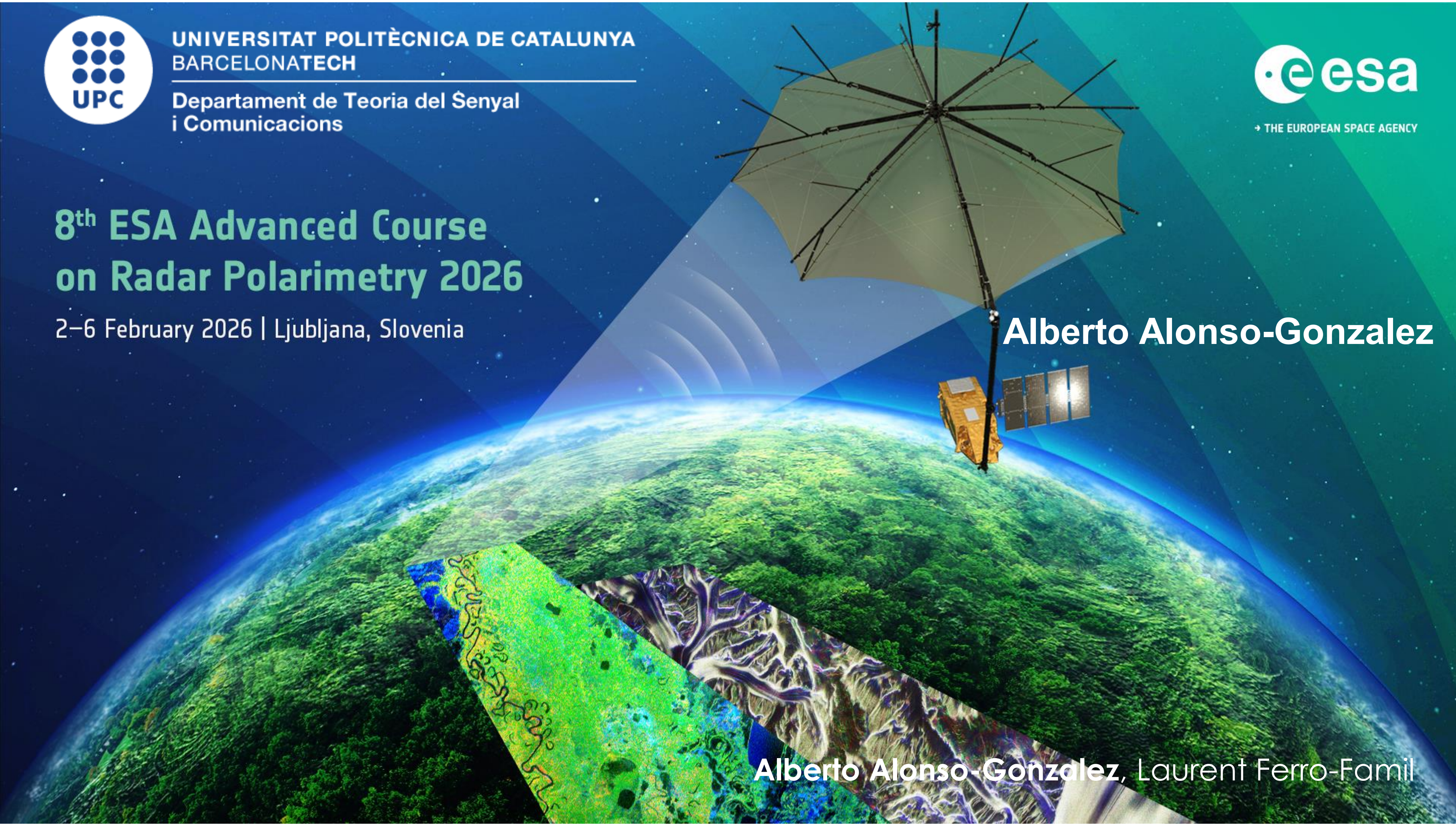


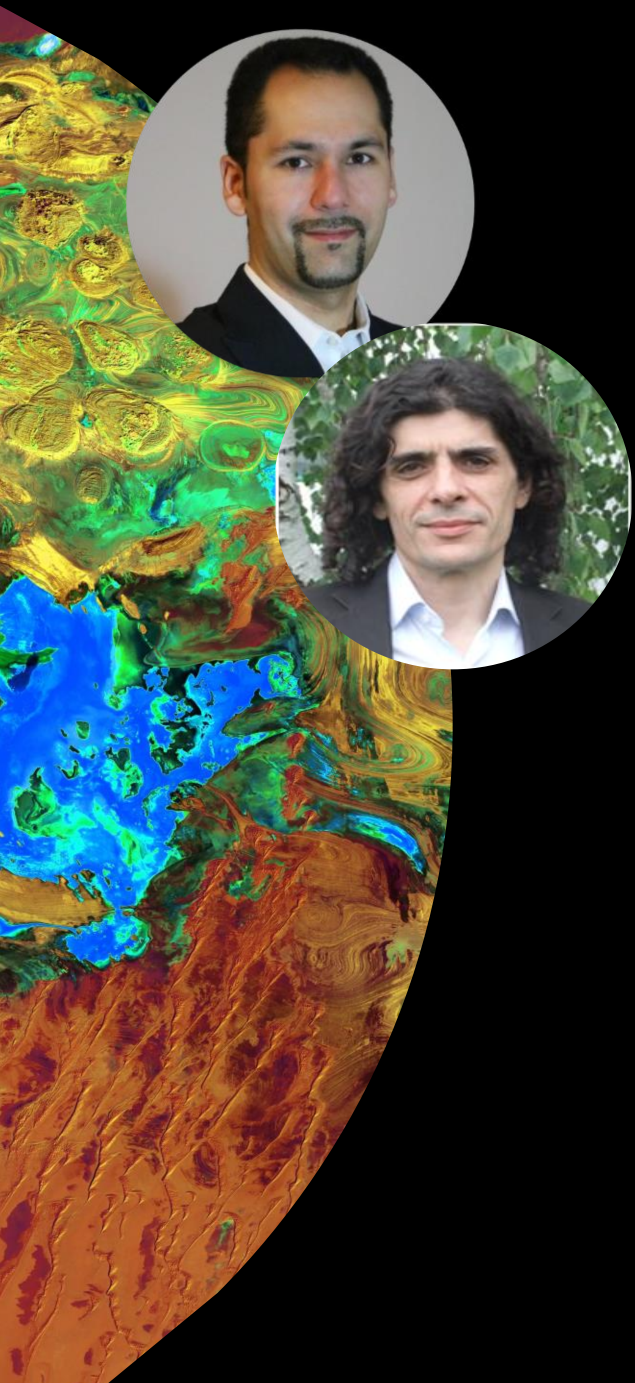
Alberto Alonso-Gonzalez

Alberto Alonso-Gonzalez, Laurent Ferro-Famil

SAR Statistics and Polarimetric decompositions

8th Advanced Training Course on Radar Polarimetry
Ljubljana, Slovenia
February 2026

