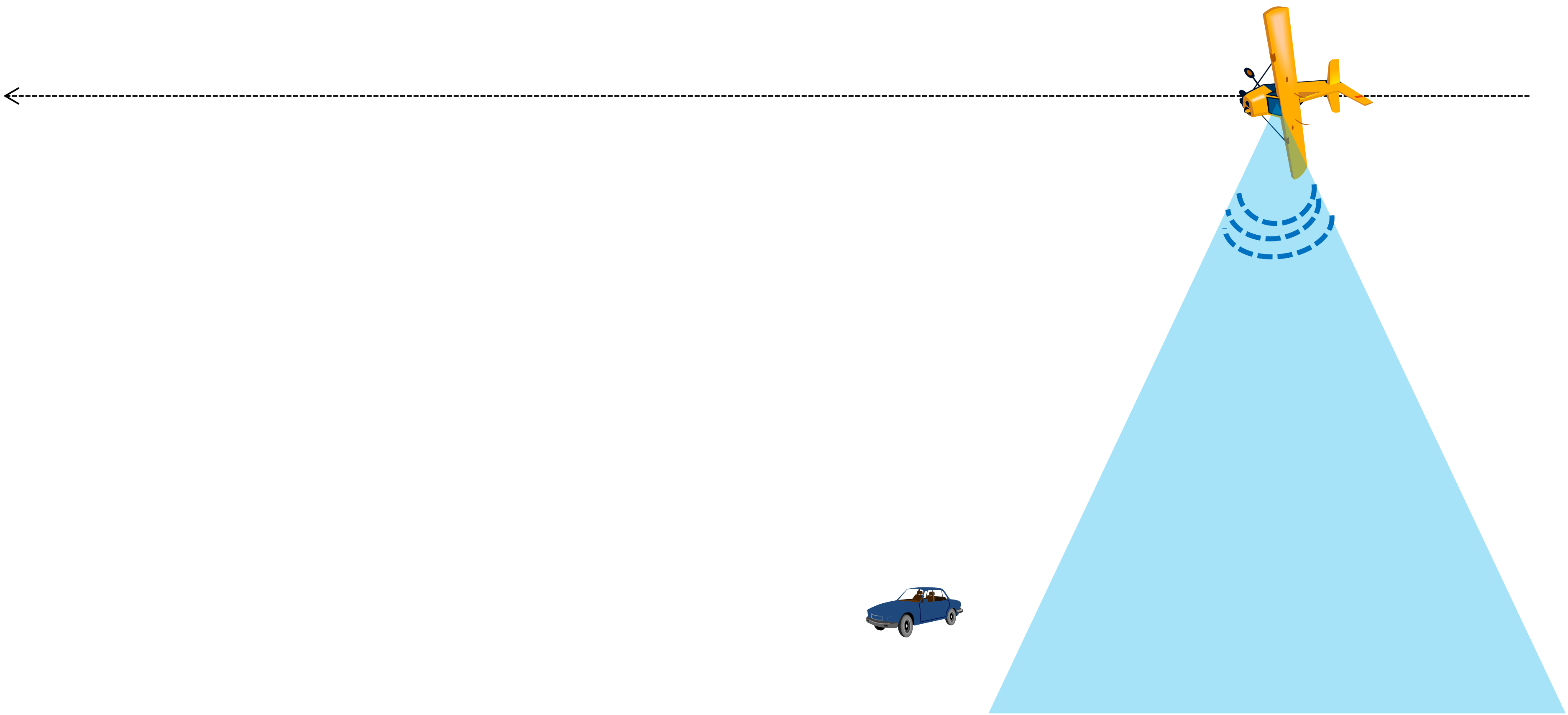
Airborne SAR at P-Band

λ = 0.7 m

$\Delta\psi \cong 20\,°$

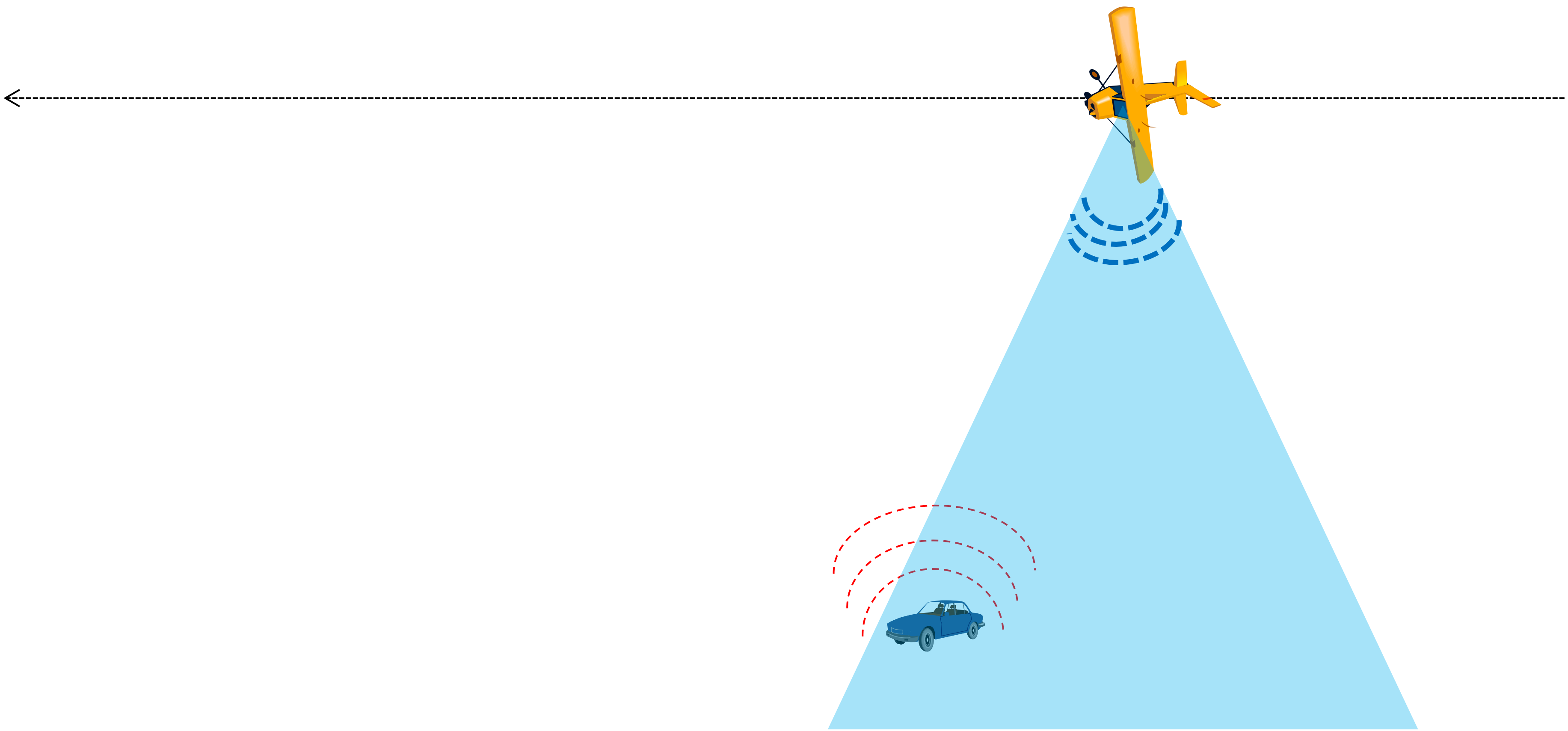
SAR imaging

⇒ ***Targets are illuminated only a fraction of the time***



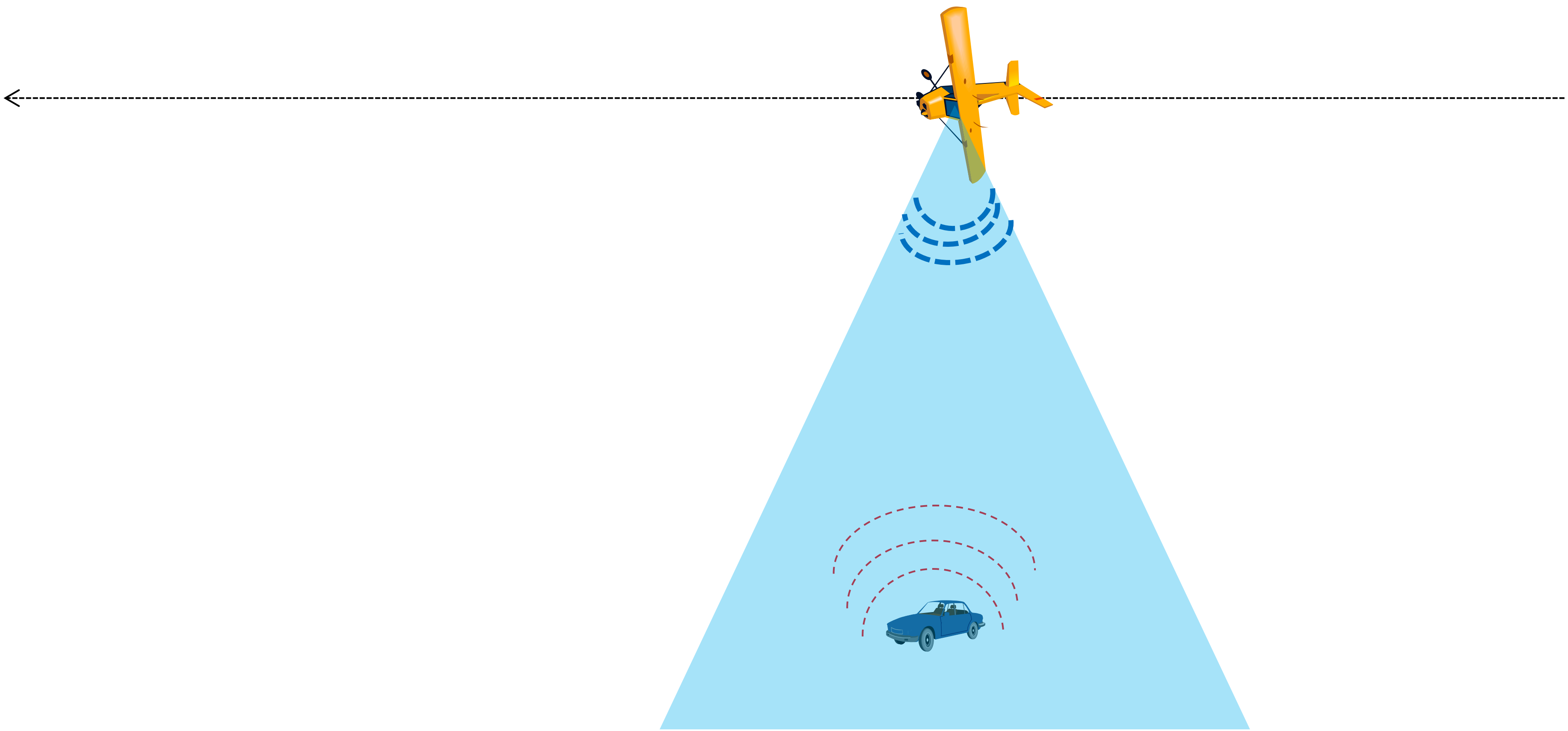
SAR imaging

⇒ ***Targets are illuminated only a fraction of the time***



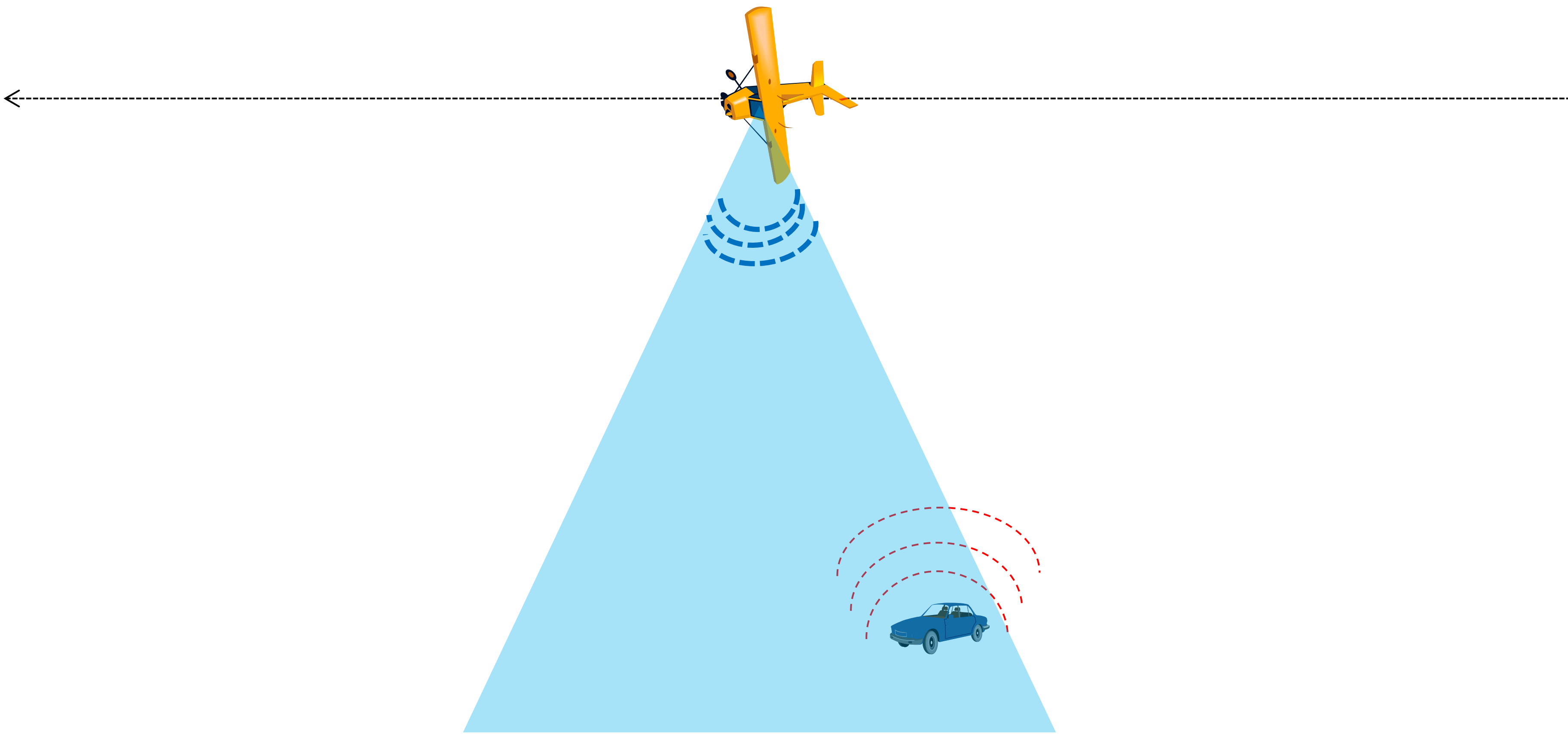
SAR imaging

⇒ ***Targets are illuminated only a fraction of the time***



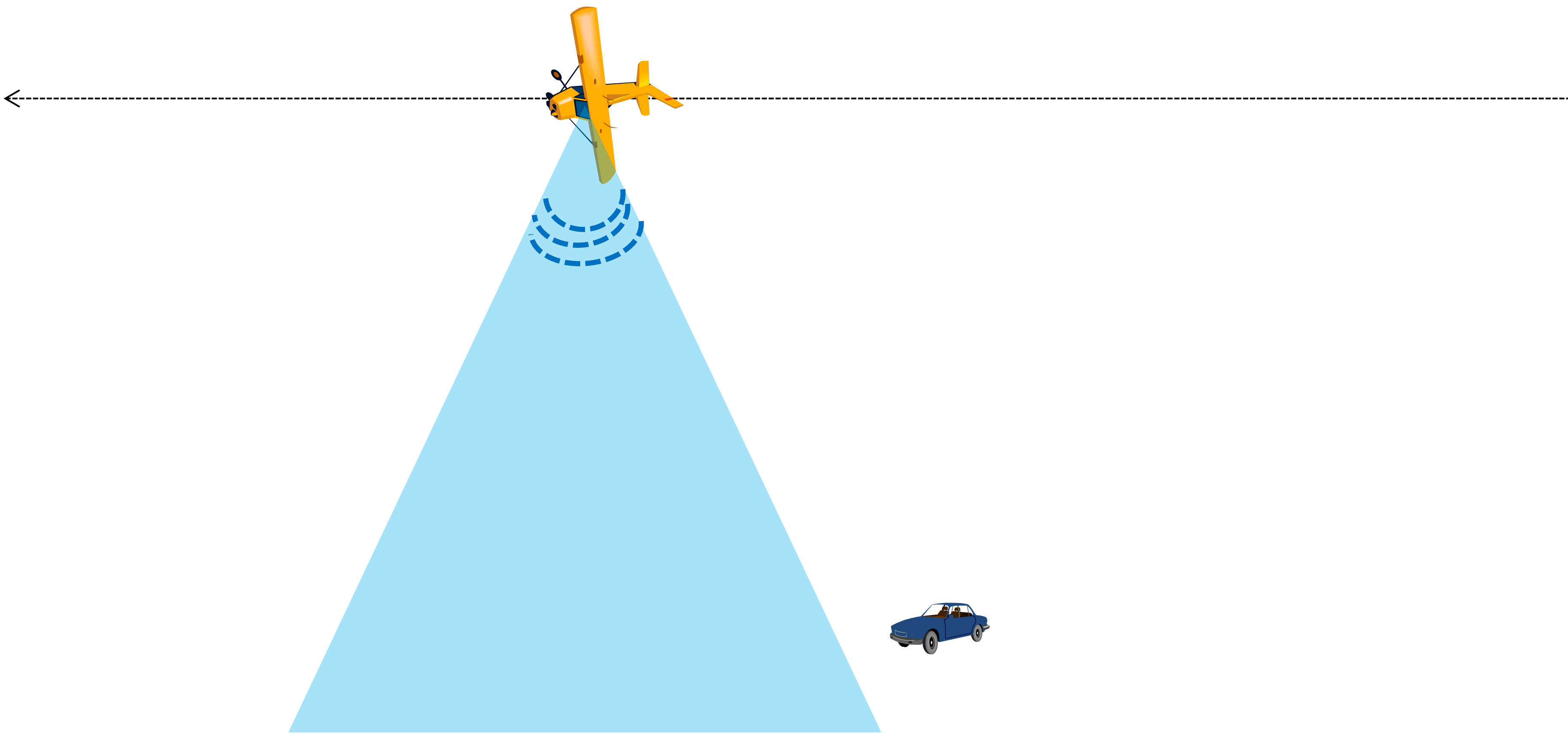
SAR imaging

⇒ ***Targets are illuminated only a fraction of the time***



SAR imaging

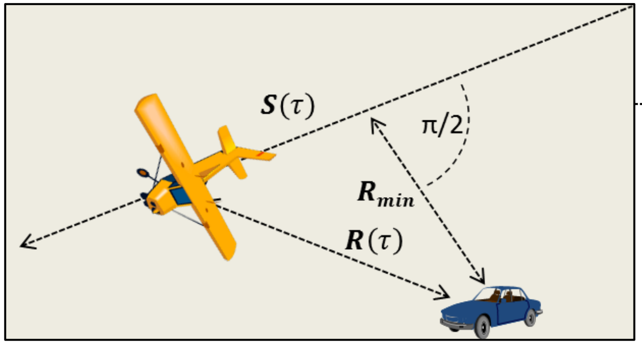
⇒ ***Targets are illuminated only a fraction of the time***

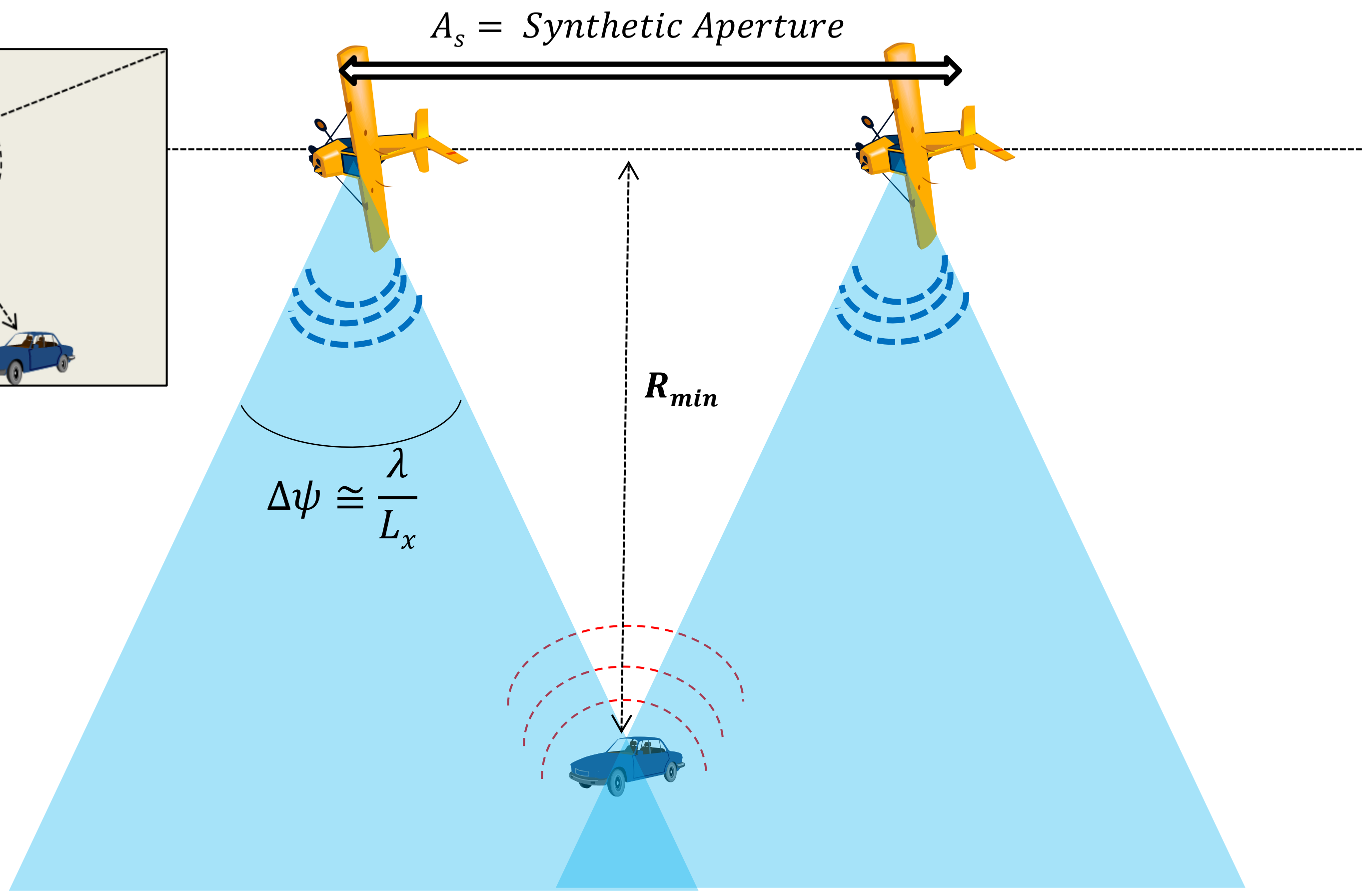


SAR imaging

Length of the synthetic aperture

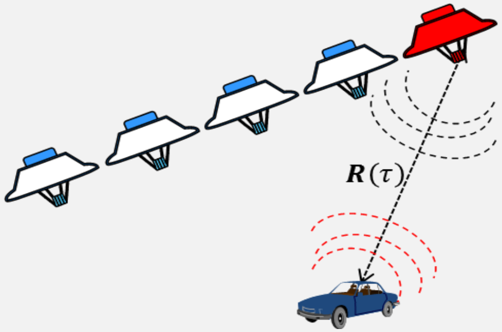
$$A_s \cong \Delta\psi \cdot \boldsymbol{R_{min}}$$