Universitat Politècnica de Catalunya – UPC
Signal Theory and Communications Department – TSC
Remote Sensing Laboratory - RSLab., Barcelona, Spain
alberto.alonso-gonzalez@upc.edu



Alberto Alonso-Gonzalez

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Departament de Teoria del Senyal
i Comunicacions

Laurent FERRO-FAMIL

ISAE SUPAERO – Institut Supérieur de l'Aéronautique et de l'Espace
CESBIO
Université Fédérale Toulouse Midi-Pyrénées

alberto.alonso-gonzalez@upc.edu
laurent.ferro-famil@isae-supaero.fr

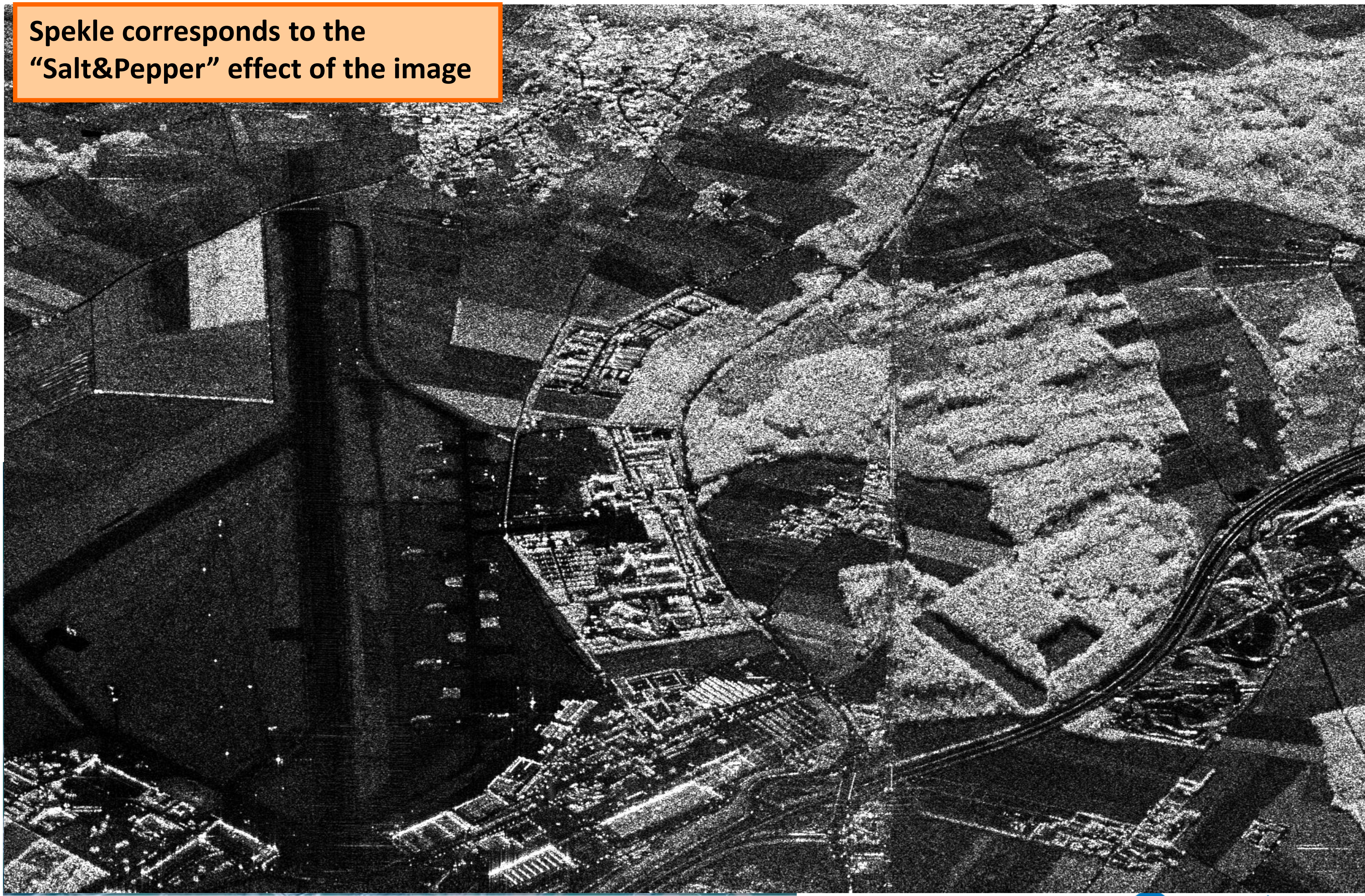
Outline

- SAR Data Statisitical Characterization
- PolSAR Data Statistical Characterization
- Information Estimation/Filtering

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory
and Communications

Speckle Noise

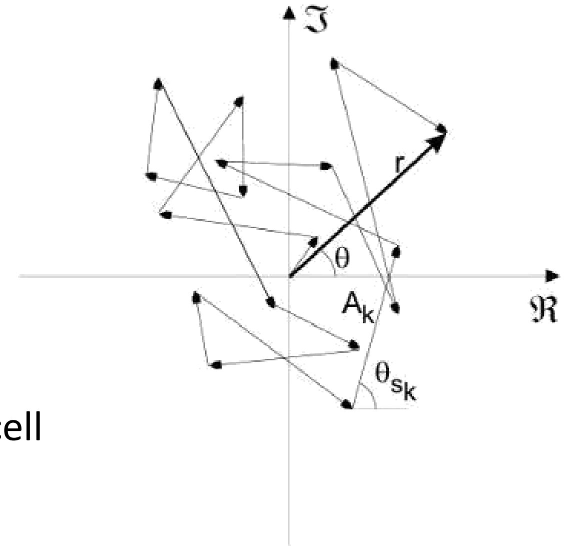
Spekle corresponds to the "Salt&Pepper" effect of the image



|Svv|

Speckle Noise

On the basis of the discrete scatter description



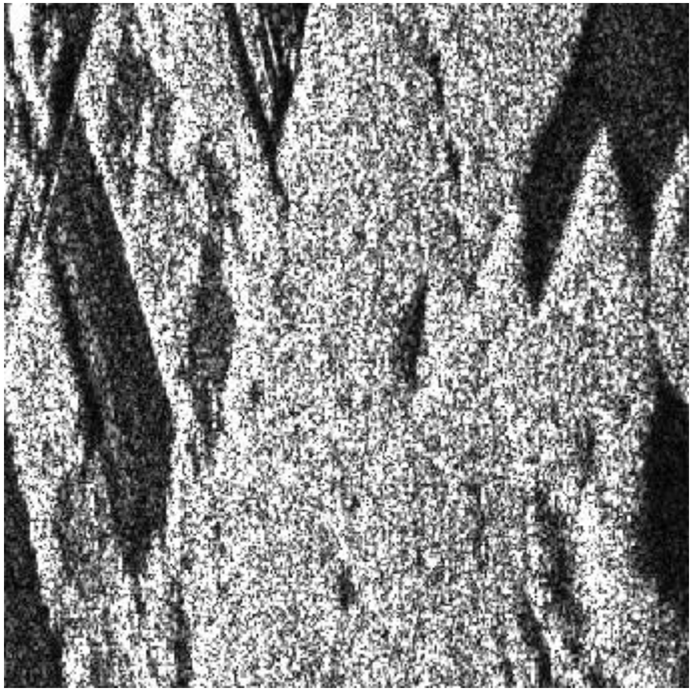
$$S(x,r) = \frac{1}{\sqrt{L}} \sum_{k=1}^{L} \sqrt{\sigma_k}\, e^{j\theta_k} h(x - x_k, r - r_k)$$

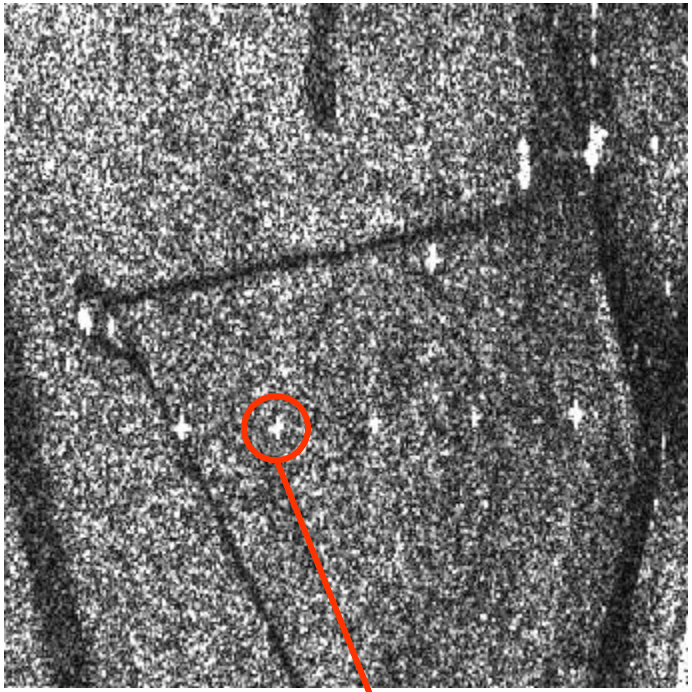
L: Number of point scatters embraced by the resolution cell

- L as a deterministic quantity
  - L = 1: or a dominating point scatter: **Deterministic scattering**
    - Rice/Rician model
  - L >1: Partially developed speckle
    - Not solved model. Even numerical solution difficult
  - L >>1: **Fully developed speckle**
    - Gaussian model
- L as a stochastic quantity
  - L characterized by a pdf: Image texture
    - K-distribution model

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory
and Communications

Fully Developed Speckle Noise





Fully Developed speckle

Bright points: Points where the interference is constructive

Dark points: Points where the interference is destructive

Corner reflector
Dominant scatter
No speckle

Speckle is an electromagnetic interference phenomenon

DLR

S_{hh} amplitude
E-SAR L-band system

Fully Developed Speckle Noise

- Completely developed Speckle (large L and no dominant scatterer)
  - When the number of scatterers is sufficiently large, and their spatial distribution is random (random phases) the central limit theorem applies

$$S = \mathrm{N}_{C^2}\left(0, \sigma^2/2\right)$$

- Central Limit Theorem
  - **Real Part**

$$p_{\Re\{S\}}\left(\Re\{S\}\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\left(\frac{\Re\{S\}}{\sigma}\right)^2\right) \quad \Re\{S\} \in (-\infty, \infty)$$ Gaussian pdf

  - **Imaginary Part**

$$p_{\Im\{S\}}\left(\Im\{S\}\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\left(\frac{\Im\{S\}}{\sigma}\right)^2\right) \quad \Im\{S\} \in (-\infty, \infty)$$ Gaussian pdf

  - Real and imaginary parts are uncorrelated $E\left\{\Re\{S\}\Im\{S\}\right\} = 0$