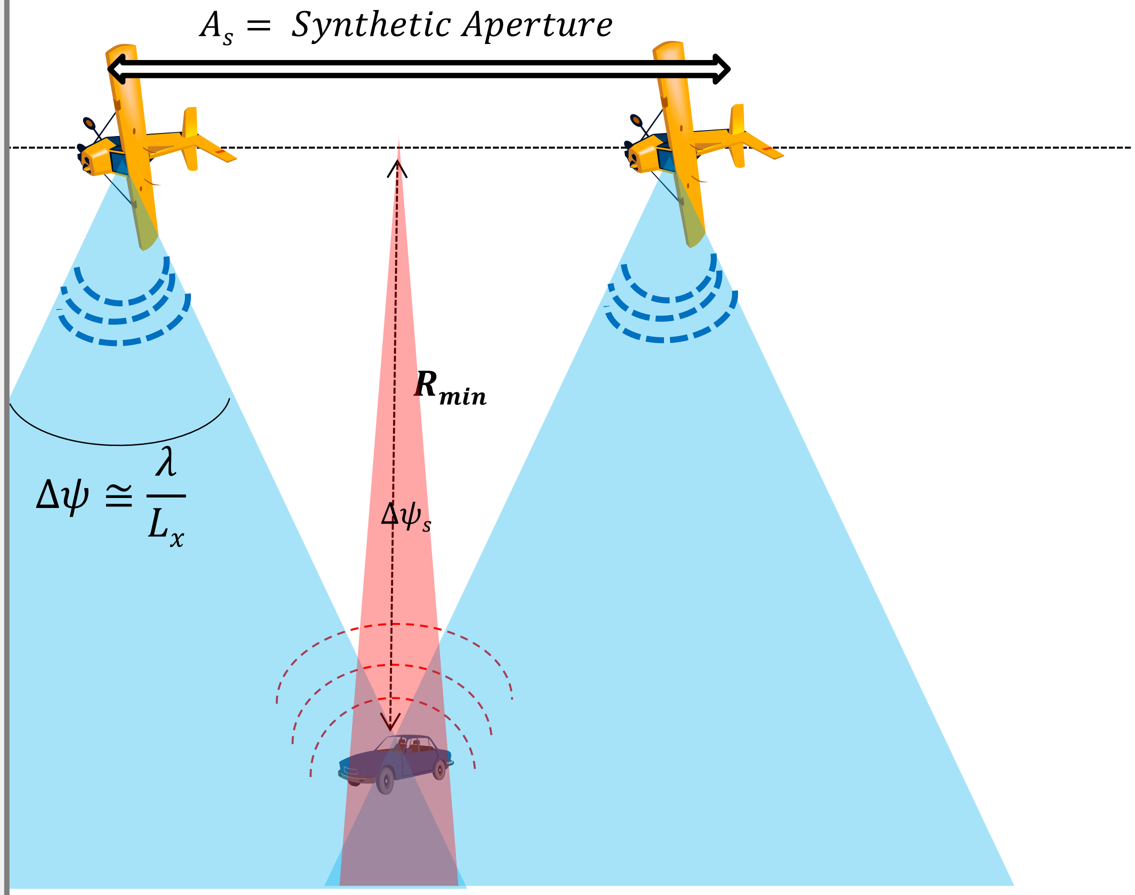
SAR imaging

Result from array theory



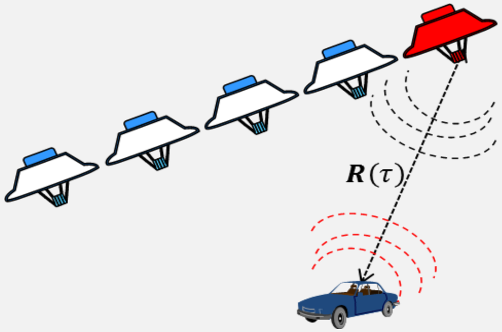
Aperture length translates to angular resolution according to

$$\Delta\psi_s \cong \frac{\lambda}{2A_s}$$



SAR imaging

Result from array theory

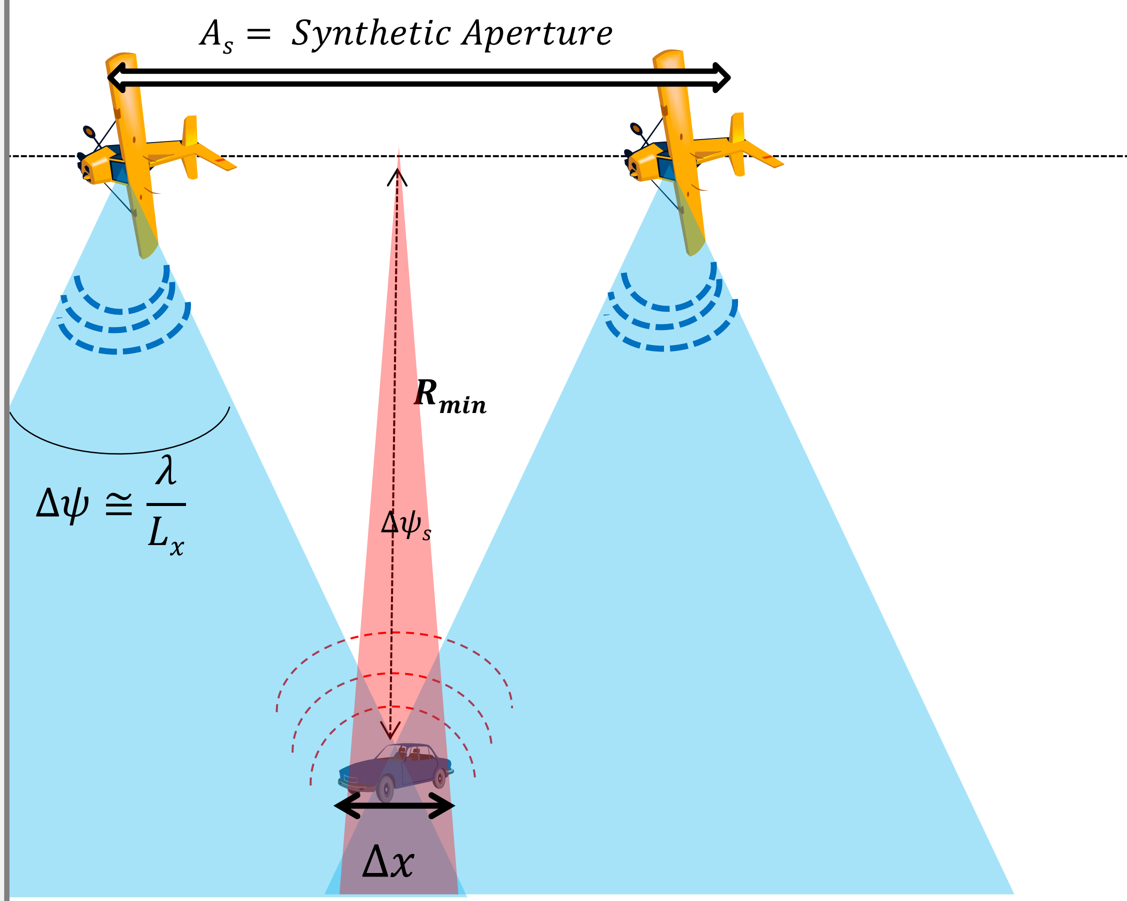


Aperture length translates to angular resolution according to

$$\Delta\psi_s \cong \frac{\lambda}{2A_s}$$

Angular resolution translates into horizontal resolution according to

$$\Delta x \cong \Delta\psi_s \cdot \boldsymbol{R}_{min}$$

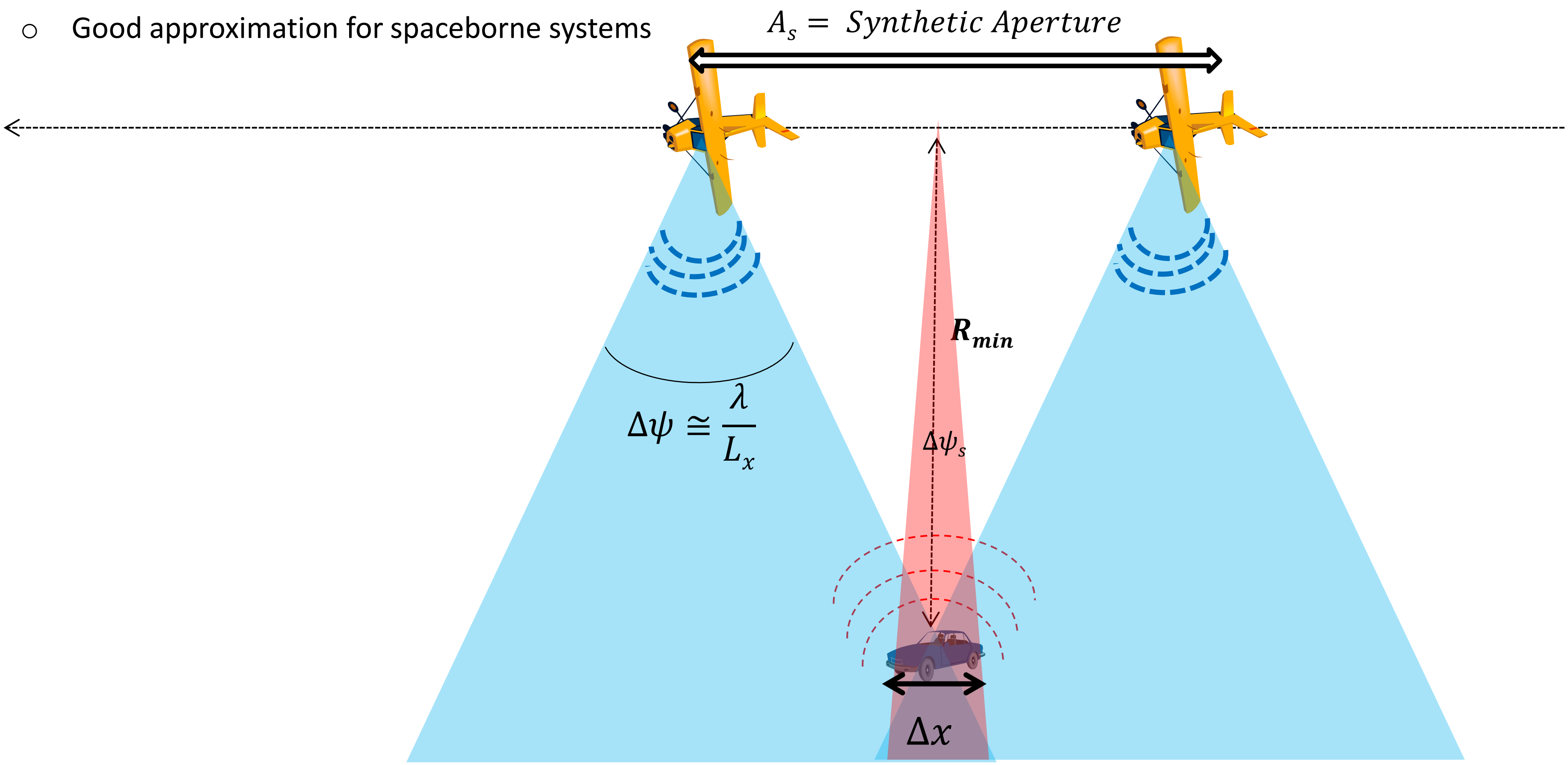


SAR imaging

Horizontal (along-track) resolution = half the antenna length

$$\Delta x \cong \frac{L_x}{2}$$

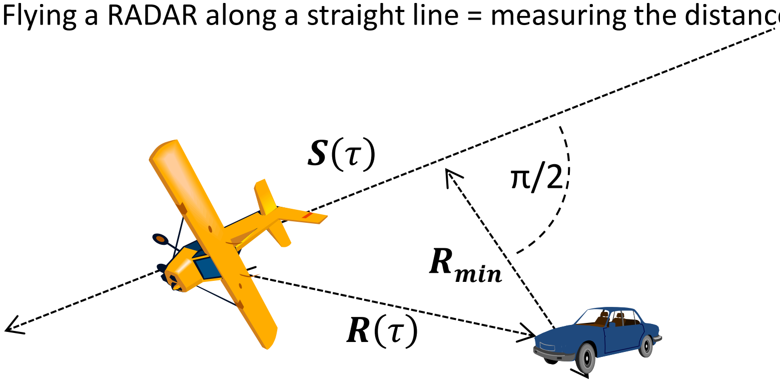
- Independent on target's distance from the trajectory
- Good approximation for spaceborne systems

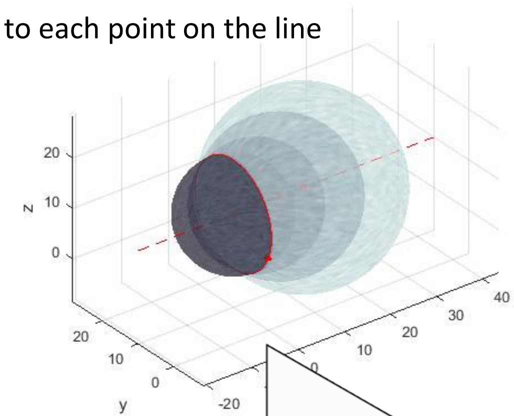


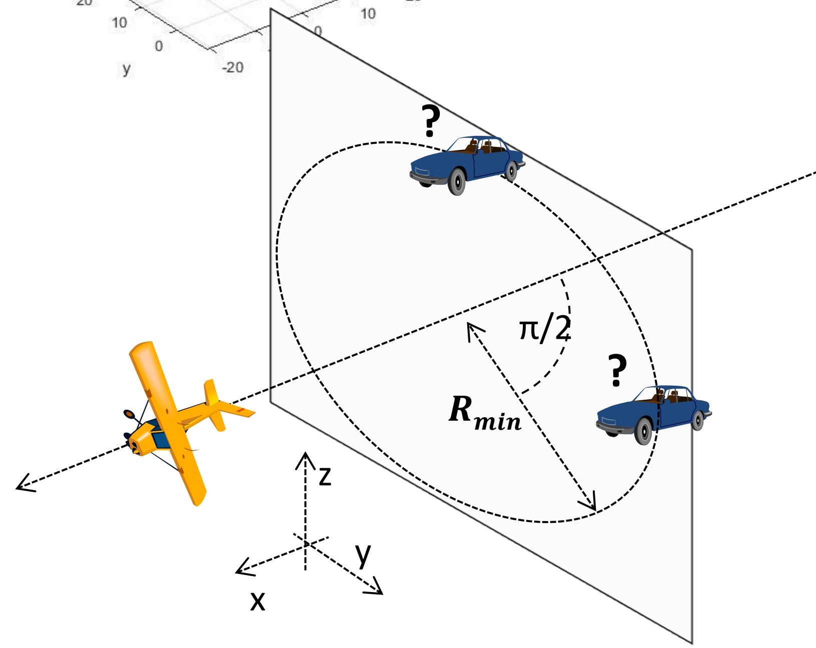
SAR imaging – geometrical interpretation

Synthetic Aperture Radars (SAR) employ a moving RADAR sensor, flown onboard a satellite or an aircraft, in order to synthesize an antenna as long as several kilometers

Flying a RADAR along a straight line = measuring the distance from the target to each point on the line







The target is bound to lie on the intersection of all the spheres:

- Centered in $\boldsymbol{S}(\tau)$
- Of radius $\boldsymbol{R}(\tau)$

$\Rightarrow$ The target is bound to lie on the circle:

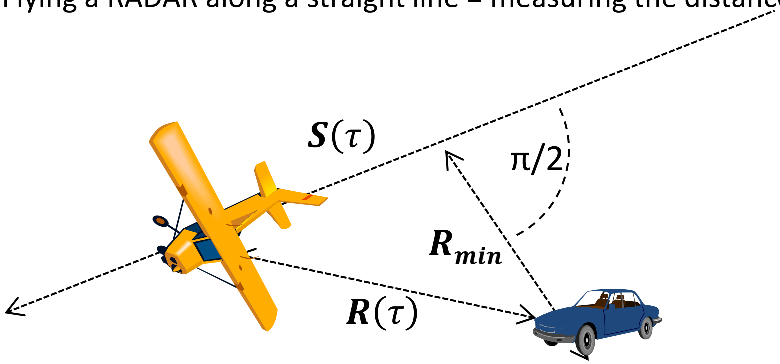
- Centered on the trajectory
- Perpendicular to the trajectory (*yz* plane)
- Of radius $\boldsymbol{R}_{min}$

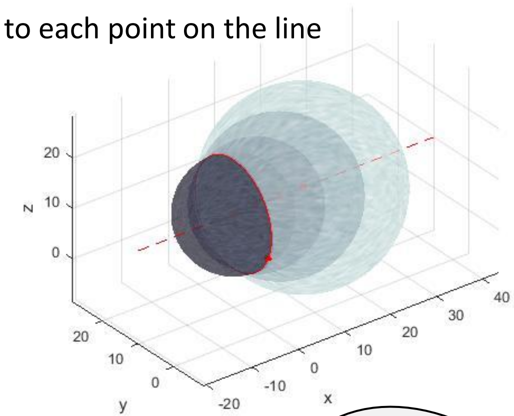
$\Rightarrow$ 2D Localization

SAR imaging – physical interpretation

Synthetic Aperture Radars (SAR) employ a moving RADAR sensor, flown onboard a satellite or an aircraft, in order to synthesize an antenna as long as several kilometers

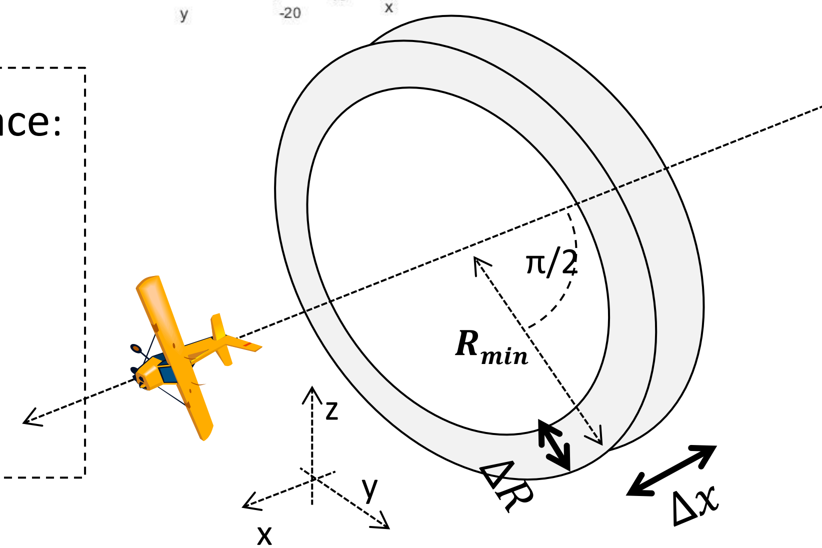
Flying a RADAR along a straight line = measuring the distance from the target to each point on the line





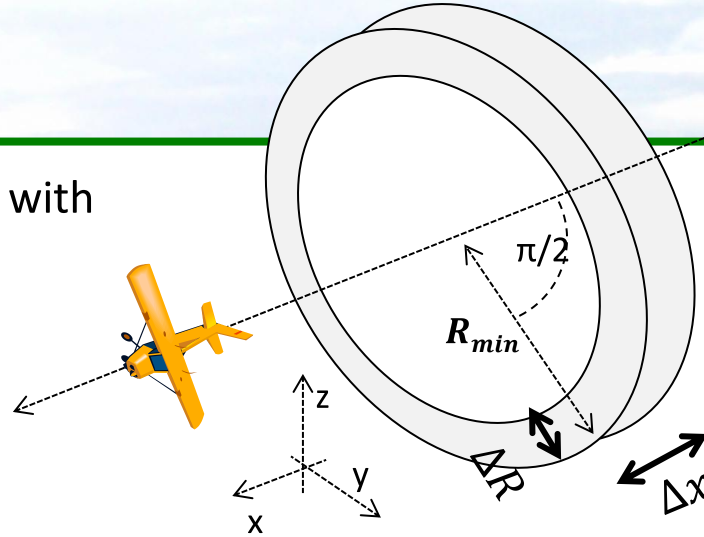
⇒ The target is bound to lie in the region of space:

- Centered on the trajectory
- Perpendicular to the trajectory (*yz* plane)
- Range thickness $\Delta R = \frac{c}{2B}$
- Along-track thickness $\Delta x = \frac{\lambda R}{2A_s} \cong \frac{L_x}{2}$

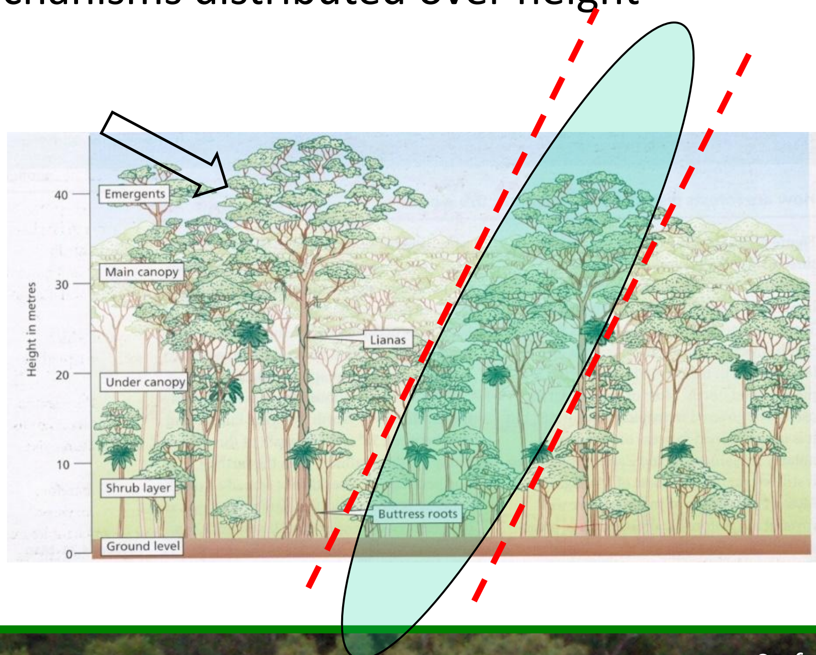


The SAR resolution cell

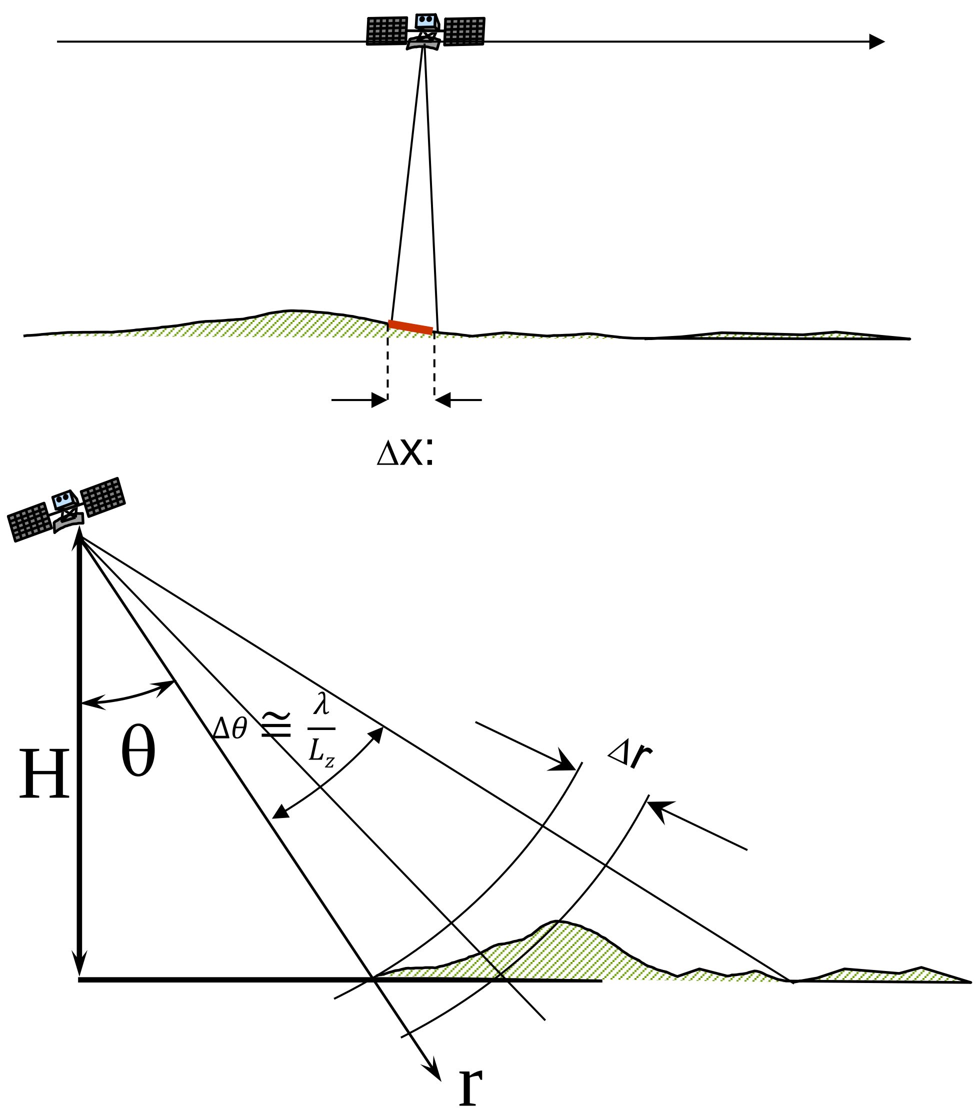
This region represents the ***resolution cell*** associated with each pixel in a SAR image

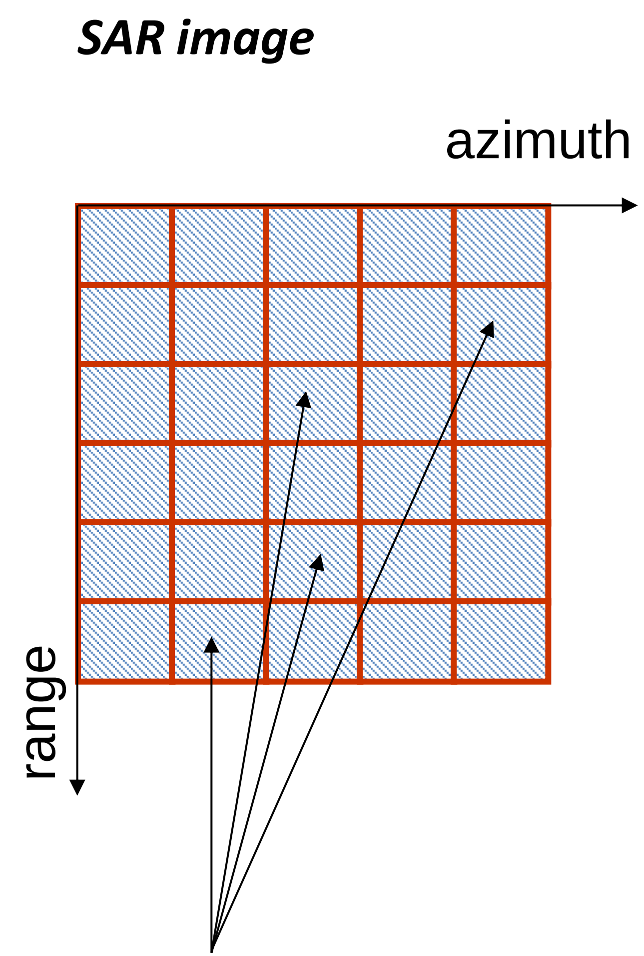


Especially when penetration occurs, we need to consider any single pixel within a SAR image as a mixture of different scattering mechanisms distributed over height



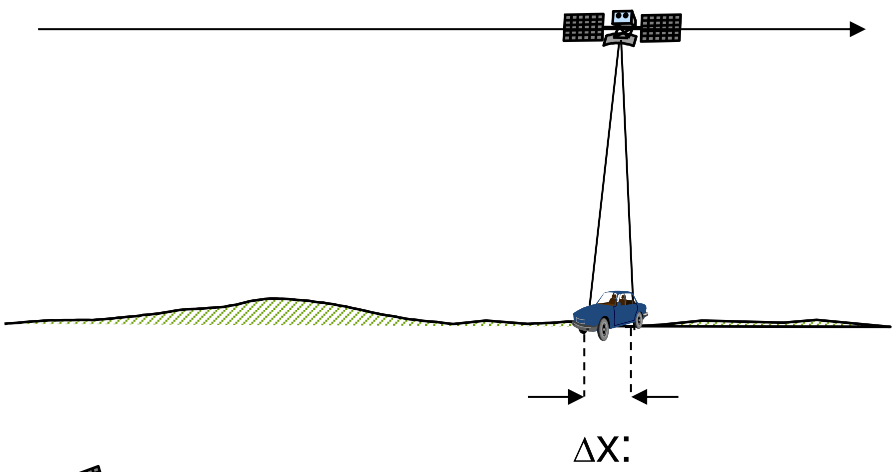
The SAR resolution cell

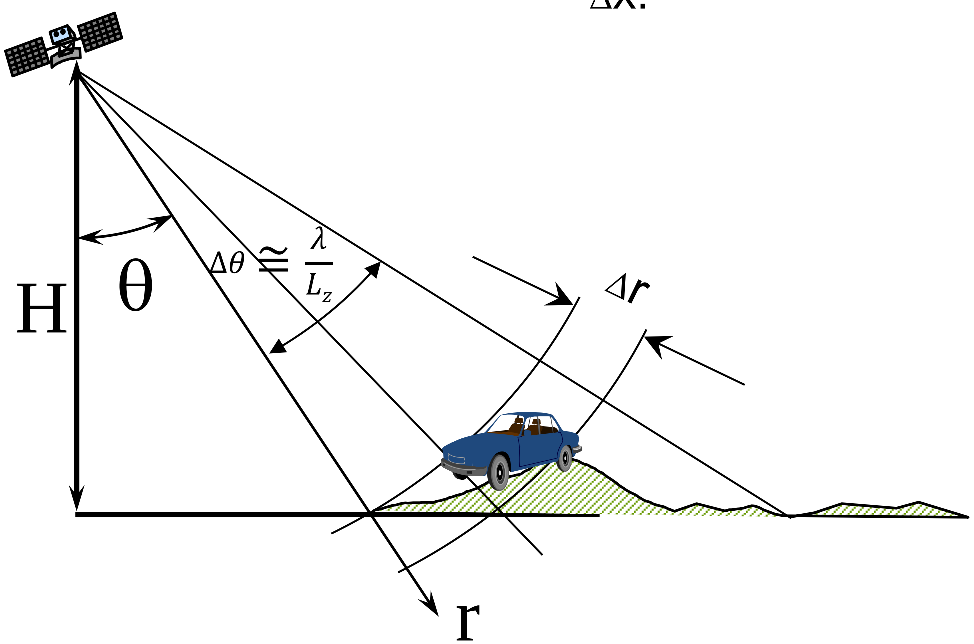




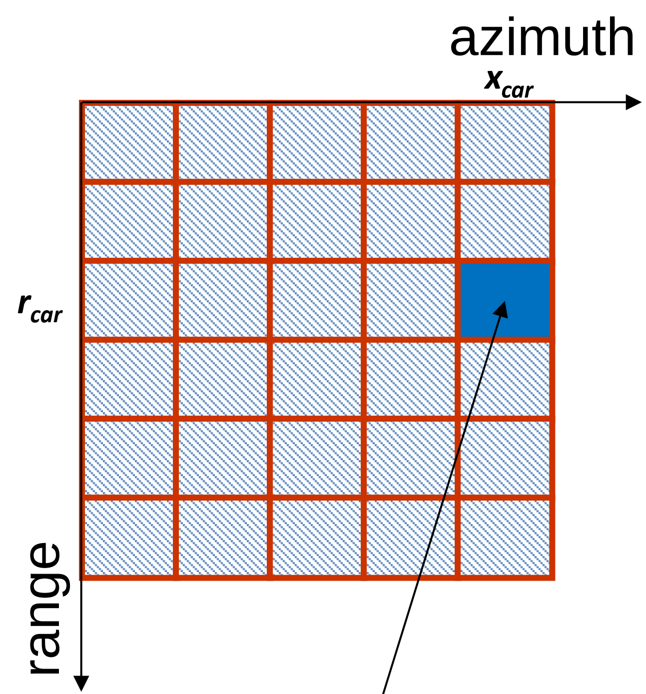
Each pixel is associated with a range/azimuth resolution cell

The SAR resolution cell





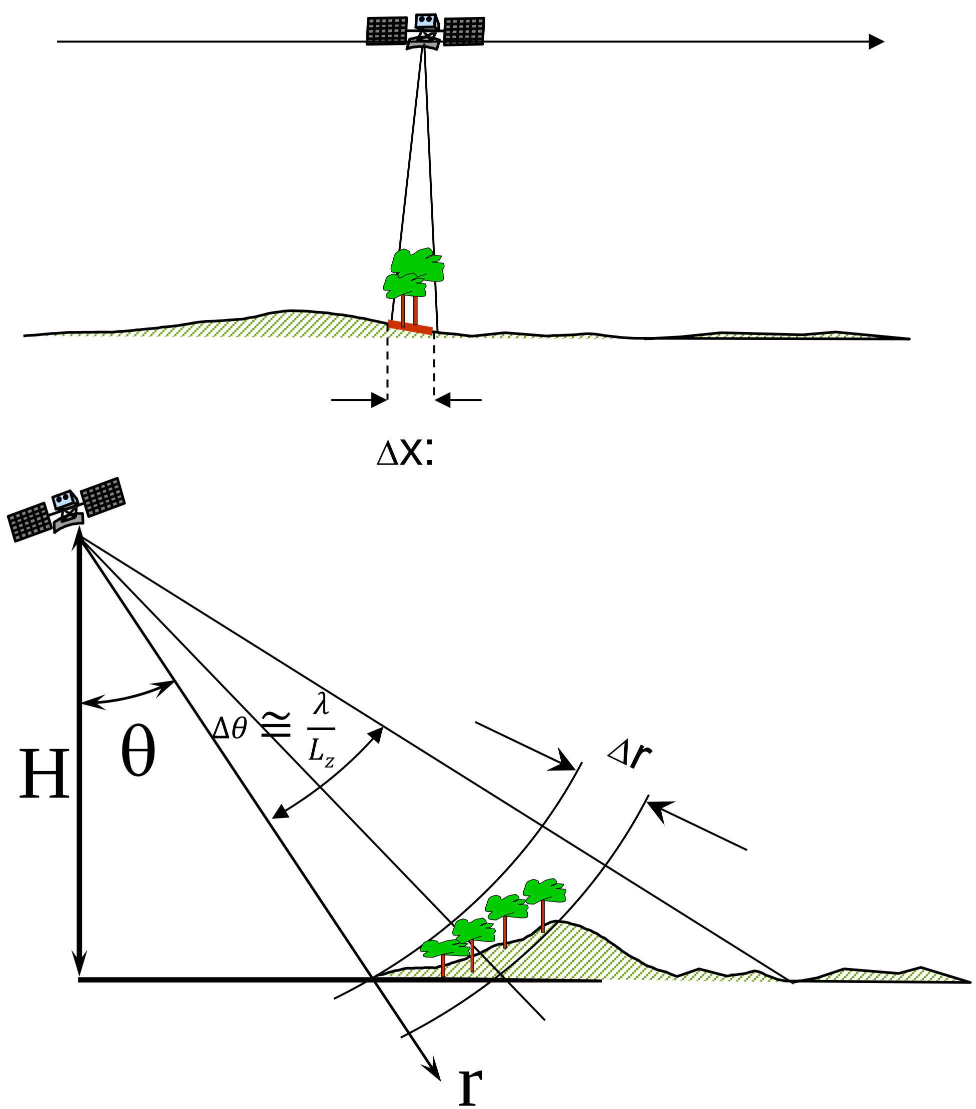
SAR image



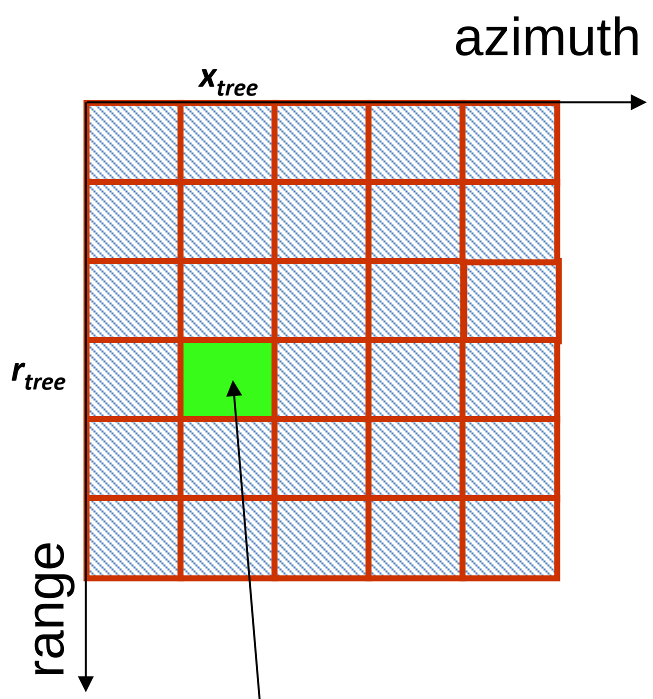
This pixel value corresponds to the target at position x_{car}, r_{car}

$$I(r_{car}, x_{car}) = A_{car} \cdot e^{-j\frac{4\pi}{\lambda}r_{car}}$$

The SAR resolution cell

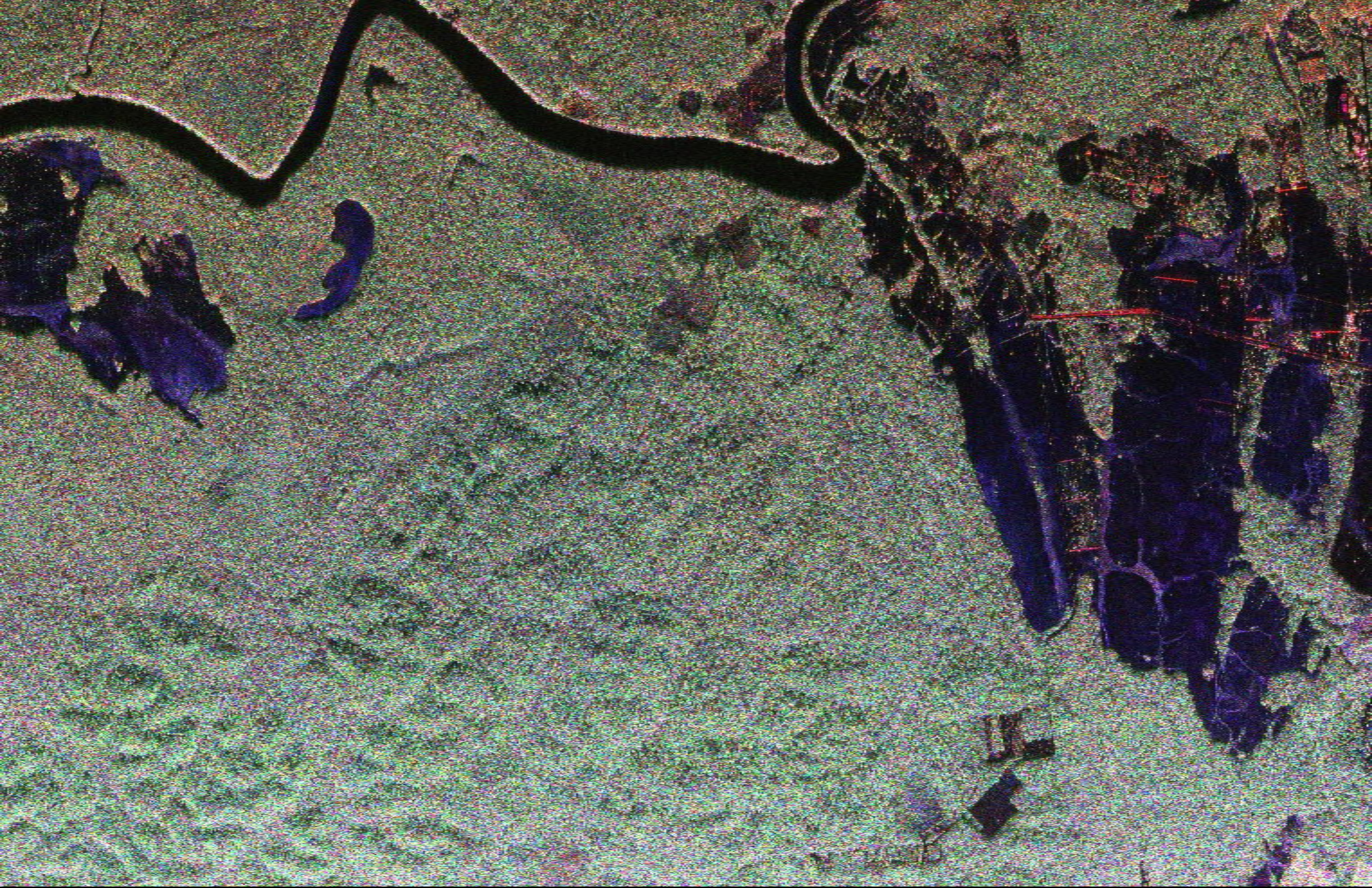


SAR image



This pixel value is arises from the interference of all trees (and terrain) within the resolution cell

$$I(r_{tree}, x_{tree}) = \sum_k A_k \cdot e^{-j\frac{4\pi}{\lambda}r_k}$$



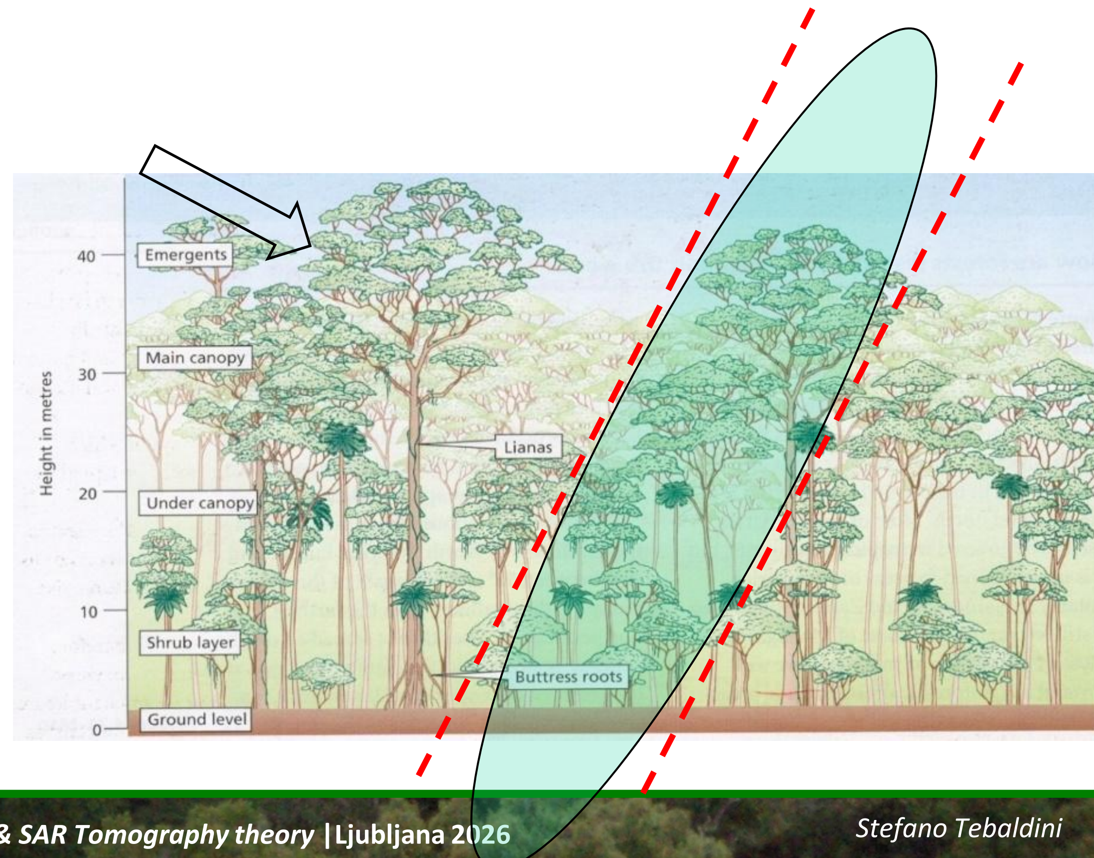
P-Band airborne SAR image of the tropical forest at Paracou, French Guiana. Image size is 4 x 9 Km at 1 x 1 m resolution. This picture is obtained as a color-composite representation of the magnutudes of three images acquired using different wave polarizations at Tx and Rx. Data gathered by ONERA during the ESA campaign TropiSAR, 2009.

P-Band airborne SAR image of the tropic ... 1 x 1 m resolution. This picture is obtained as a color-composite representation of the magnitudes of three images acquired using different wave polarizations at Tx and Rx. Data gathered by ONERA during the ESA campaign TropiSAR, 2009.