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

Fully Developed Speckle Noise

- **Amplitude**: Rayleigh pdf

$$p_r(r) = \frac{r}{\sigma^2} \exp\left(-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2\right) \quad r \in [0, \infty)$$

$$E\{r\} = \sqrt{\frac{\pi}{2}}\sigma$$

$$E\{r^2\} = 2\sigma^2$$

$$\sigma_r^2 = E\{r^2\} - E^2\{r\} = \left(2 - \frac{\pi}{2}\right)\sigma^2$$

- **Intensity** ($I=r^2$): Exponential pdf

$$p_I(I) = \frac{1}{2\sigma^2} \exp\left(-\frac{I}{2\sigma^2}\right) \quad I \in [0, \infty)$$

$$E\{I\} = 2\sigma^2 \equiv \sigma$$

$$E\{I^2\} = 2\left(2\sigma^2\right)^2$$

$$\sigma_I^2 = E\{I^2\} - E^2\{I\} = \left(2\sigma^2\right)^2$$

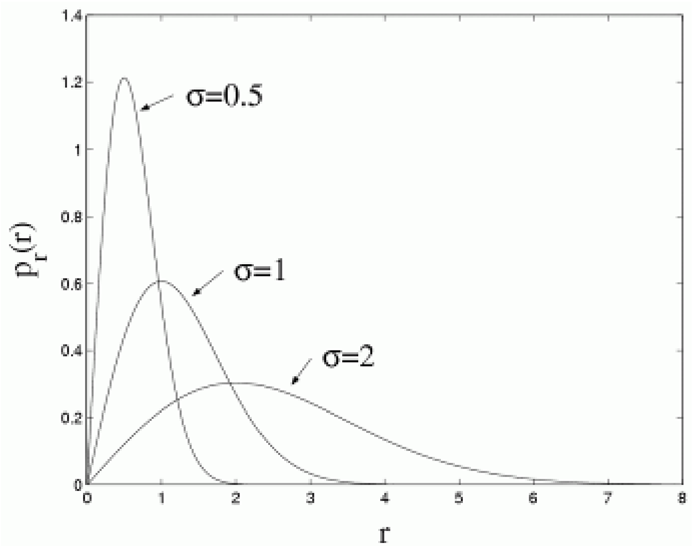
- **Phase**: Uniform pdf. Contains NO information

$$p_\theta(\theta) = \frac{1}{2\pi} \quad \theta \in [-\pi, \pi)$$

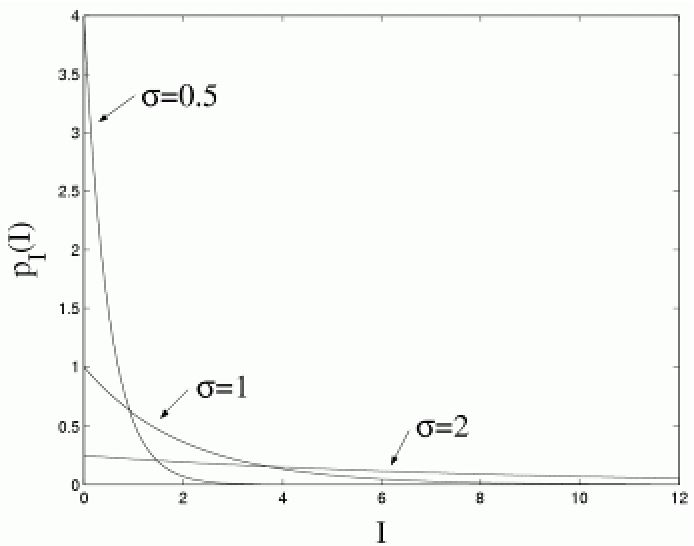
- Amplitude and phase are uncorrelated

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

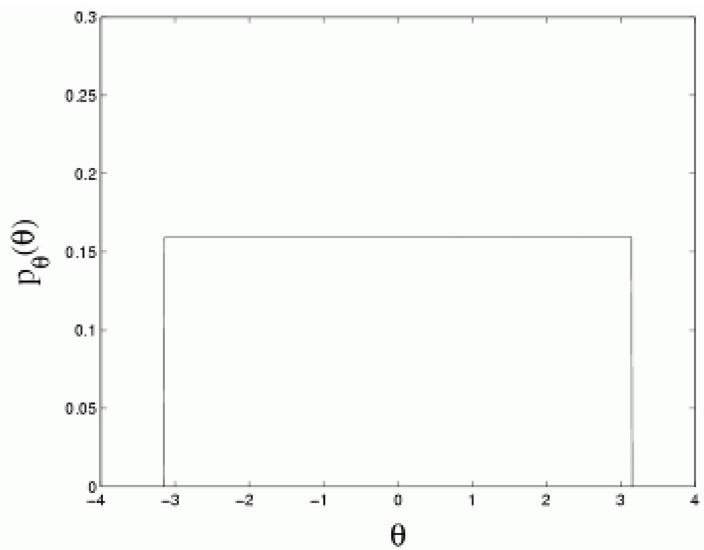
Fully Developed Speckle Noise



Amplitude: Rayleigh pdf



Intensity: Exponential pdf



Phase: Uniform pdf

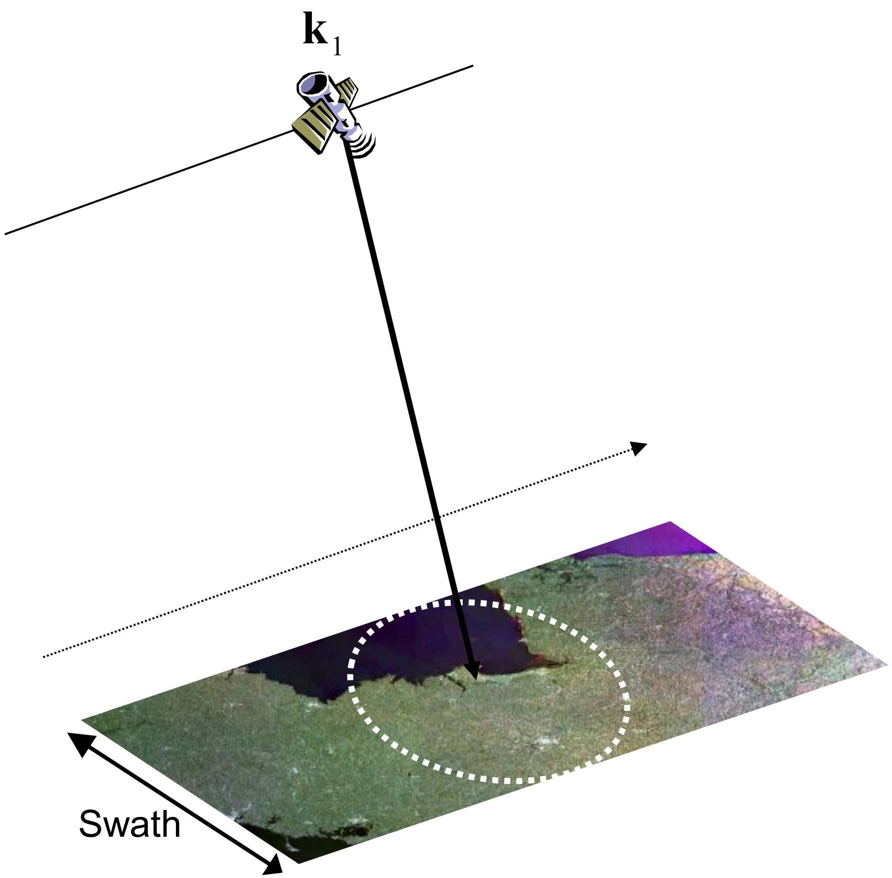
UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

Polarimetric SAR Systems

The Polarimetric SAR system acquires 3 complex SAR images

Scattering matrix $\mathbf{S} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{hv} & S_{vv} \end{bmatrix}$

Scattering vector $\mathbf{k} = [S_{hh}, 2S_{hv}, S_{vv}]^T$



The properties of the scattering vector follow from the properties of a single SAR image

- **k** is deterministic for point scatters.
- **k** is a multidimensional random variable for distributed scatters due to speckle.

SAR images characterized through second order moments

- Second order moments in multidimensional SAR data are matrix quantities

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

Multidimensional Gaussian pdf Properties

Characterization of random variables

- Probability Density Function (pdf)
- Statistical moments (mean, power, kurtosis, skewness…)

Zero-mean multidimensional complex Gaussian pdf

$$p_{\mathbf{k}}(\mathbf{k}) = \frac{1}{\pi^3 |\mathbf{C}|} \exp\left(-\mathbf{k}^H \mathbf{C}^{-1} \mathbf{k}\right)$$

- First order moment $E\{\mathbf{k}\} = \mathbf{0}$
- Second order moment: Covariance matrix

$$\mathbf{C} = E\{\mathbf{k}\mathbf{k}^H\} = \begin{bmatrix} E\{|S_{hh}|^2\} & E\{S_{hh}S_{hv}^*\} & E\{S_{hh}S_{vv}^*\} \\ E\{S_{hv}S_{hh}^*\} & E\{|S_{hv}|^2\} & E\{S_{hv}S_{vv}^*\} \\ E\{S_{vv}S_{hh}^*\} & E\{S_{vv}S_{hv}^*\} & E\{|S_{vv}|^2\} \end{bmatrix}$$

$$E\{S_k S_l^*\} \neq 0 \quad k,l \in \{1,\ldots,m\}, k \neq l$$

⬇

Correlated SAR images

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

Mathematical Representation

The covariance matrix contains the correlation structure of the set of *m* SAR images

$$\mathbf{C} = E\left\{\mathbf{k}\mathbf{k}^{H}\right\} = \begin{bmatrix} E\left\{\left|S_{hh}\right|^{2}\right\} & E\left\{S_{hh}S_{hv}^{*}\right\} & E\left\{S_{hh}S_{vv}^{*}\right\} \\ E\left\{S_{hv}S_{hh}^{*}\right\} & E\left\{\left|S_{hv}\right|^{2}\right\} & E\left\{S_{hv}S_{vv}^{*}\right\} \\ E\left\{S_{vv}S_{hh}^{*}\right\} & E\left\{S_{vv}S_{hv}^{*}\right\} & E\left\{\left|S_{vv}\right|^{2}\right\} \end{bmatrix}$$

Information

- Diagonal elements: Power information

$$E\left\{S_{k}S_{k}^{H}\right\} = E\left\{\left|S_{k}\right|^{2}\right\} \quad k \in \left\{1,2,\ldots,m\right\}$$

- Off-diagonal elements: Correlation information

$$E\left\{S_{k}S_{l}^{H}\right\} \quad k,l \in \left\{1,2,\ldots,m\right\}, k \neq l \qquad \rho_{k,l} = \frac{E\left\{S_{k}S_{l}^{*}\right\}}{\sqrt{E\left\{\left|S_{k}\right|^{2}\right\} \cdot E\left\{\left|S_{l}\right|^{2}\right\}}} = \left|\rho_{k,l}\right| e^{j\theta_{k,l}}$$

Mathematical Representation

Multidimensional SAR data descriptors for distributed scatterers

First order moment

Second order moment

$$\mathbf{S} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{hv}^H & S_{vv} \end{bmatrix}$$

Scattering matrix

Description for PolSAR data

$$\mathbf{k}\mathbf{k}^H = \begin{bmatrix} S_{hh}S_{hh}^H & \sqrt{2}S_{hh}S_{hv}^H & S_{hh}S_{vv}^H \\ \sqrt{2}S_{hv}S_{hh}^H & 2S_{hv}S_{hv}^H & \sqrt{2}S_{hv}S_{vv}^H \\ S_{vv}S_{hh}^H & \sqrt{2}S_{vv}S_{hv}^H & S_{vv}S_{vv}^H \end{bmatrix}$$

Covariance matrix

Point scatterers characterization

$$\mathbf{k} = \left[S_1, S_2, \ldots, S_m\right]^T$$

Scattering vector

Description for generalized multidimensional SAR data

$$\mathbf{k}\mathbf{k}^H = \begin{bmatrix} S_1S_1^H & S_1S_2^H & \cdots & S_1S_m^H \\ S_2S_1^H & S_2S_2^H & \cdots & S_2S_m^H \\ \vdots & \vdots & \ddots & \vdots \\ S_mS_1^H & S_mS_2^H & \cdots & S_mS_m^H \end{bmatrix}$$

$$E\{\mathbf{k}\} = \mathbf{0}$$

Characterizes completely the data distribution

Distributed scatterers characterization

$$E\{\mathbf{k}\mathbf{k}^H\} = \mathbf{C}$$

Covariance matrix

$$p_{\mathbf{k}}(\mathbf{k}) = \frac{1}{\pi^m |\mathbf{C}|} \exp\left(-\mathbf{k}^H \mathbf{C}^{-1} \mathbf{k}\right)$$

Sample Covariance Matrix

Covariance matrix estimation by means of a MultiLook (BoxCar)