Speckle!

The term ***speckle*** refers to a random salt-and-pepper amplitude modulation of the intensities of SAR pixels due to the interference of different targets within the same resolution cell

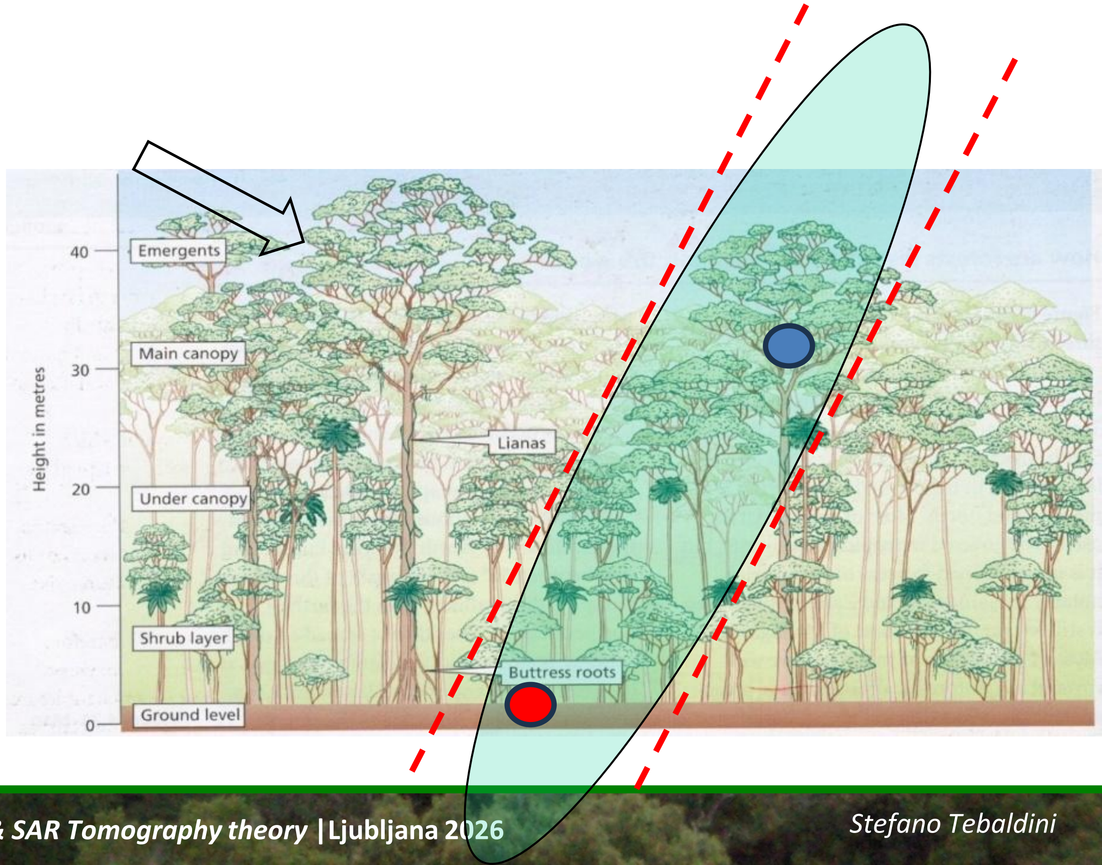
$$I(r_{tree}, x_{tree}) = \sum_k A_k \cdot e^{-j\frac{4\pi}{\lambda}r_k}$$



Speckle!

To see how it works, let's consider a simple example with only two targets with approximately the same amplitudes

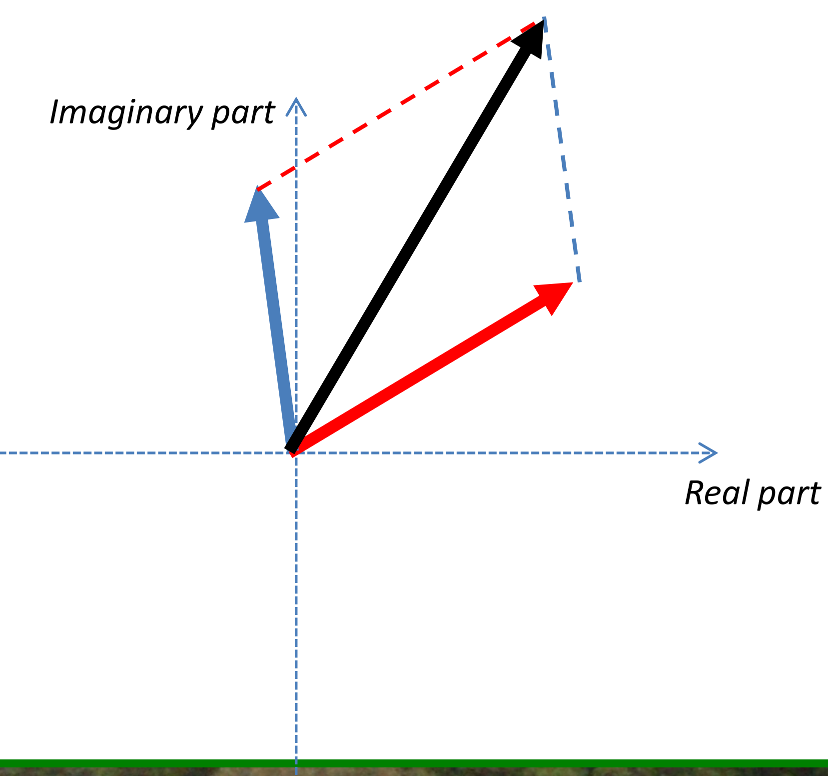
$$I(r_{tree}, x_{tree}) = A \cdot e^{-j\frac{4\pi}{\lambda}r_1} + A \cdot e^{-j\frac{4\pi}{\lambda}r_2}$$



Speckle!

To see how it works, let's consider a simple example with only two targets with approximately the same amplitudes

$$I(r_{tree}, x_{tree}) = A \cdot e^{-j\frac{4\pi}{\lambda}r_1} + A \cdot e^{-j\frac{4\pi}{\lambda}r_2}$$

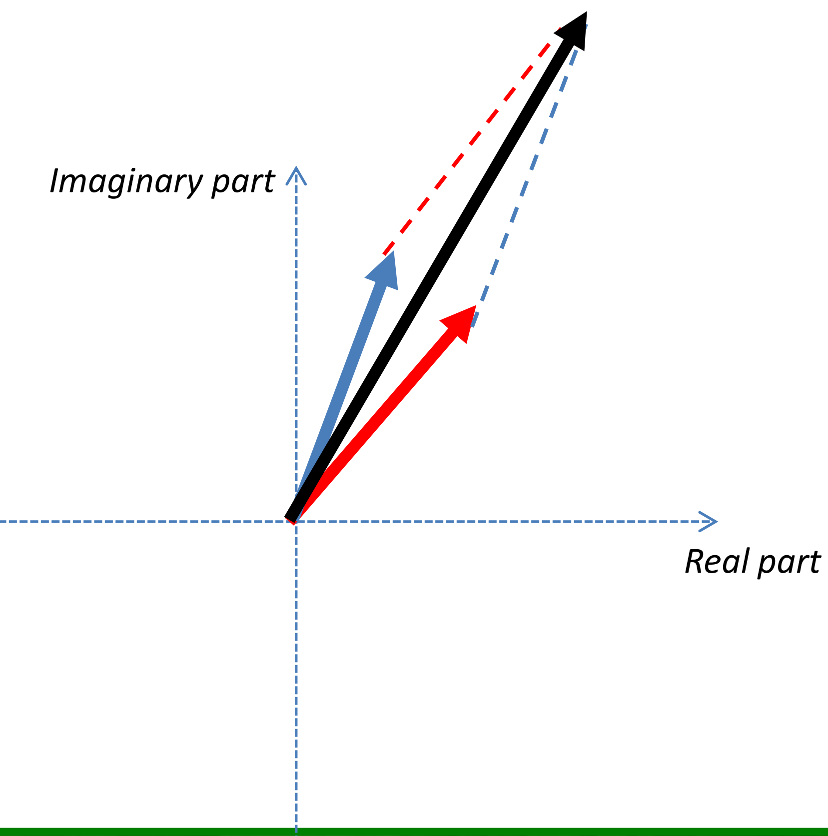


The magnitude of the resulting signal depends on whether the two vectors are aligned or not

Speckle!

To see how it works, let's consider a simple example with only two targets with approximately the same amplitudes

$$I(r_{tree}, x_{tree}) = A \cdot e^{-j\frac{4\pi}{\lambda}r_1} + A \cdot e^{-j\frac{4\pi}{\lambda}r_2}$$



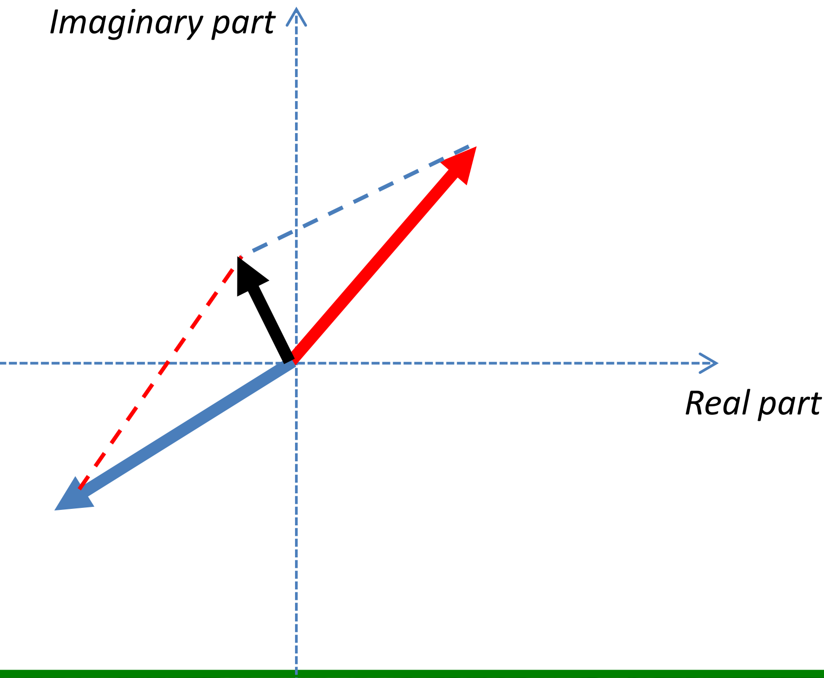
The magnitude of the resulting signal depends on whether the two vectors are aligned or not

Constructive interference occurs when the two vectors are aligned ⇔ magnitude is increased

Speckle!

To see how it works, let's consider a simple example with only two targets with approximately the same amplitudes

$$I(r_{tree}, x_{tree}) = A \cdot e^{-j\frac{4\pi}{\lambda}r_1} + A \cdot e^{-j\frac{4\pi}{\lambda}r_2}$$



The magnitude of the resulting signal depends on whether the two vectors are aligned or not

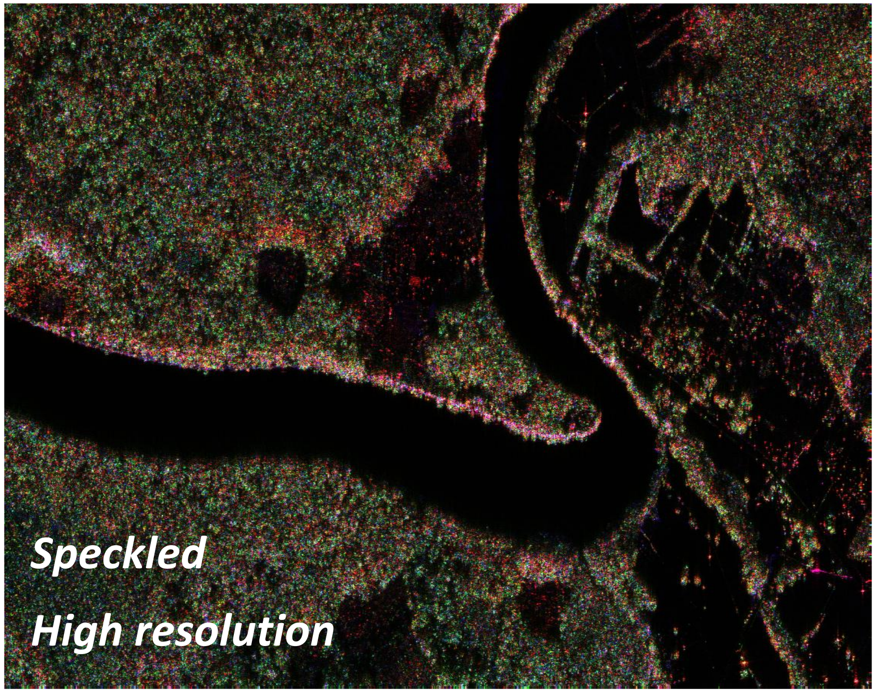
Constructive interference occurs when the two vectors are aligned ⇔ magnitude is increased

Destructive interference occurs when the two vectors are not aligned ⇔ magnitude is reduced

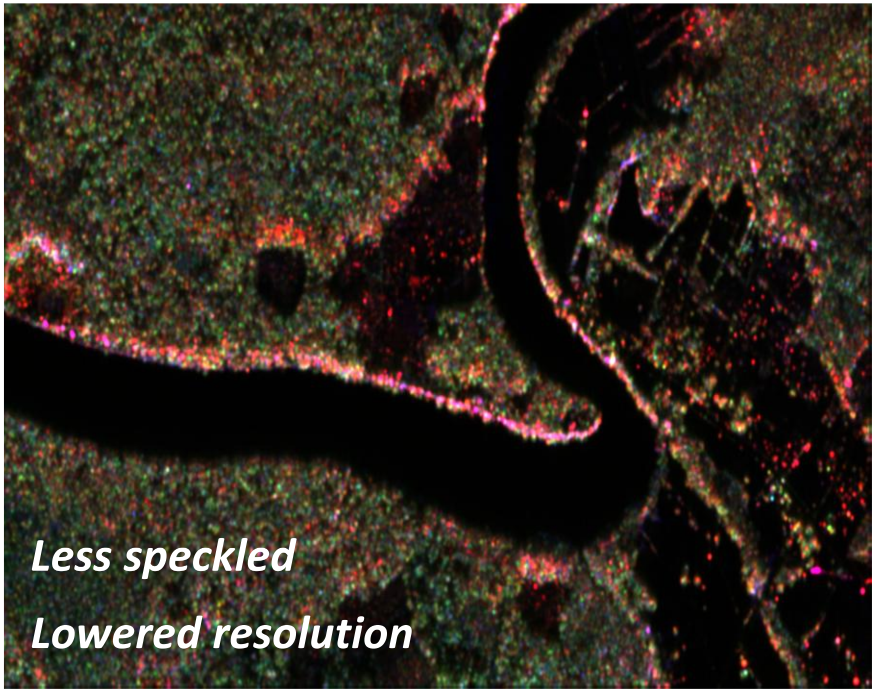
De-Speckle!

The standard procedure to mitigate speckle effects is to average (a.k.a. *filter*) the image magnitudes over a small moving window

Original amplitude

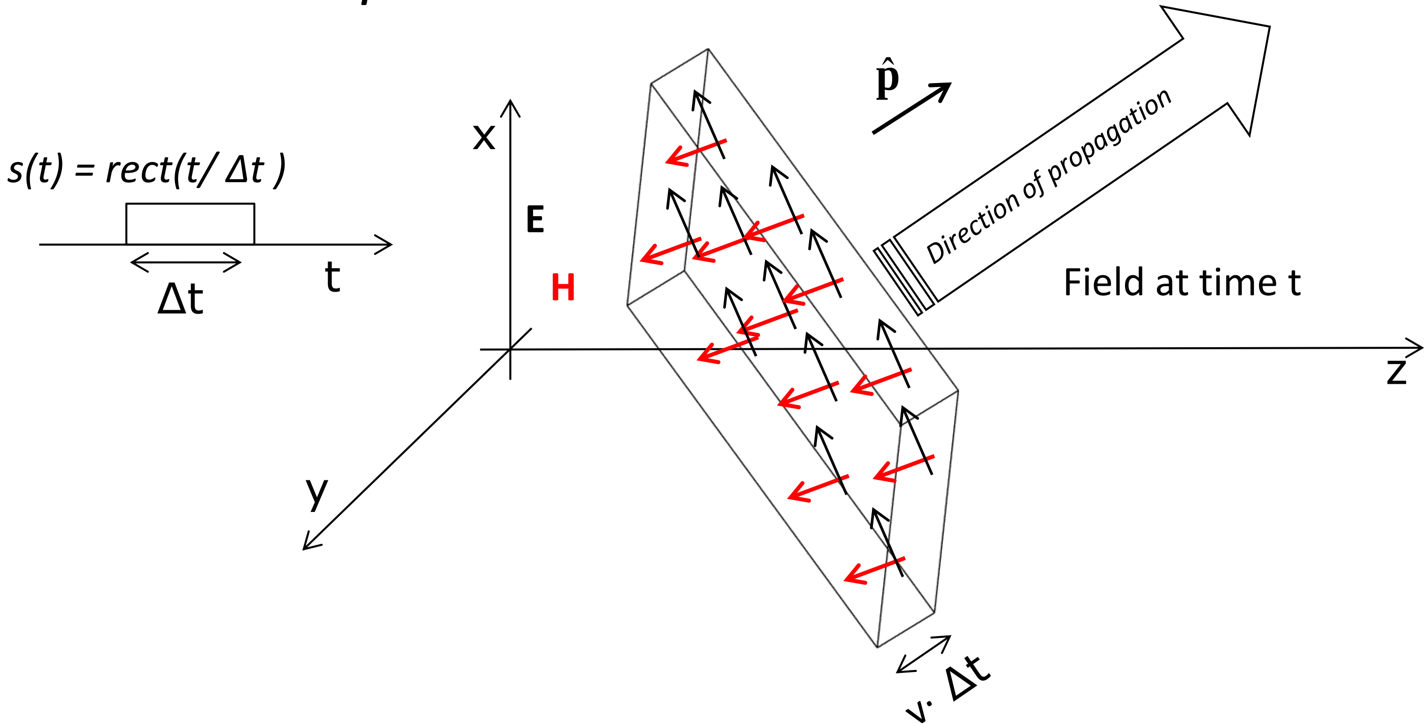


Filtered amplitude



EM waves are polarized!

- The **electric** (**E**) and **magnetic** (**H**) fields are always orthogonal to the propagation direction ⇔ **transverse** fields
- **E** and **H** are orthogonal to each other
- Propagation direction is obtained as the vector product **E** X **H**
- The ratio between **E** and **H** is called intrinsic impedance of the medium (377 ohm for vacuum)
- The direction of ***E*** is called ***wave polarization***



SAR Polarimetry

SAR sensors can transmit and receive EM waves for which the electric field is directed both horizontally (H) and vertically (V) with respect to the Radar antenna

$\Rightarrow$ In this way, polarimetric SAR sensor produces up to four images per track

- HH ⇔ transmit H, receive H
- HV ⇔ transmit V, receive H
- VH ⇔ transmit H, receive V
- VV ⇔ transmit V, receive V

By electromagnetic reciprocity HV = VH

$\Rightarrow$ Each acquisition produces up to ***three*** independent SAR images

The three polarimetric combinations are often represented considering:

The lexicographic basis

$$\begin{bmatrix} HH \\ \sqrt{2}\cdot HV \\ VV \end{bmatrix}$$

The Pauli basis

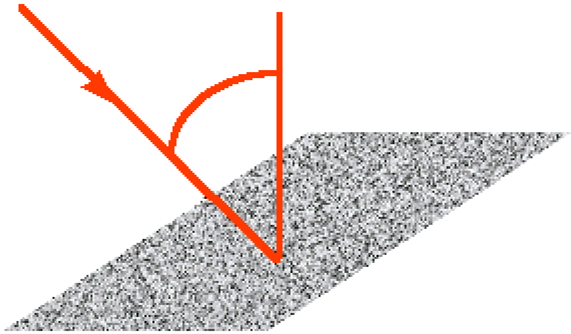
$$\frac{1}{\sqrt{2}}\cdot\begin{bmatrix} HH+VV \\ HH-VV \\ 2\cdot HV \end{bmatrix}$$

SAR Polarimetry

The signal in different polarimetric combinations is associated with different features.

Typical signatures:

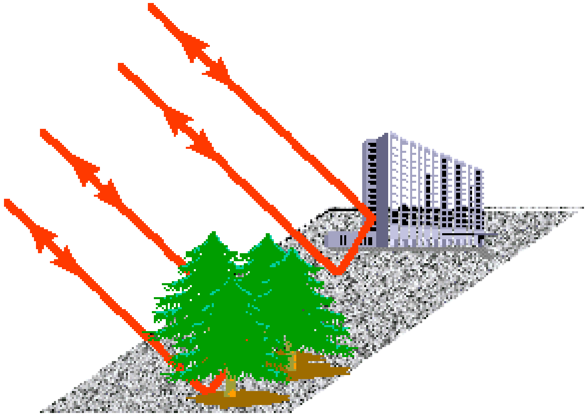
SINGLE BOUNCE
(ROUGH SURFACE)



Ideal features:

- *HH and VV only*
- *HH and VV are highly correlated*
- *HH-VV phase ≈ 0°*

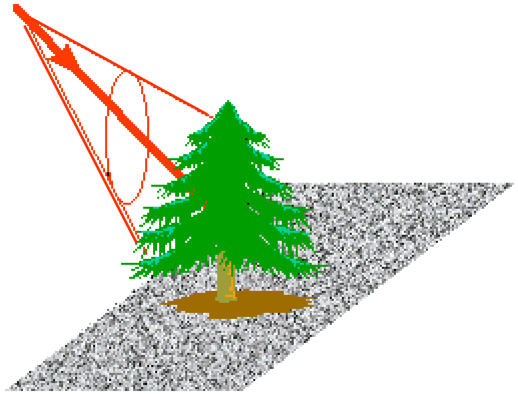
DOUBLE BOUNCE
(dihedral-like)



Ideal features:

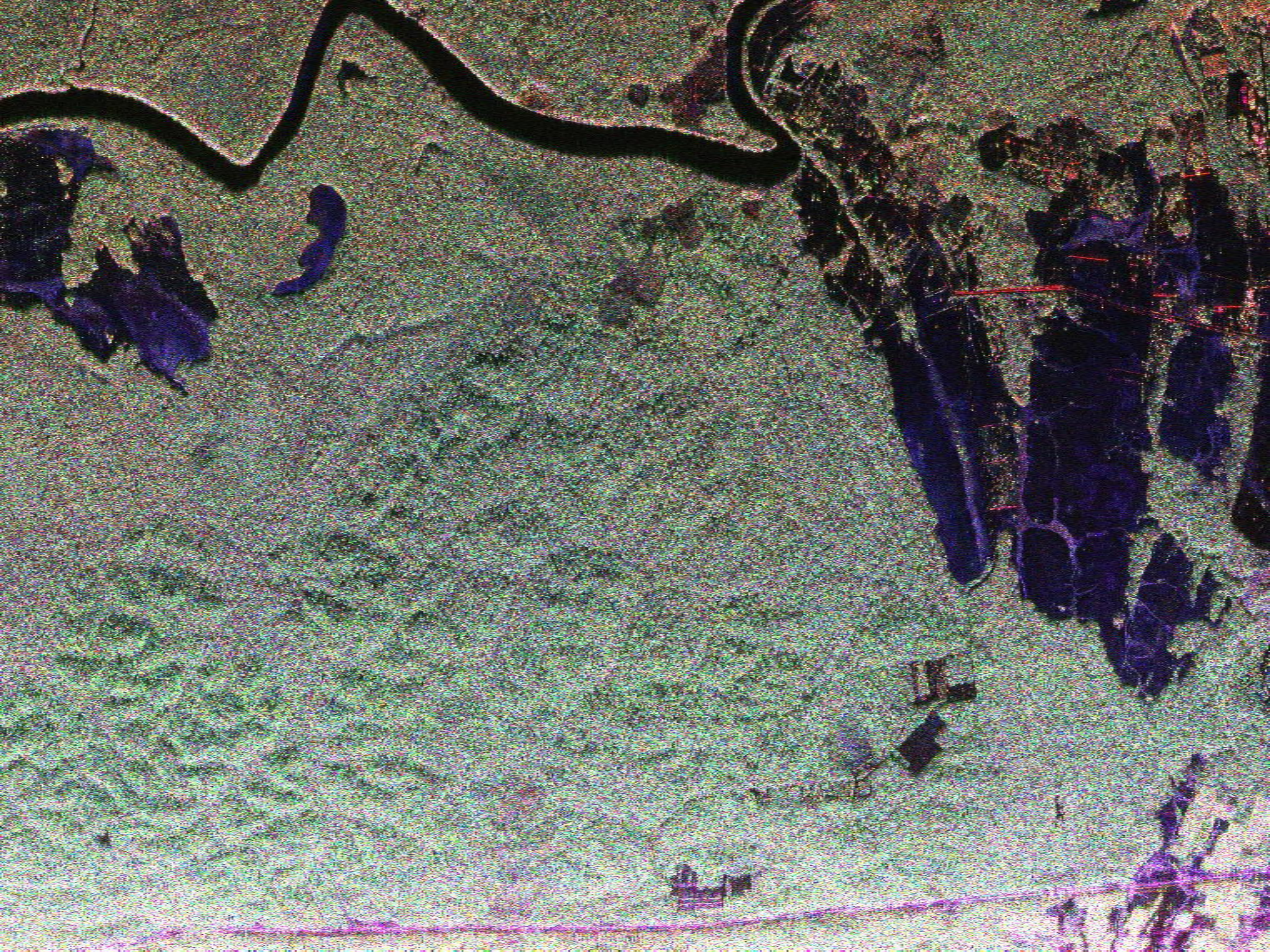
- *HH and VV only*
- *HH and VV are highly correlated*
- *HH-VV phase ≈ 180°*

VOLUME SCATTERING
(random)



Ideal features:

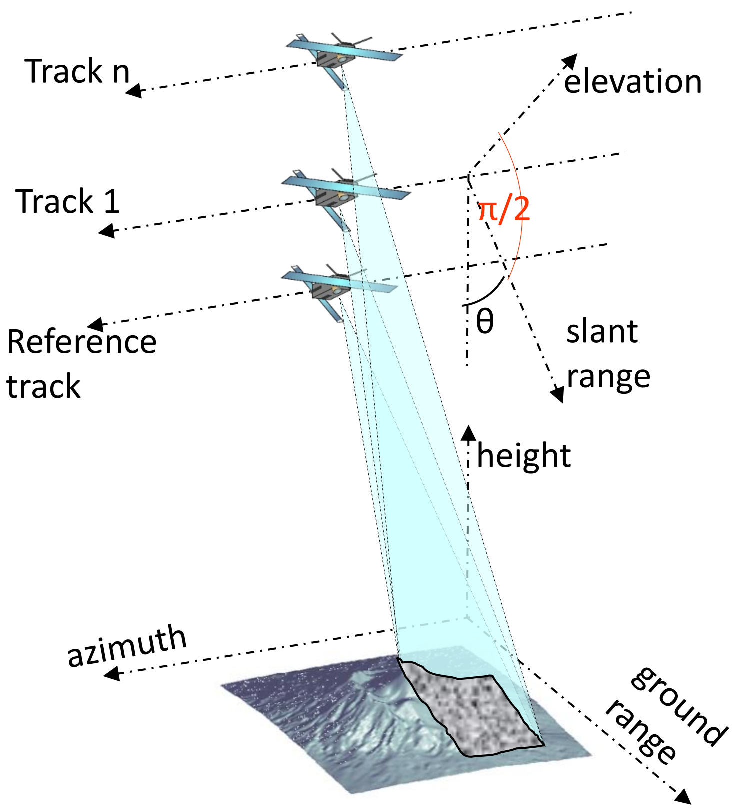
- *Random scattering ⇔ All polarization*
- *No correlation between different polarimetric channels*



TomoSAR Imaging

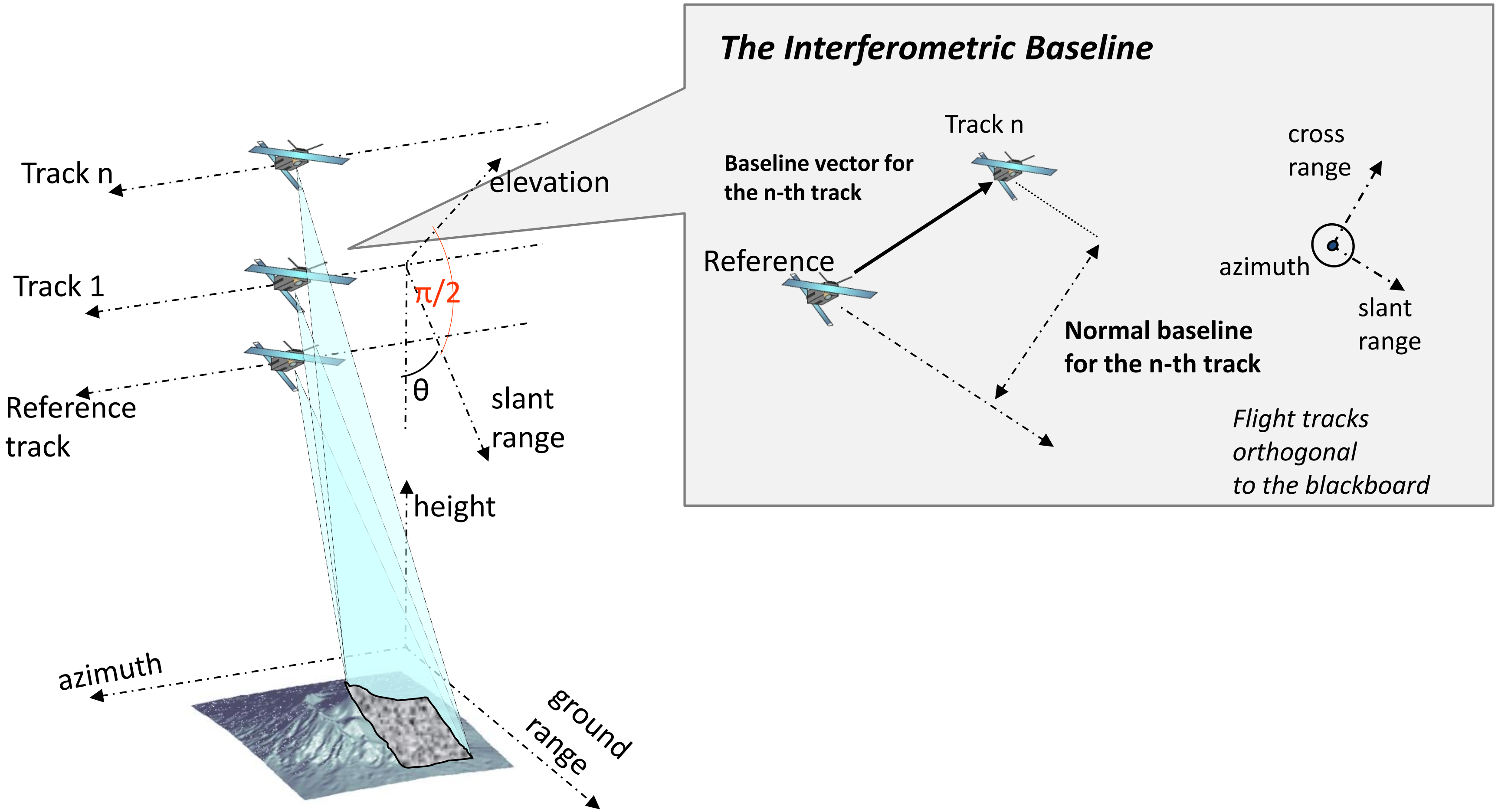
TomoSAR Imaging

Multiple baselines ⇔ Illumination from multiple points of view



TomoSAR Imaging

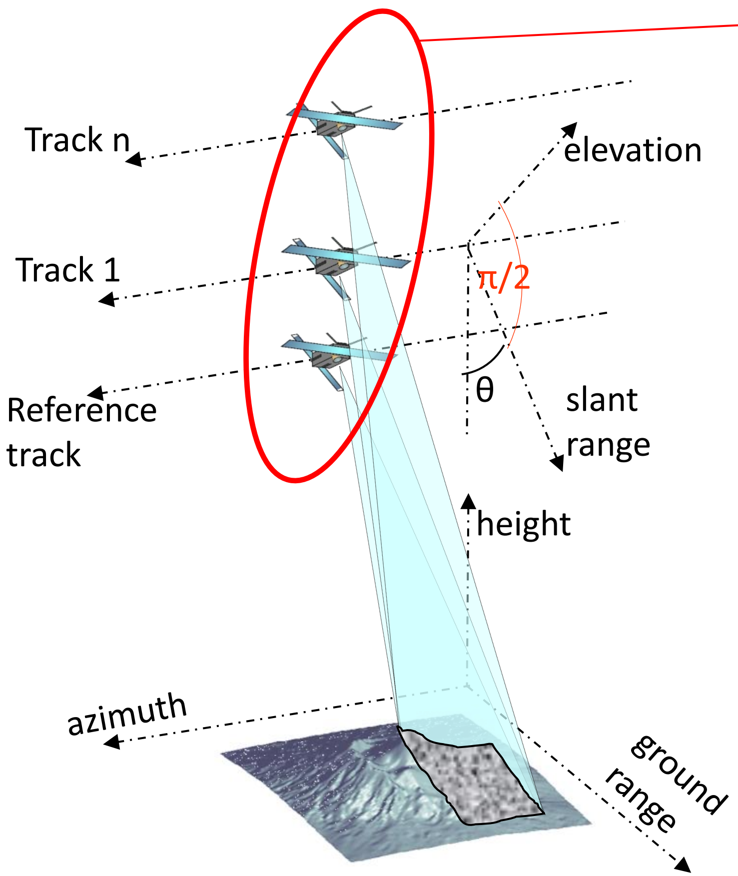
Multiple baselines ⇔ Illumination from multiple points of view

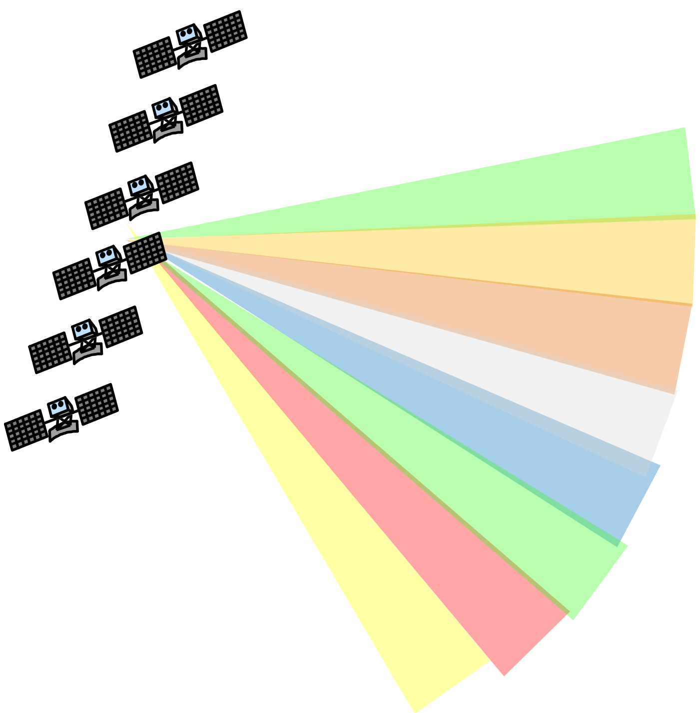


TomoSAR Imaging

Multiple baselines ⇔ Illumination from multiple points of view

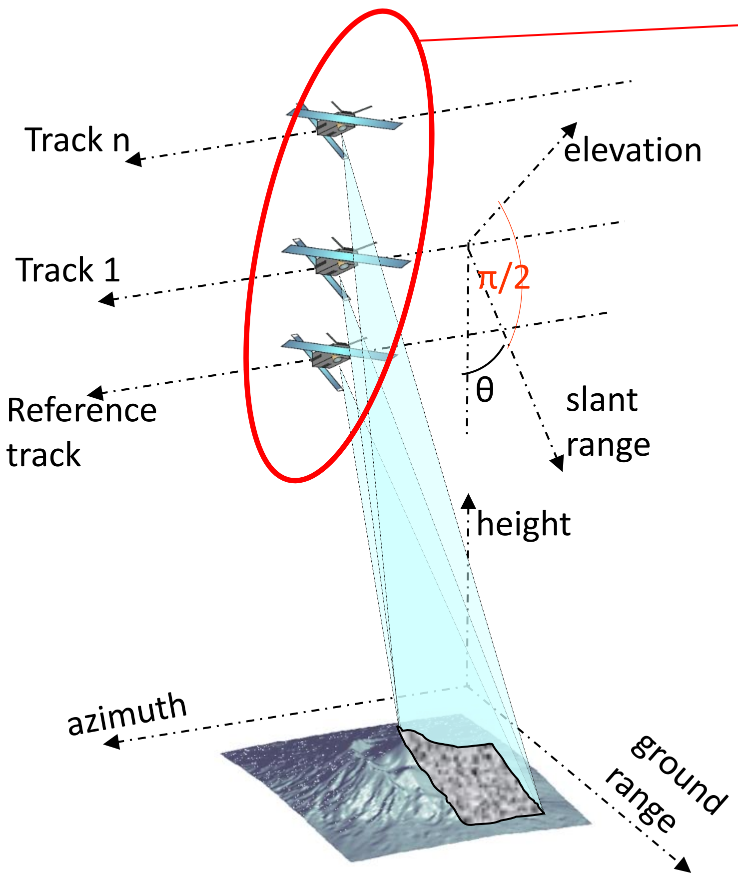
An antenna array is formed at each azimuth position





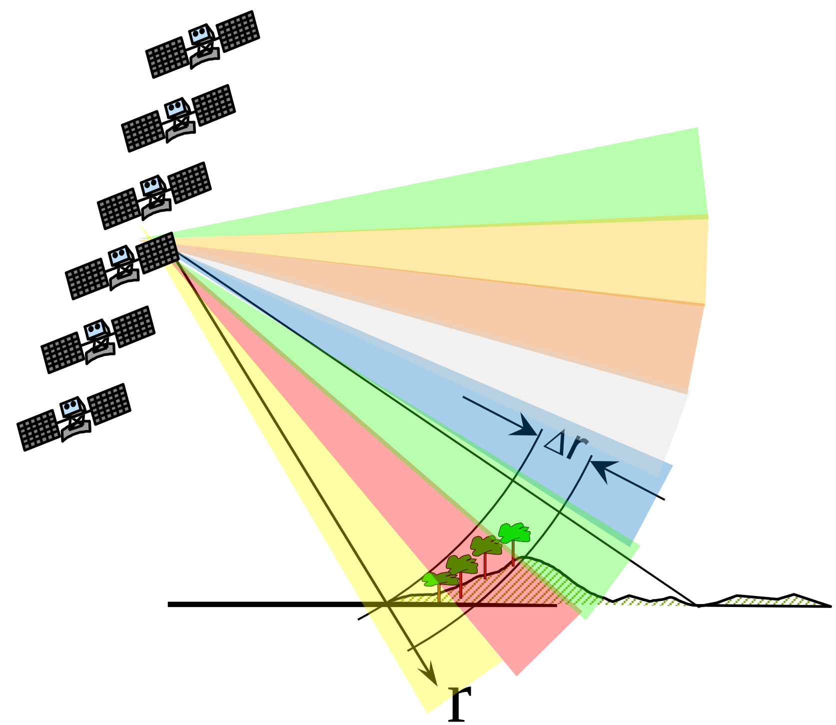
TomoSAR Imaging

Multiple baselines ⇔ Illumination from multiple points of view

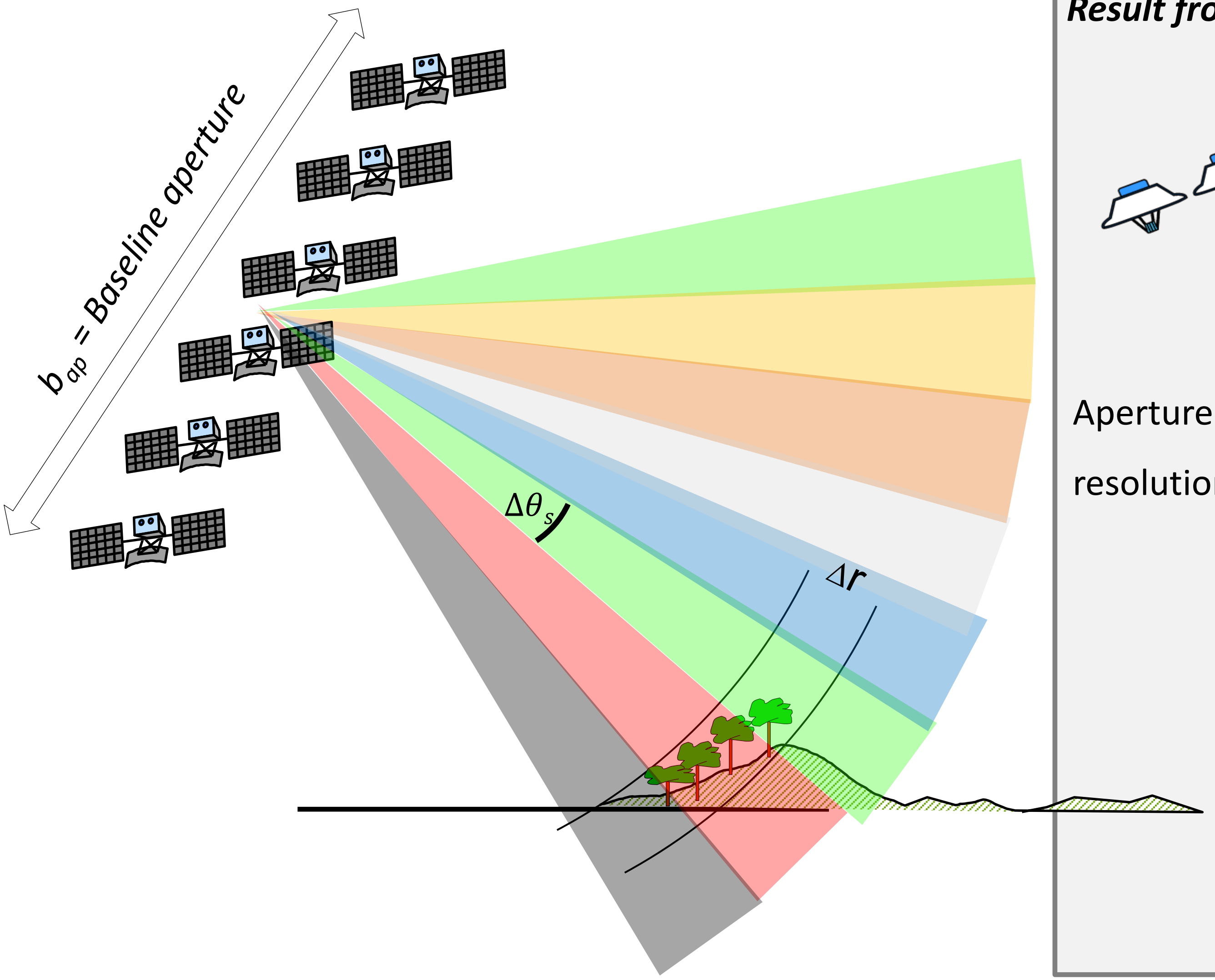


An antenna array is formed at each azimuth position

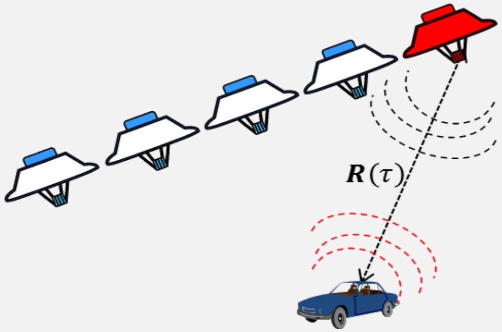
⇔ Resolution of targets at different elevation within each SAR range/azimuth resolution cell