- Maximum likelihood estimator: Sample covariance matrix

$$\mathbf{Z}_n = \frac{1}{n}\sum_{k=1}^{n} \mathbf{k}\mathbf{k}^H$$

Averaging usually done in the spatial domain (assuming homogeneous samples)

- n represents the total number of samples employed to estimate the covariance matrix, taken a region (square, rectangular, adapted…)
- $\mathbf{Z}_n$ as estimator of V
  - Does not consider signal morphology/heterogeneity
  - Loss of spatial resolution

The sample covariance matrix $\mathbf{Z}_n$ is itself a multidimensional random variable

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH
Department of Signal Theory and Communications

Sample Covariance Matrix Distribution

The sample covariance matrix $\mathbf{Z}_n$ is characterized by the complex Wishart distribution

$$\mathbf{Z}_n \sim \mathrm{W}\left(n, \mathbf{C}\right)$$

$$p_{\mathbf{Z}_n}\left(\mathbf{Z}_n\right) = \frac{n^{mn} \left|\mathbf{Z}_n\right|^{n-m}}{\left|\mathbf{C}\right|^n \tilde{\Gamma}_m\left(n\right)} \operatorname{etr}\left(-n\mathbf{C}^{-1}\mathbf{Z}_n\right) \qquad \tilde{\Gamma}_m\left(n\right) = \pi^{m(m-1)/2} \prod_{i=1}^{m} \Gamma\left(n-i+1\right)$$

- Multidimensional data distribution
- Valid for $n \geq m$, otherwise $|\mathbf{Z}_n|^{n-m}$ is equal to zero and the Wishart pdf is undetermined

Multidimensional SAR Data Description

$$S_k = \mathrm{N}_{C^2}\left(0, \sigma^2/2\right)$$

Single SAR image

→

$$\mathbf{k} = \left[S_1, S_2, \ldots, S_m\right]^T$$

Multidimensional SAR dataset

→

$$p_{\mathbf{k}}\left(\mathbf{k}\right) = \frac{1}{\pi^m \left|\mathbf{C}\right|} \exp\left(-\mathbf{k}^H \mathbf{C}^{-1} \mathbf{k}\right)$$

Multidimensional complex Gaussian pdf

↓

$$\mathbf{Z}_n = \frac{1}{n} \sum_{i=1}^{n} \mathbf{k}_i \mathbf{k}_i^H$$

$$\mathbf{Z}_n \sim \mathrm{W}\left(n, \mathbf{C}\right)$$

Sample covariance matrix

↓

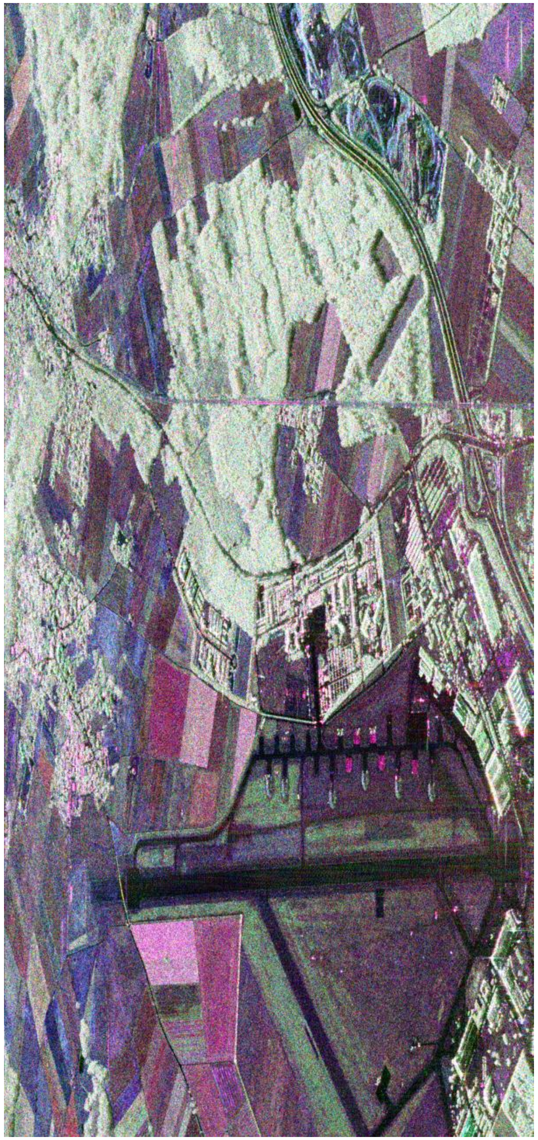
$$p_{\mathbf{Z}_n}\left(\mathbf{Z}_n\right) = \frac{n^{mn} \left|\mathbf{Z}_n\right|^{n-m}}{\left|\mathbf{C}\right|^n \tilde{\Gamma}_m\left(n\right)} \mathrm{etr}\left(-n \mathbf{C}^{-1} \mathbf{Z}_n\right)$$