TomoSAR Resolution



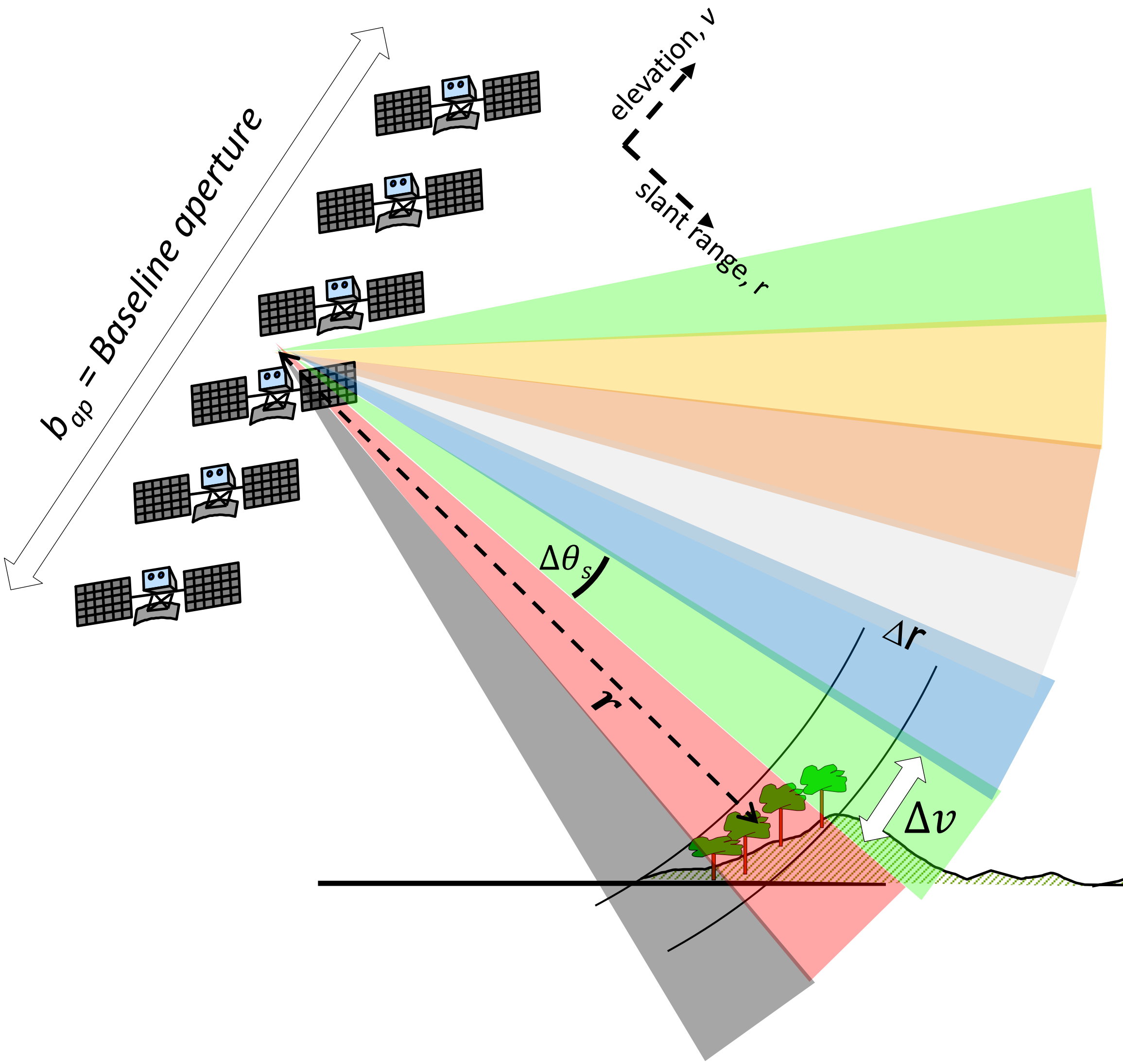
Result from array theory



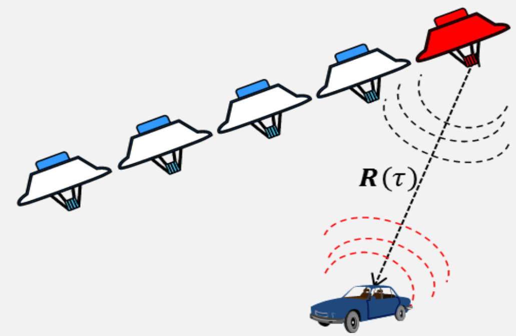
Aperture length translates to angular resolution according to

$$\Delta\theta_s \cong \frac{\lambda}{2b_{ap}}$$

TomoSAR Resolution



Result from array theory



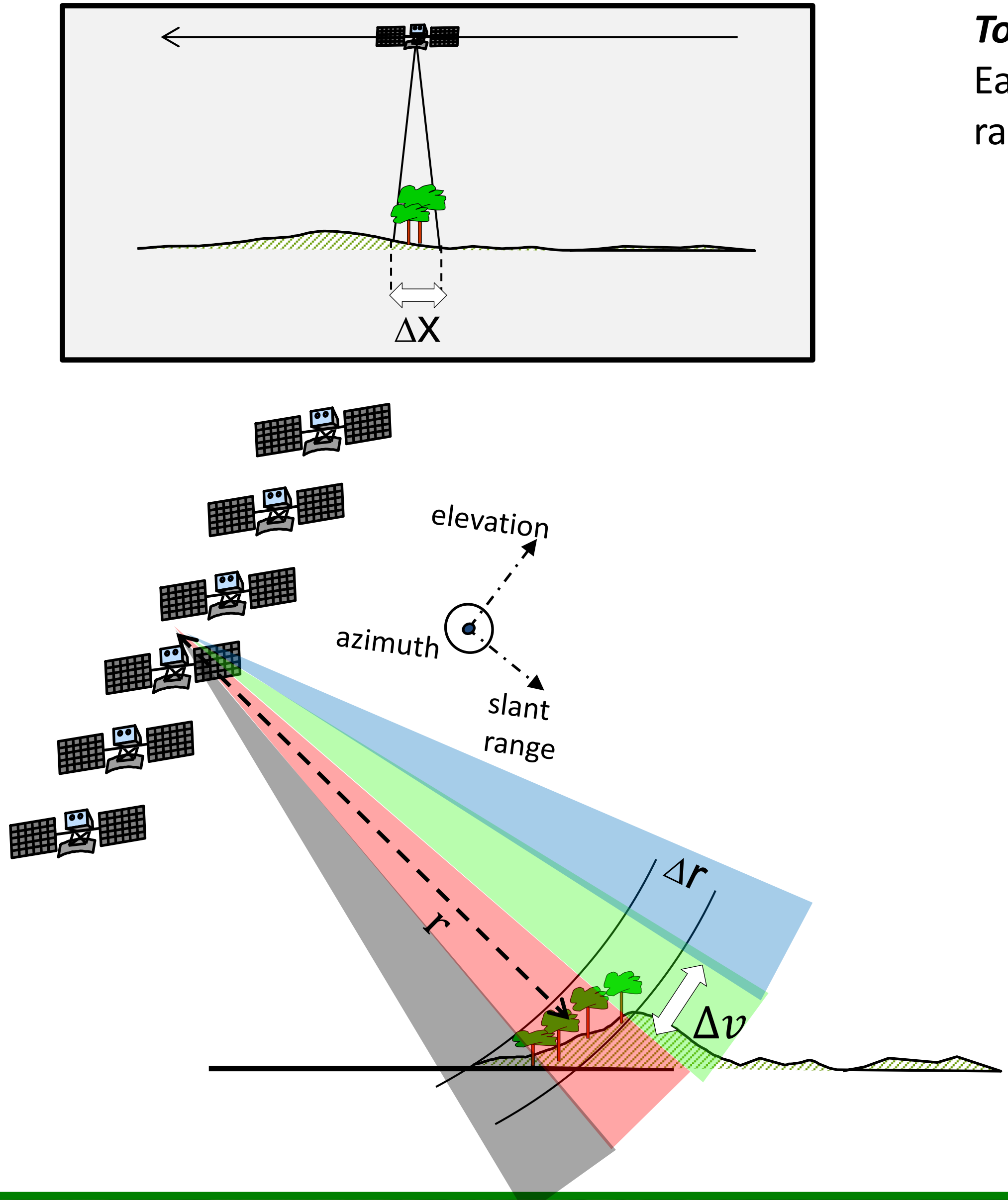
Aperture length translates to angular resolution according to

$$\Delta\theta_s \cong \frac{\lambda}{2b_{ap}}$$

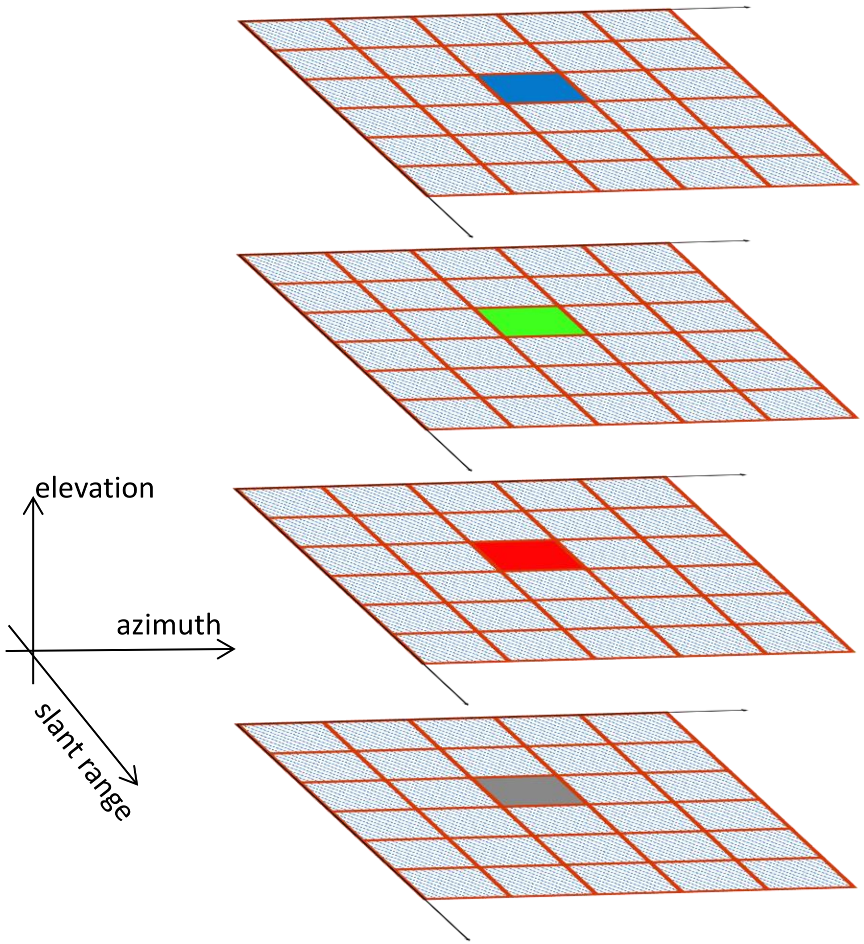
Angular resolution translates into **elevation** resolution according to

$$\Delta v \cong \Delta\theta_s \cdot r$$

TomoSAR Resolution Cell



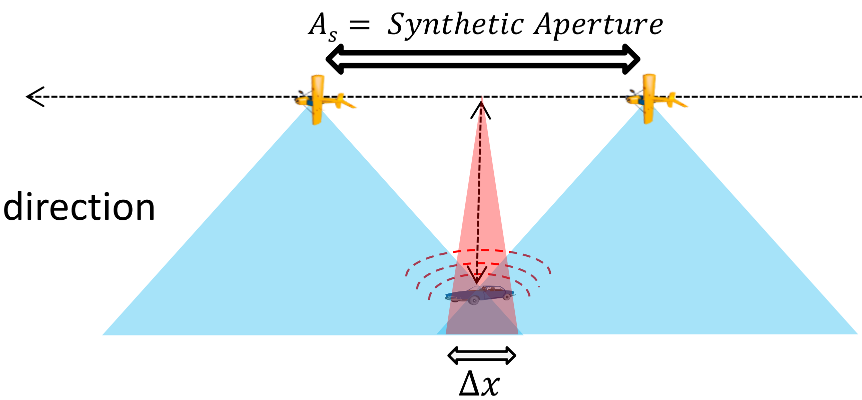
TomoSAR cube
Each voxel is associated with a range/azimuth/elevation resolution cell

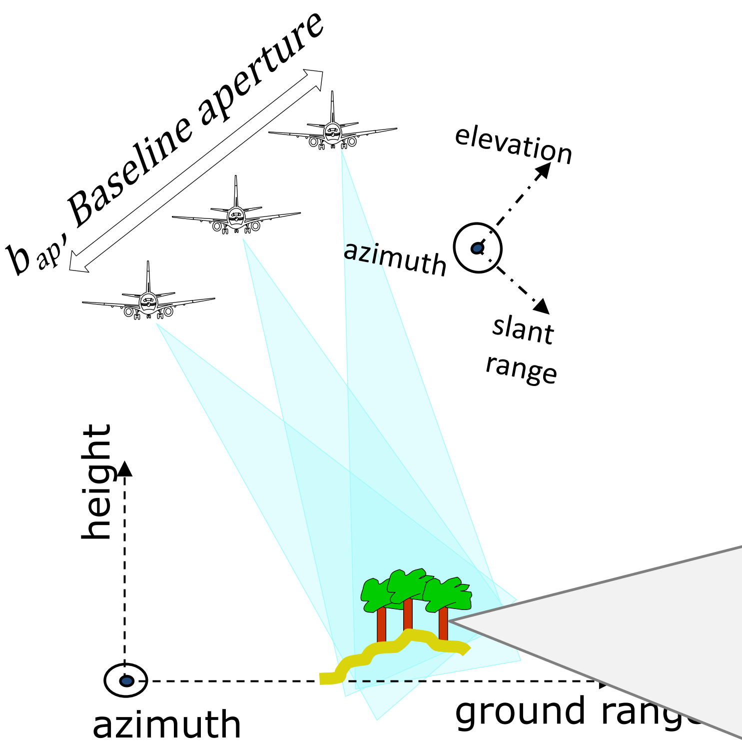


TomoSAR Resolution Cell

Resolution is determined by

- Pulse bandwidth along the slant range direction
- Along-track synthetic aperture length in the azimuth direction
- Baseline aperture in the elevation direction



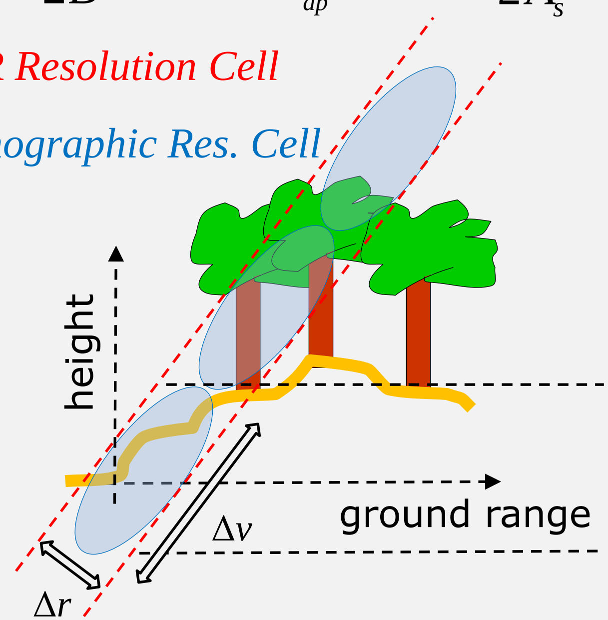


$$\Delta r = \frac{c}{2B} \qquad \Delta v = \frac{\lambda r}{2 b_{ap}} \qquad \Delta x = \frac{\lambda r}{2 A_s}$$

B: pulse bandwidth
λ: carrier wavelength

SAR Resolution Cell

Tomographic Res. Cell



For most systems:

$$\Delta v >> \Delta r, \Delta x$$

$$\Delta z \cong \Delta v \cdot \sin(\theta)$$

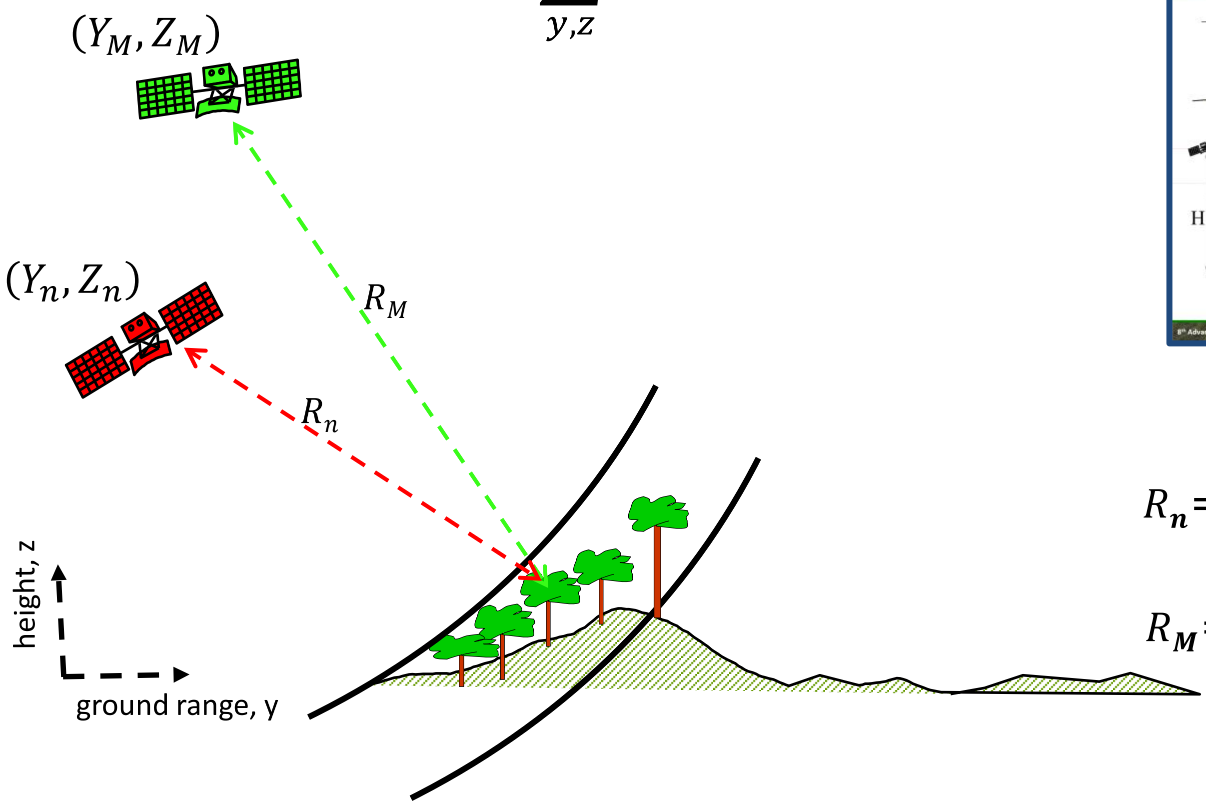
TomoSAR Processing

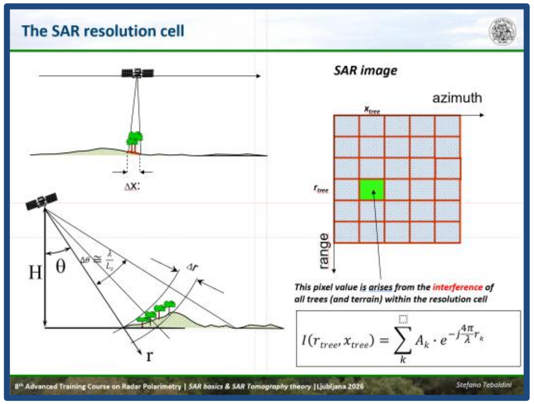
SAR pixel – multiple baseline model

SAR pixel = Sum of all elementary scatterer at different elevations within the same range/azimuth resolution cell

- Each elementary scatterer is phase-rotated according to its distance from the Radar

$$I_n(r,x) = \sum_{y,z} A(y,z) \cdot e^{-j\frac{4\pi}{\lambda}R_n(y,z)}$$





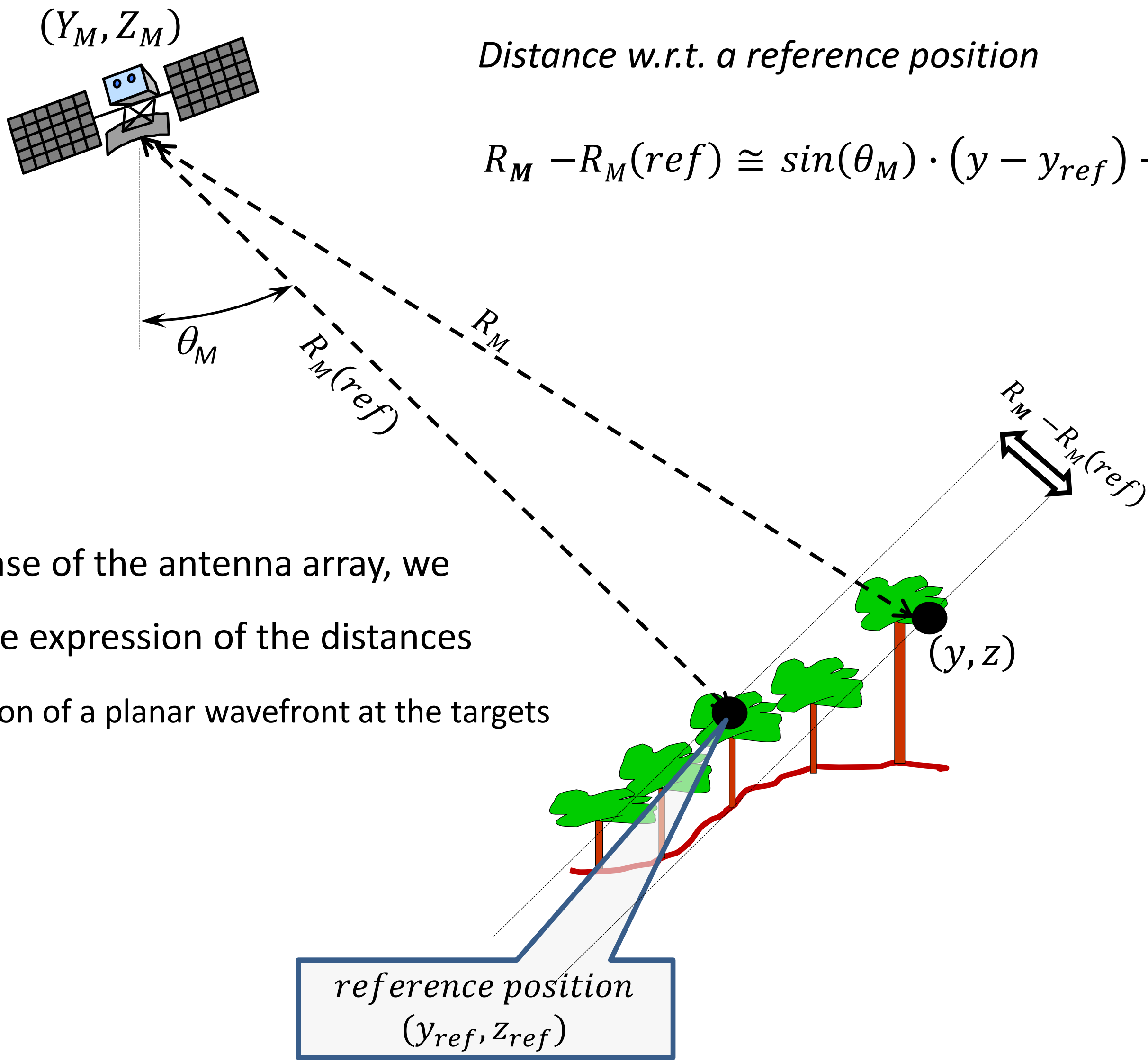
$$R_{\boldsymbol{n}} = \sqrt{(Y_n - y)^2 + (Z_n - z)^2}$$

$$R_{\boldsymbol{M}} = \sqrt{(Y_M - y)^2 + (Z_M - z)^2}$$

SAR pixel – multiple baseline model

Distance w.r.t. a reference position

$$R_{\boldsymbol{M}} - R_M(ref) \cong sin(\theta_M) \cdot (y - y_{ref}) - cos(\theta_M) \cdot (z - z_{ref})$$



As in the case of the antenna array, we linearize the expression of the distances