Complex Wishart pdf

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

Multilook/Boxcar Example

|Shh-Svv| 2|Shv| | Shh +Svv|



Original data

7x7 MLT data

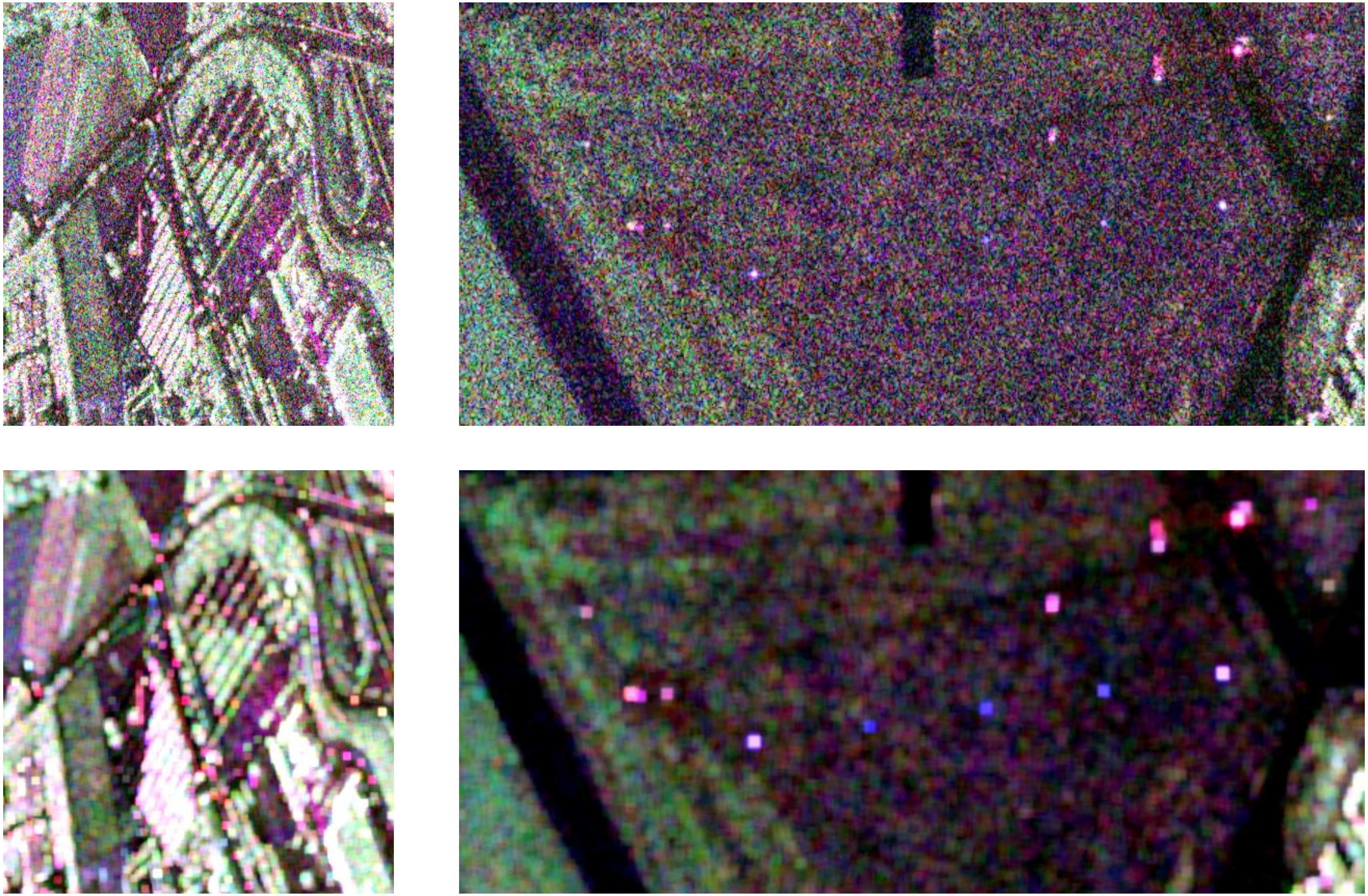
L-band (1.3 GHz) fully PolSAR data
E-SAR system. Oberpfaffenhofen test area (D)