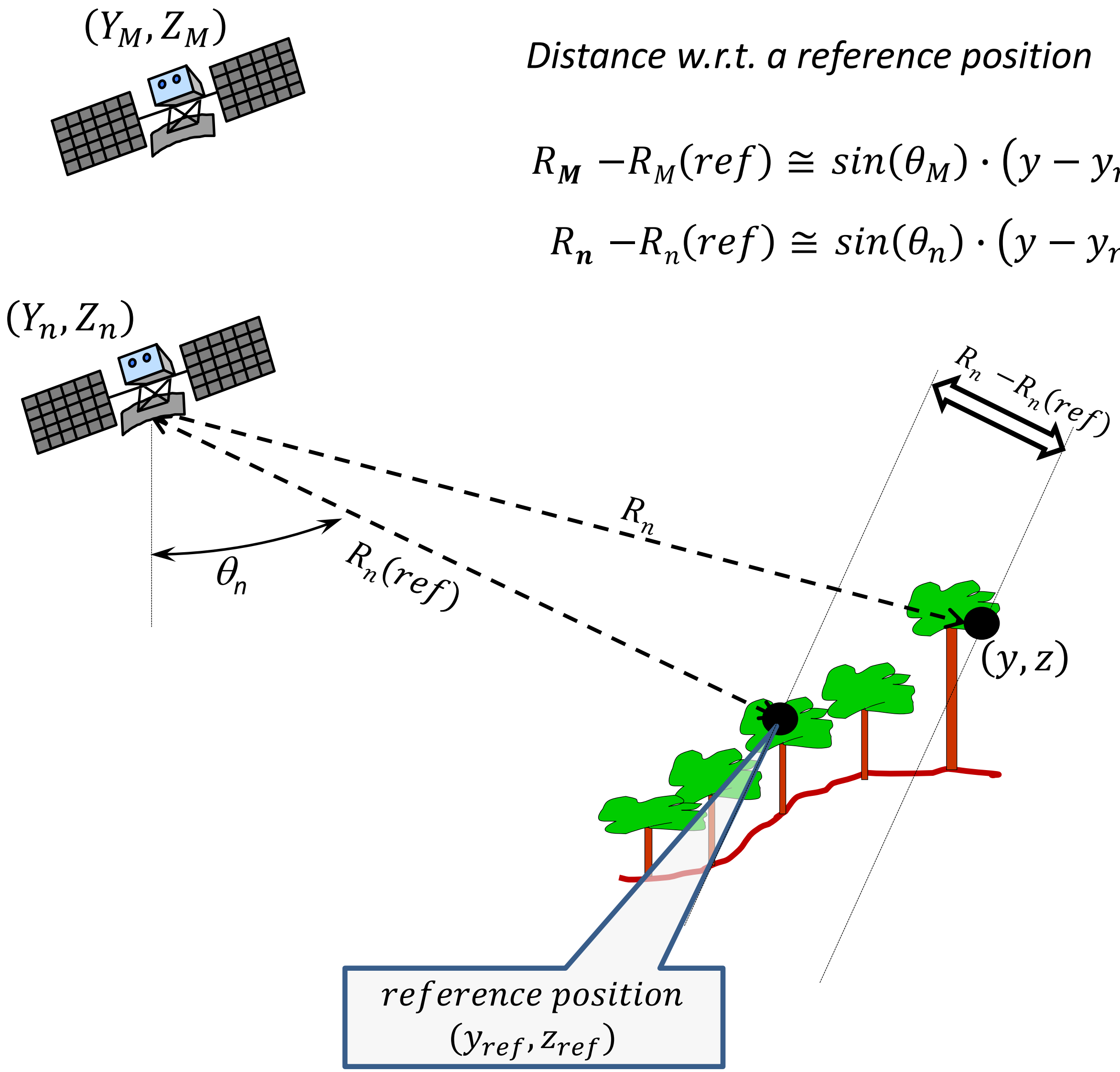
⬄ Assumption of a planar wavefront at the targets

SAR pixel – multiple baseline model

Distance w.r.t. a reference position

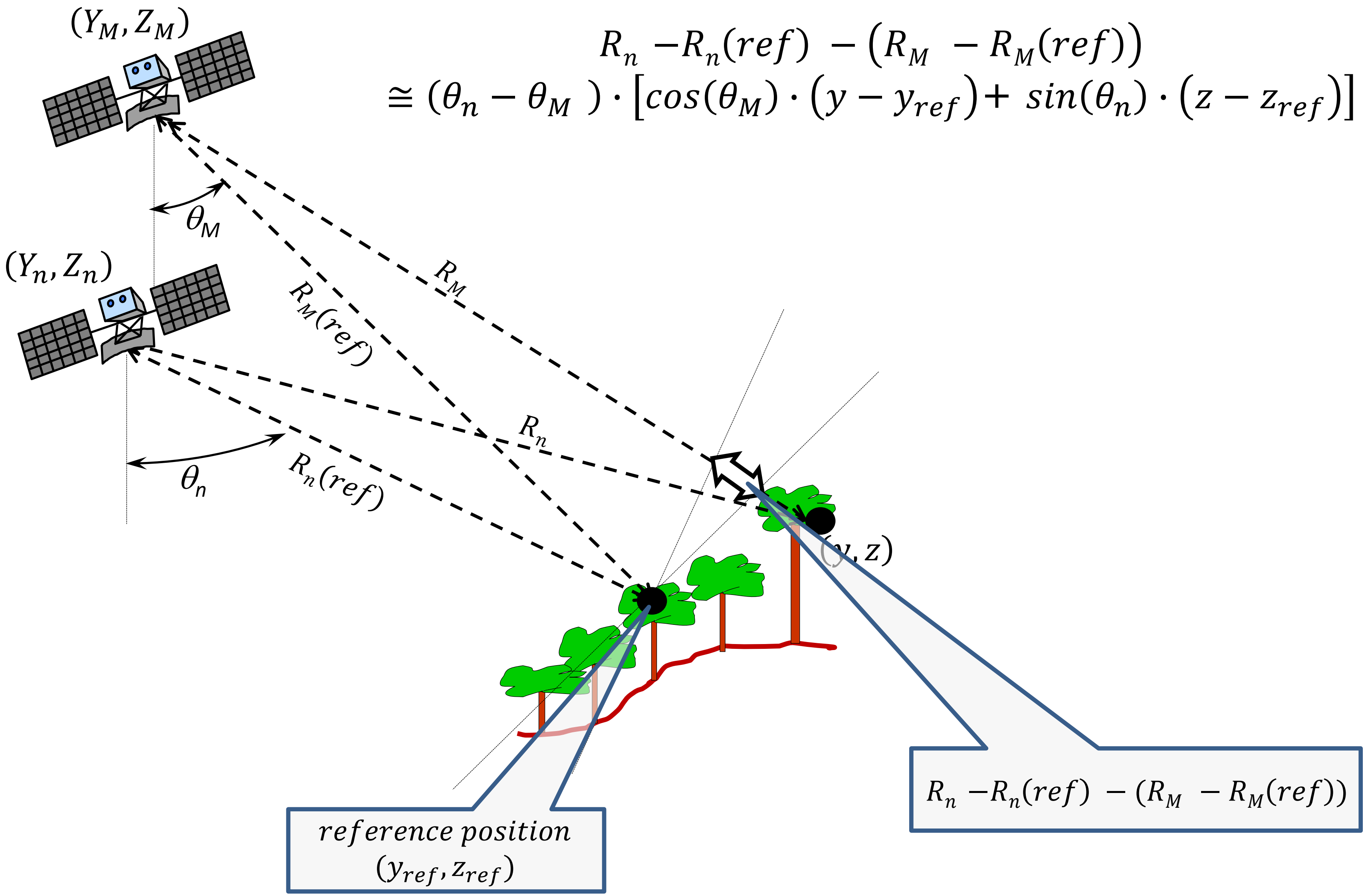
$$R_{\boldsymbol{M}} - R_M(ref) \cong sin(\theta_M) \cdot (y - y_{ref}) - cos(\theta_M) \cdot (z - z_{ref})$$

$$R_{\boldsymbol{n}} - R_n(ref) \cong sin(\theta_n) \cdot (y - y_{ref}) - cos(\theta_n) \cdot (z - z_{ref})$$



SAR pixel – multiple baseline model

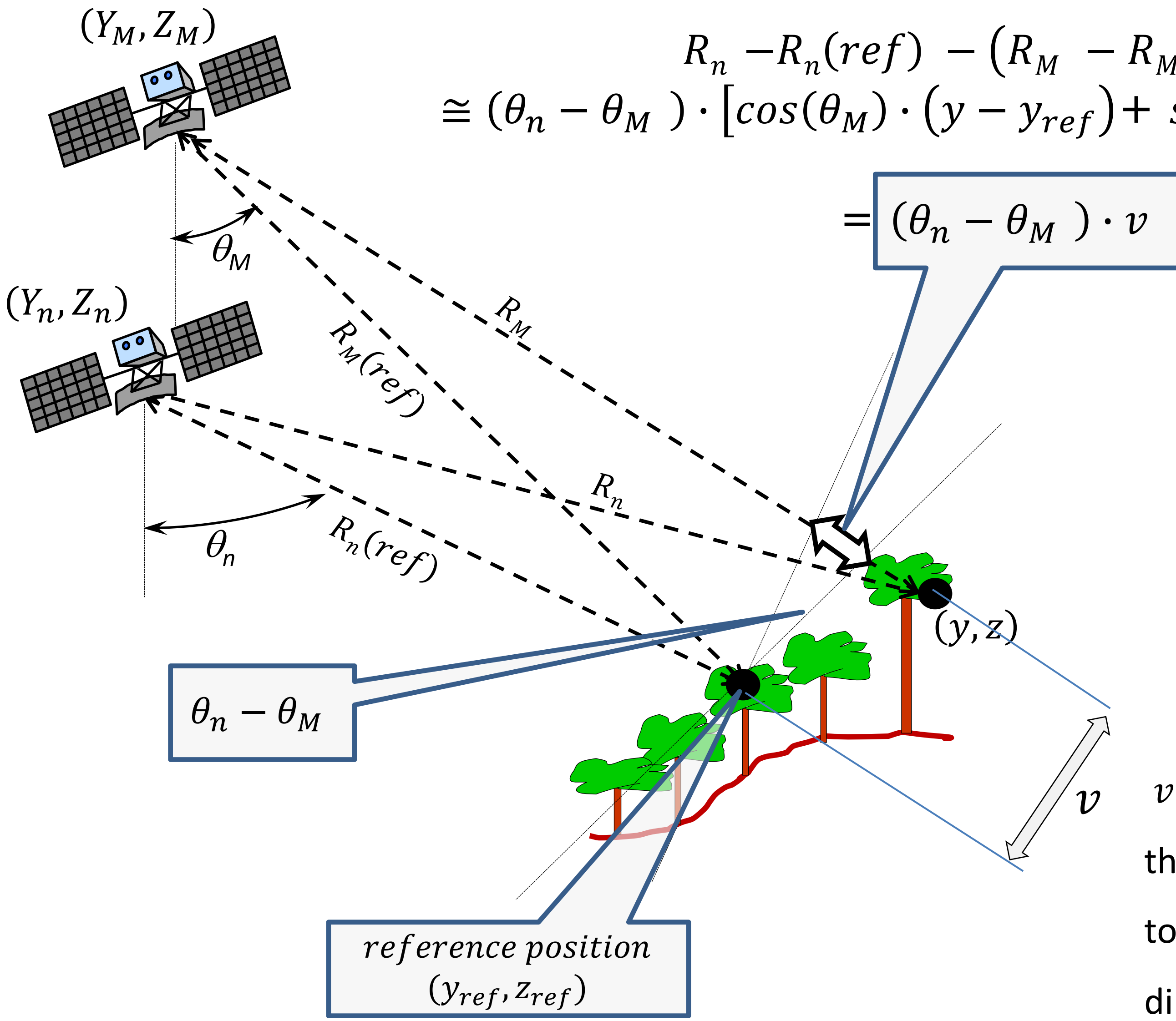
$$R_n - R_n(ref) - (R_M - R_M(ref))$$
$$\cong (\theta_n - \theta_M) \cdot [cos(\theta_M) \cdot (y - y_{ref}) + sin(\theta_n) \cdot (z - z_{ref})]$$



Stefano Tebaldini

SAR pixel – multiple baseline model

$$R_n - R_n(ref) - \left(R_M - R_M(ref)\right)$$
$$\cong (\theta_n - \theta_M) \cdot \left[\cos(\theta_M) \cdot (y - y_{ref}) + \sin(\theta_n) \cdot (z - z_{ref})\right]$$



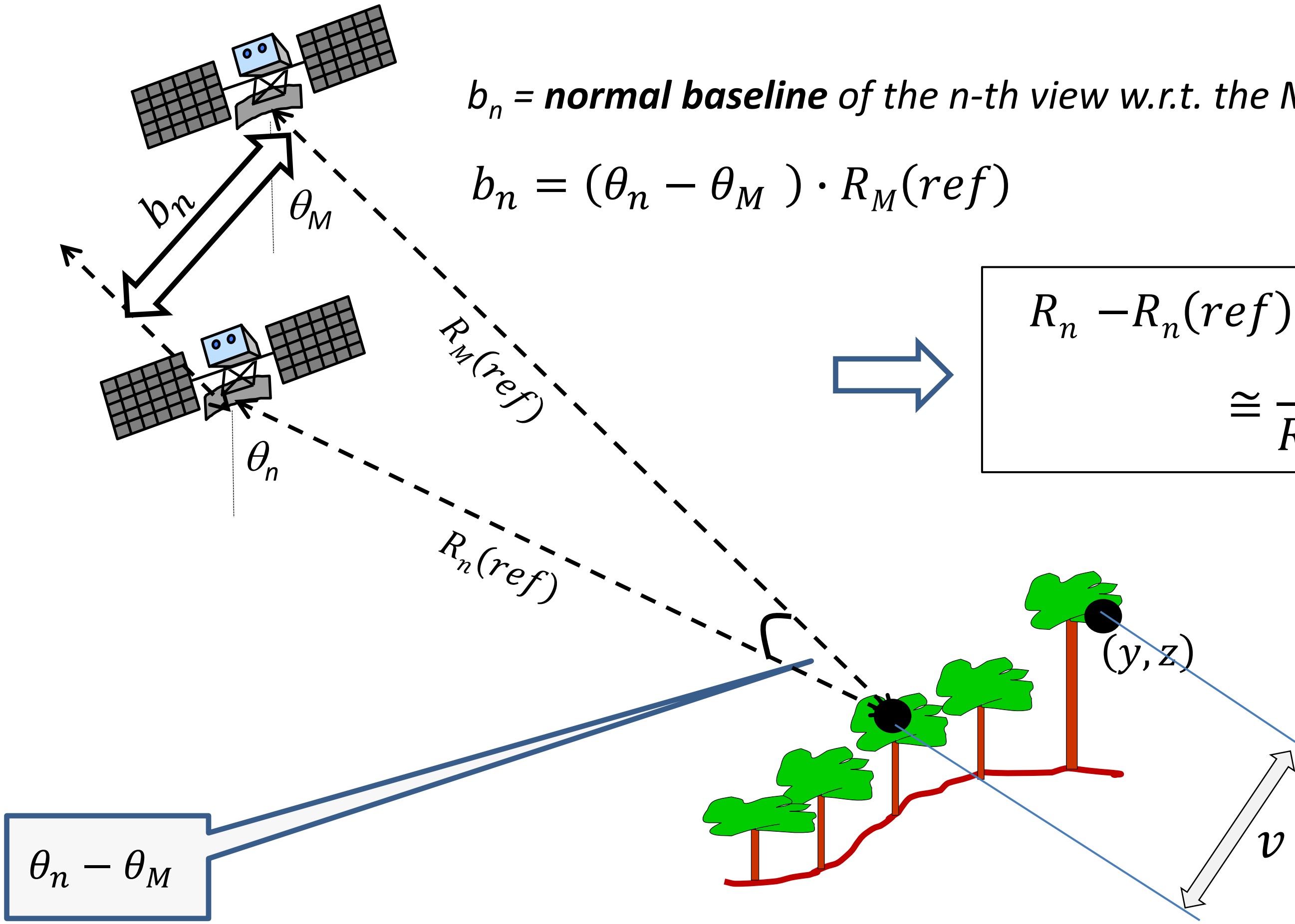
v is the displacement of the target at (y, z) w.r.t. to (y_{ref}, z_{ref}) in the direction of elevation

SAR pixel – multiple baseline model

b_n = ***normal baseline** of the n-th view w.r.t. the Master view*

$$b_n = (\theta_n - \theta_M) \cdot R_M(ref)$$

$$R_n - R_n(ref) - \left(R_M - R_M(ref)\right) \cong \frac{b_n}{R_M(ref)} \cdot v$$

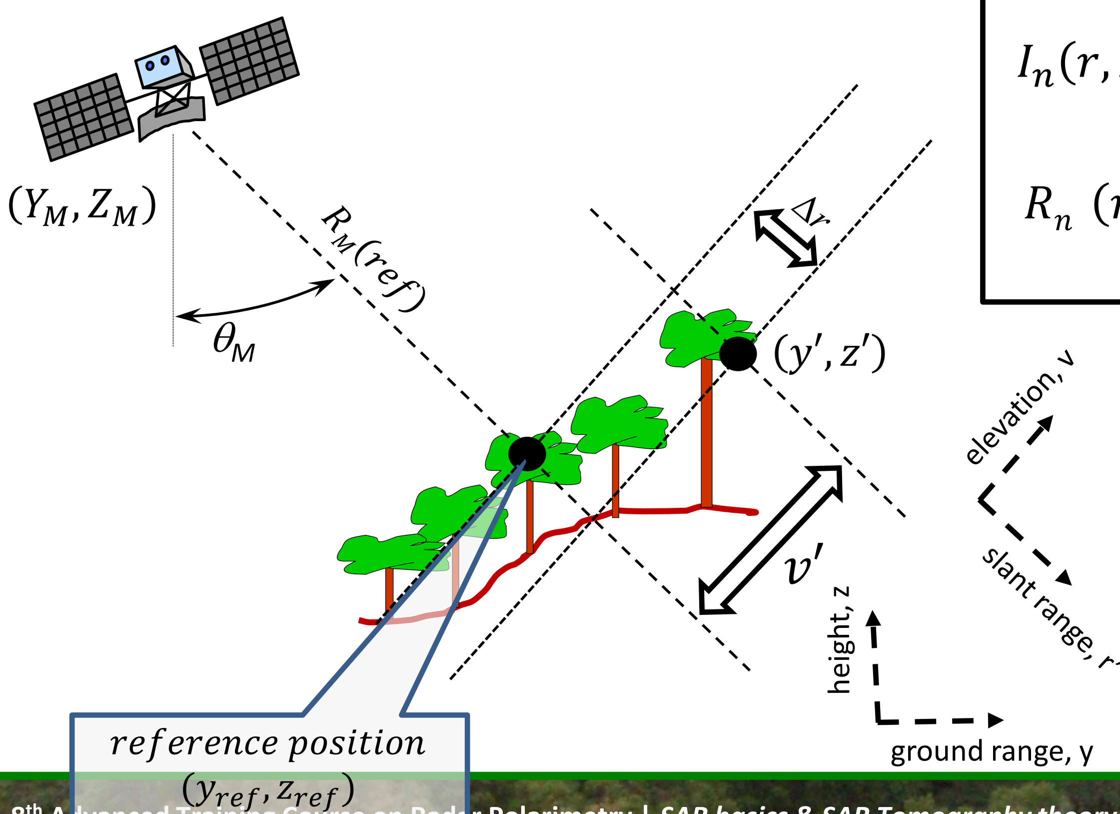


SAR pixel – multiple baseline model

The approximations above allow restating the SAR model in a new Cartesian coordinate system defined by

slant range, elevation with respect to a reference point and a reference orbit

- The reference position is typically taken as the *(x,y,z)* position of the SAR pixel when projected onto a given Digital Terrain Model
- The choice of the reference orbit is largely arbitrary



$$I_n(r,x) = \sum_{r',v} A(r',v) \cdot e^{-j\frac{4\pi}{\lambda}R_n(r',v)}$$

$$R_n(r',v) \cong R_n(ref) + r' + \frac{b_n}{R_M(ref)} \cdot v$$

SAR pixel – multiple baseline model

$$I_n(r,x) = exp\left\{-j\frac{4\pi}{\lambda}R_n(ref)\right\} \cdot \sum_{r',v} A(r',v) exp\left\{-j\frac{4\pi}{\lambda}r'\right\} \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v\right\}$$

SAR pixel – multiple baseline model

$$I_n(r,x) = exp\left\{-j\frac{4\pi}{\lambda}R_n(ref)\right\} \cdot \sum_{r',v} A(r',v)exp\left\{-j\frac{4\pi}{\lambda}r'\right\} \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v\right\}$$

Common terms for all baselines

SAR pixel – multiple baseline model

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Common terms for all baselines

Summing over r' we get

$$I_n(r,x) = exp\left\{-j\frac{4\pi}{\lambda}R_n(ref)\right\} \cdot \sum_{v} s(v) \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v\right\}$$

SAR pixel – multiple baseline model

$$I_n(r,x) = exp\left\{-j\frac{4\pi}{\lambda}R_n(ref)\right\}\cdot\sum_{r',v}A(r',v)exp\left\{-j\frac{4\pi}{\lambda}r'\right\}\cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v\right\}$$

Common terms for all baselines

Summing over r' we get

$$I_n(r,x) = exp\left\{-j\frac{4\pi}{\lambda}R_n(ref)\right\}\cdot\sum_{v}s(v)\cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v\right\}$$

Phase offset to be removed based on knowledge

of the acquisition geometry (terrain flattening)

SAR pixel – multiple baseline model

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Common terms for all baselines

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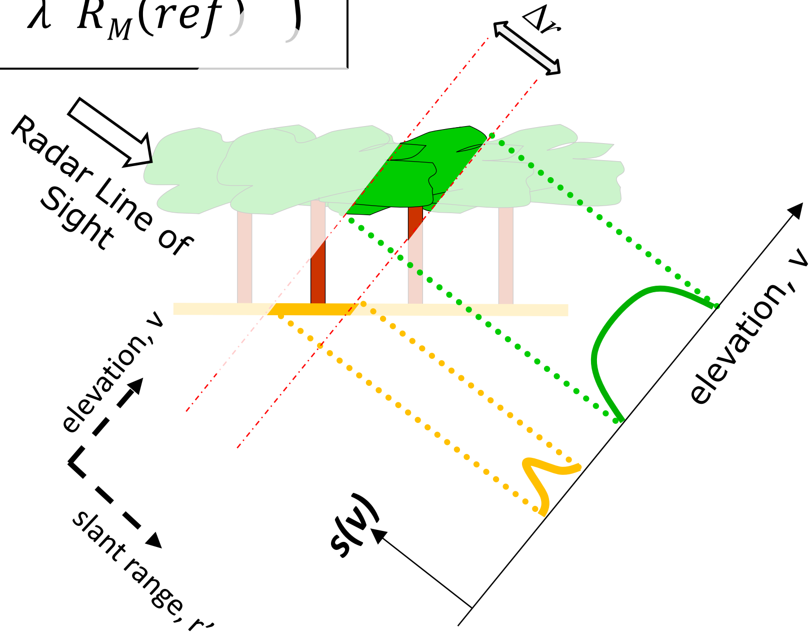
*Phase offset to be **removed** based on knowledge of the acquisition geometry (terrain flattening)*

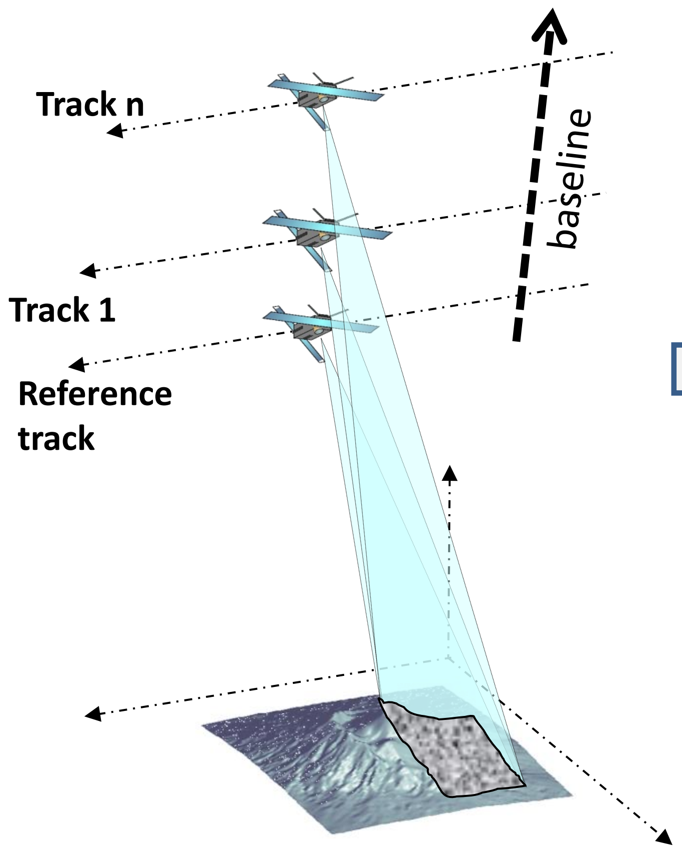
s(v) = projection of the scatterers along elevation

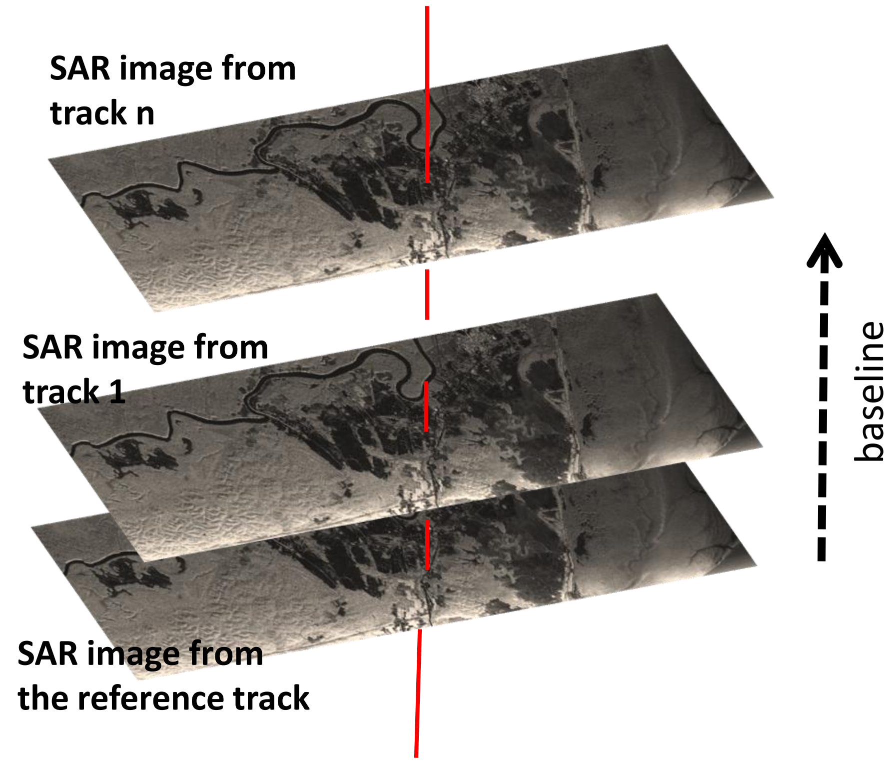


TomoSAR forward model

The SAR pixel in the *n-th* image can finally be expressed in a simple form as follows:

$$I_n(r,x) = \sum_v s(v) \cdot exp\{-j2\pi f_v b_n\} \quad with\ f_v = \frac{2}{\lambda}\frac{v}{R_M(ref)}$$





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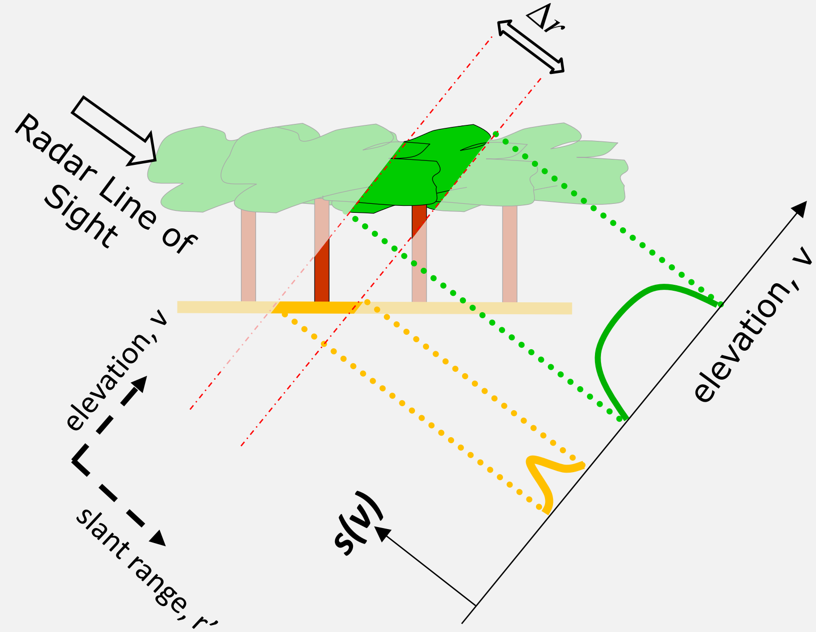
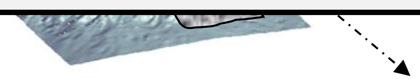
The signal obtained by taking the pixels at the same (r,x) location in a stack of SAR images is contributed by a sum of complex sinusoids

- The frequencies of the sinusoids correspond to the elevations v at which the targets are found
- The complex amplitude of the sinusoids are obtained by projecting the scatterers within the SAR resolution cell along elevation





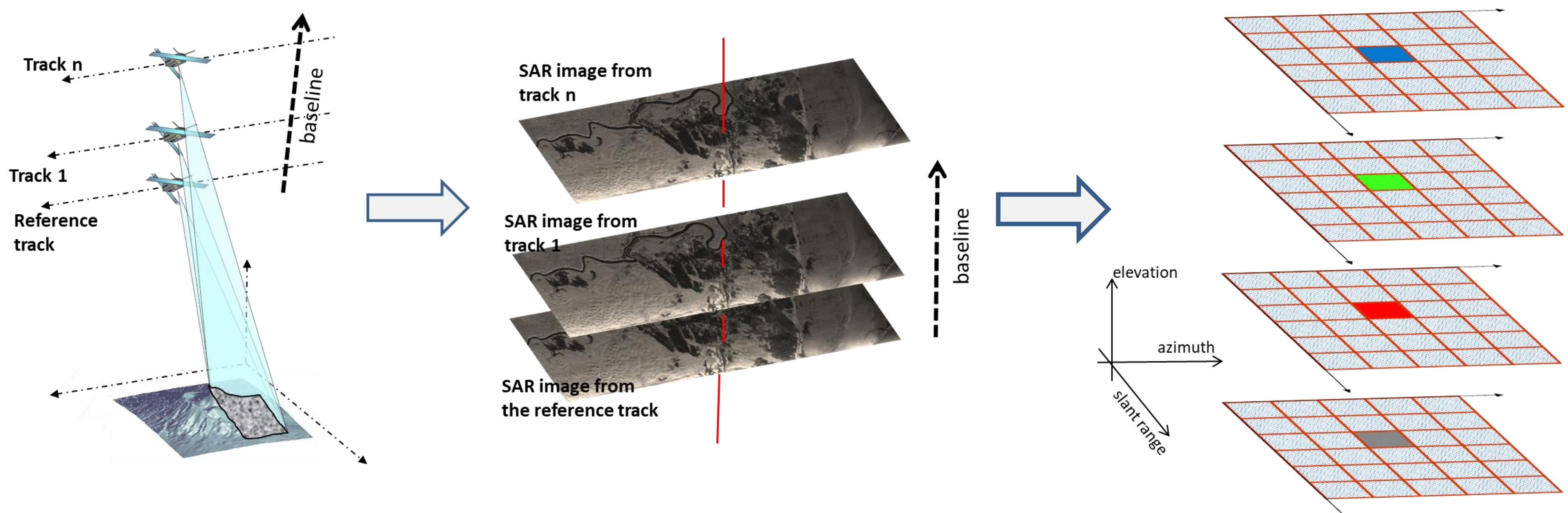
TomoSAR focusing algorithm

Tomographic focusing consists in retrieving the amplitudes $s(v)$ from the signal $I_n(r,x)$

$\Rightarrow$ As always, this is done by computing a Fourier Transform

$$T(r,x,v) = \sum_n I_n(r,x) \cdot exp\left\{ j \frac{2}{\lambda} \frac{v}{R_M(ref)} \right\}$$

Acquisition ***Stack of SAR images*** ***Tomographic voxels***



TomoSAR inversion w.r.t. height

TomoSAR forward model

$$I_n(r,x) = \sum_v s(v) \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v\right\}$$

$I_n(r,x)$: SLC pixel in the *n-th* image

s(r,x,v): projection of the scatterers along elevation

b_n : normal baseline for the *n-th* image

λ : carrier wavelength

Change of variable from cross range to height

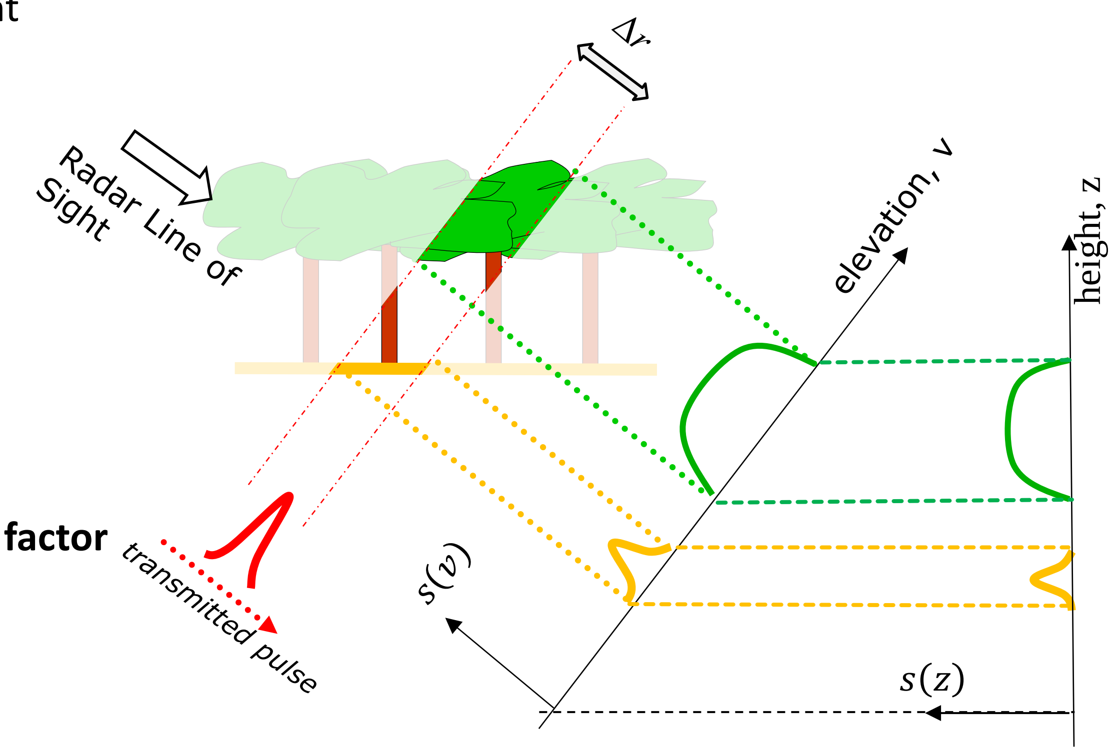
$$z = v \cdot \sin\theta$$

$$\Downarrow$$

$$I_n(r,x) = \sum_z s(z) \cdot exp\{-jk_z(n)z\}$$

k_z is usually referred to as **interferometric wavenumber** or **phase to height conversion factor**

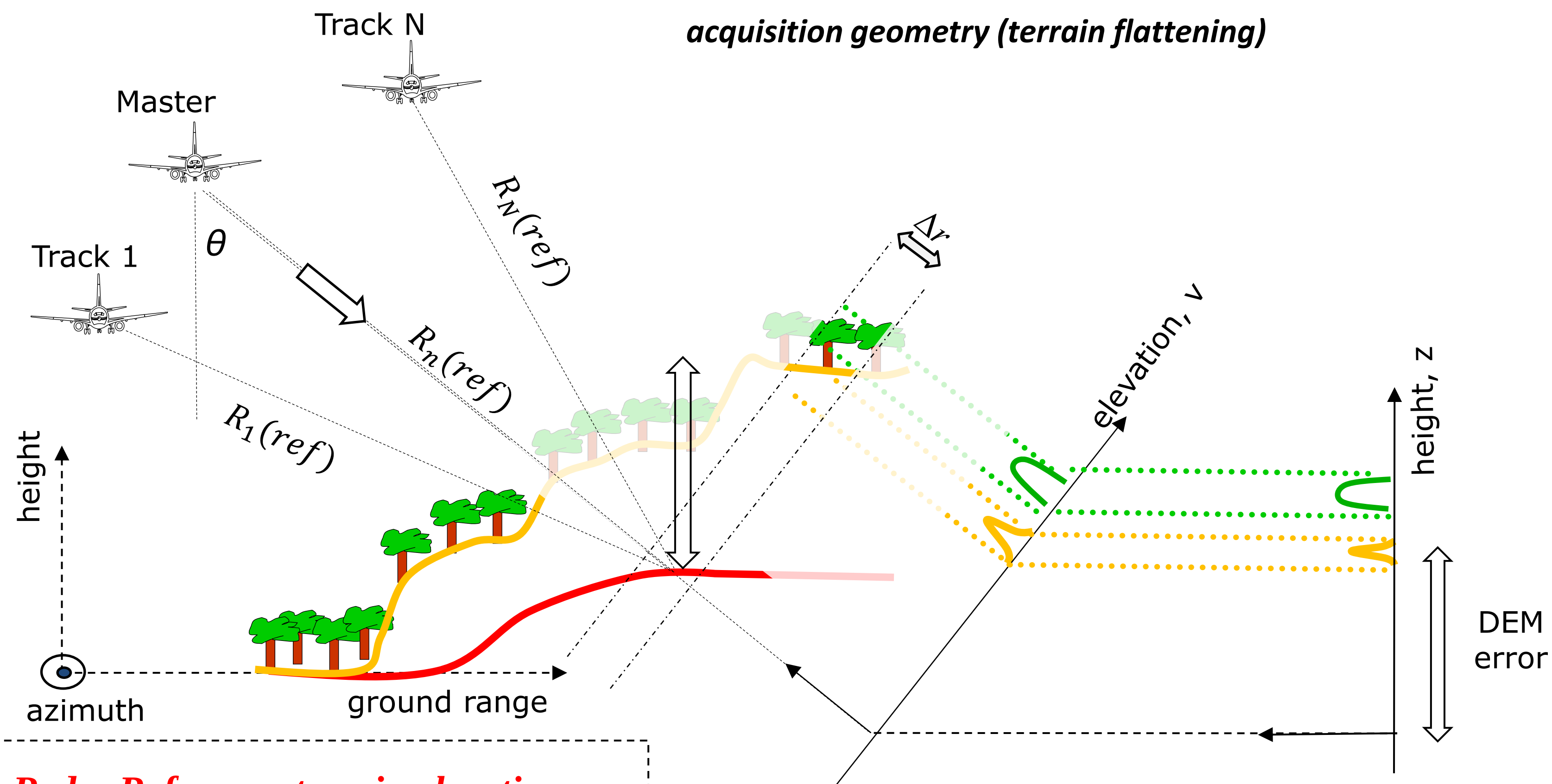
$$k_z(n) = \frac{4\pi}{\lambda R_M(ref)}\frac{b_n}{\sin\theta}$$



Terrain flattening

$$I_n(r,x) = exp\left\{-j\frac{4\pi}{\lambda}R_n(ref)\right\}\cdot\sum_z s(z)\cdot exp\{-jk_z(n)z\}$$

Phase offset to be removed based on knowledge of the acquisition geometry (terrain flattening)



Red = Reference terrain elevation

Orange = True terrain elevation

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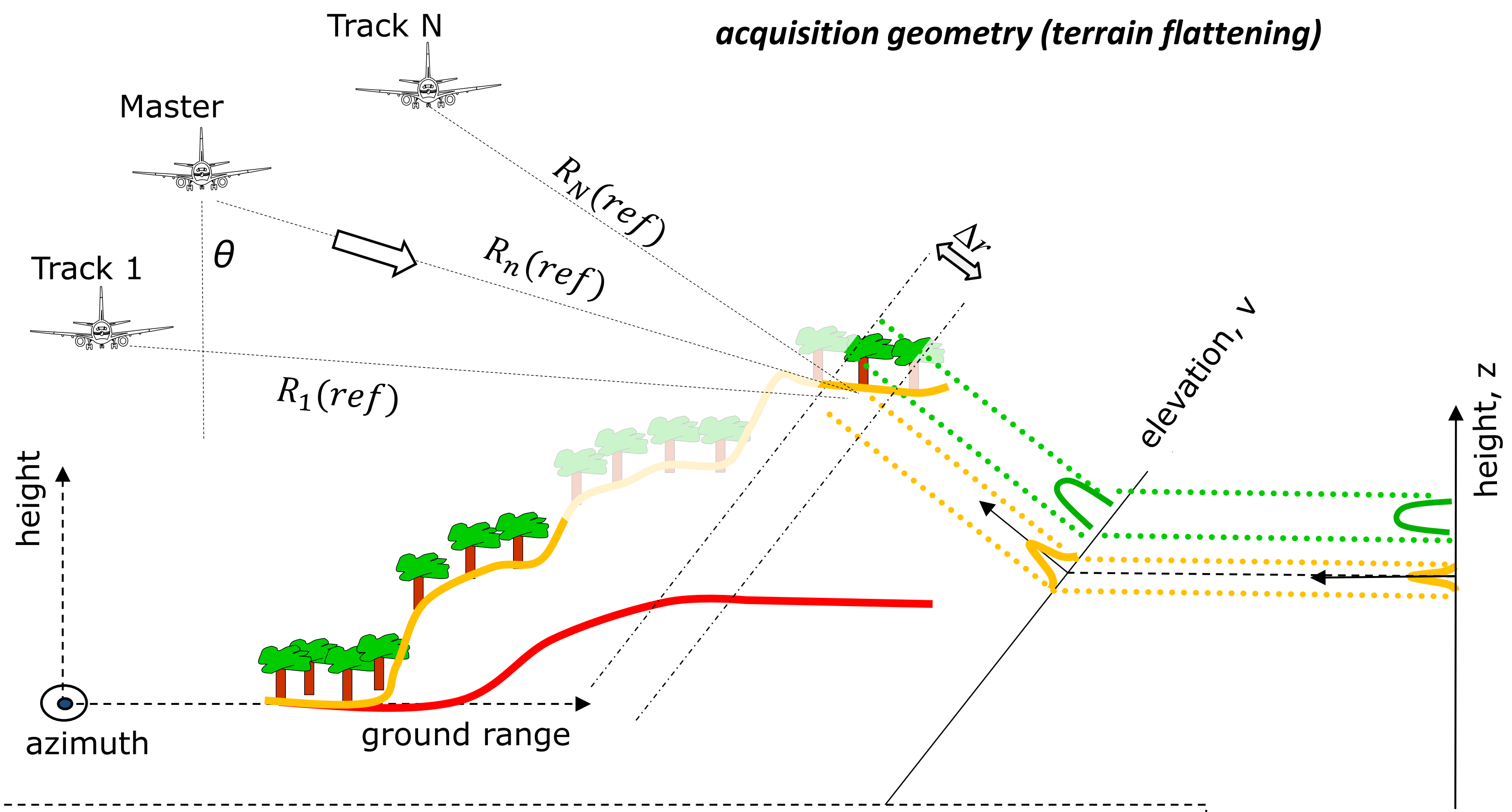
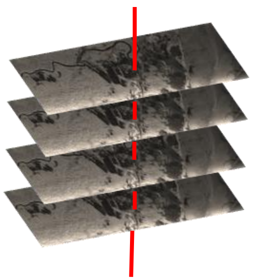


Orange = Reference terrain elevation = True terrain elevation

Advanced TomoSAR

Current paradigm for forested areas: **retrieve the *vertical distribution of backscattered power* based on the observed *InSAR coherences***

SLC data



Complex data vector (Nx1)

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_N \end{bmatrix}$$

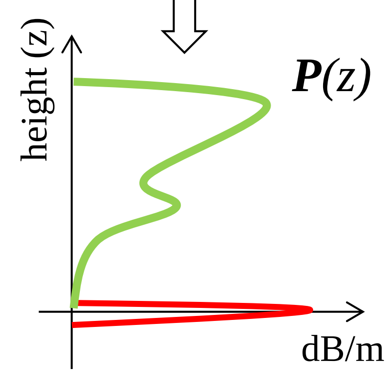
Coherence estimation (spatial multilooking)

$$\gamma_{nm} = \frac{\langle d_n {d_m}^* \rangle_L}{\sqrt{\langle |d_n|^2 \rangle_L \langle |d_m|^2 \rangle_L}}$$

Coherence matrix (NxN)

$$\mathbf{R} = \begin{bmatrix} 1 & \gamma_{12} & \cdots & \gamma_{1N} \\ \gamma_{21} & 1 & & \\ \vdots & & \ddots & \vdots \\ \gamma_{N1} & & \cdots & 1 \end{bmatrix}$$

TomoSAR processing



Why ?

- Equivalent to Fourier processing if inversion is carried out using linear methods
- **Non-linear methods can be used to achieve:**
  - **Super-resolution** (super = finer than the limit from baseline aperture)
  - **Better imaging quality** (lower sidelobes, SNR, etc…)

Advanced TomoSAR

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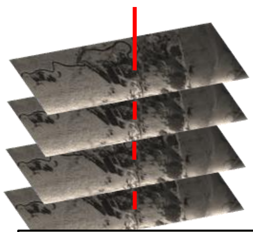

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Later in this course

W

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