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

Multilook/Boxcar Example

Original data

7x7 MLT data



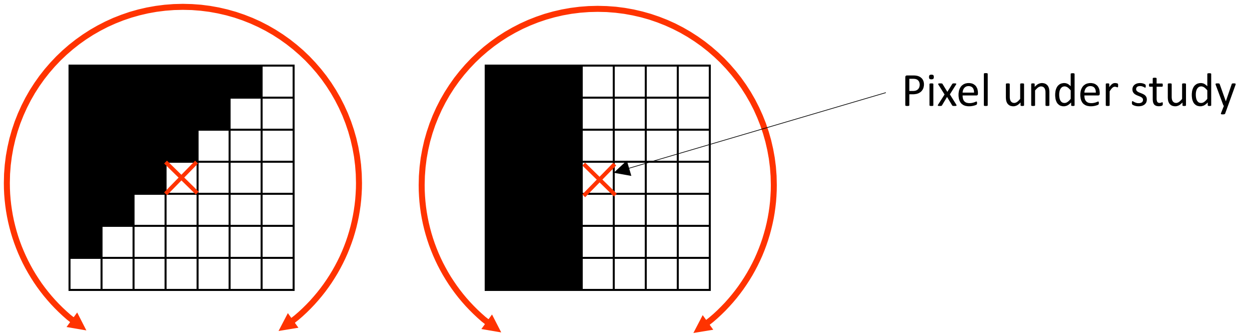
|Shh-Svv| 2|Shv| | Shh +Svv|

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

Local Statistics Linear Filter

Refined Lee filter

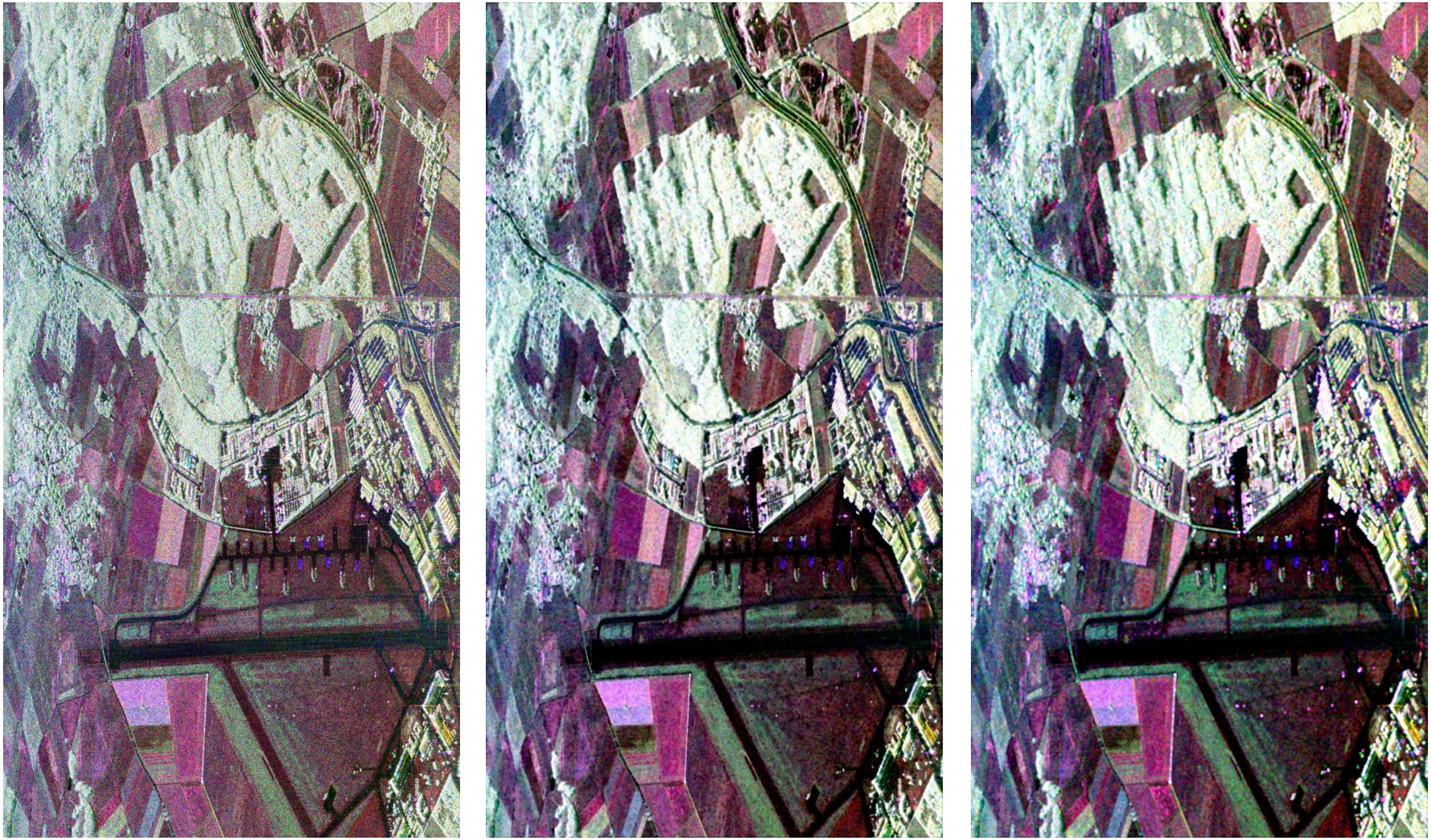
Statistics estimation in windows selected according to the signal morphology in order to retain edges, spatial feature and point targets



The directional windows try to adapt to image structure

➡ Decision needs to take into account statistical distribution to avoid the introduction of bias or distortion

Local Statistics Linear Filter

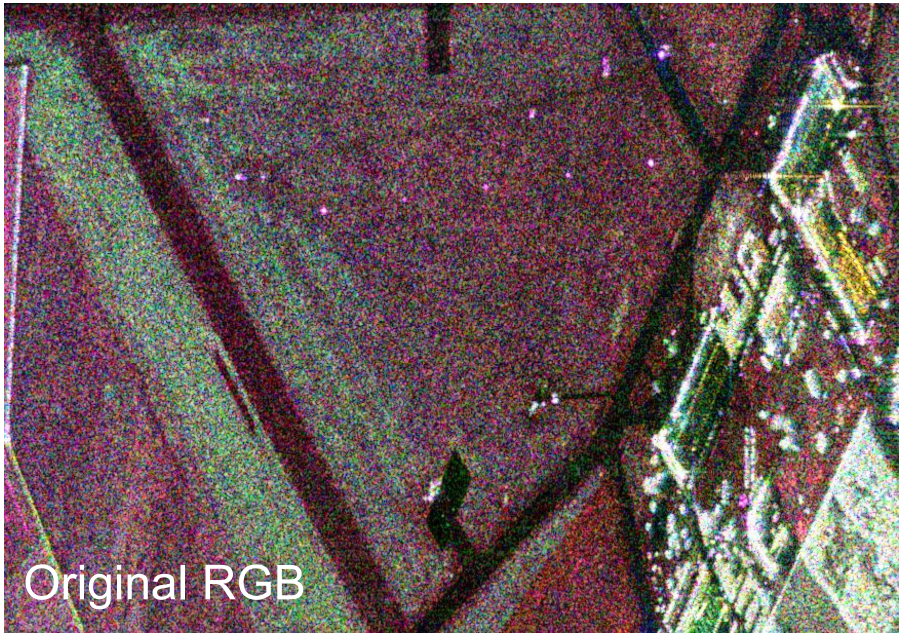


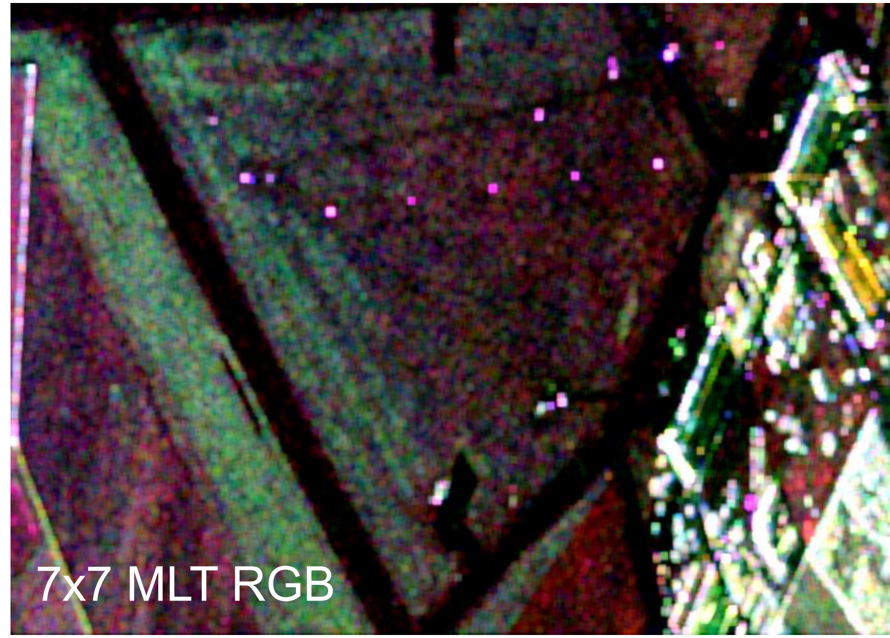
|Shh| |Shv| |Svv|

L-band (1.3 GHz) fully PolSAR data
E-SAR system. Oberpfaffenhofen test area (D)

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

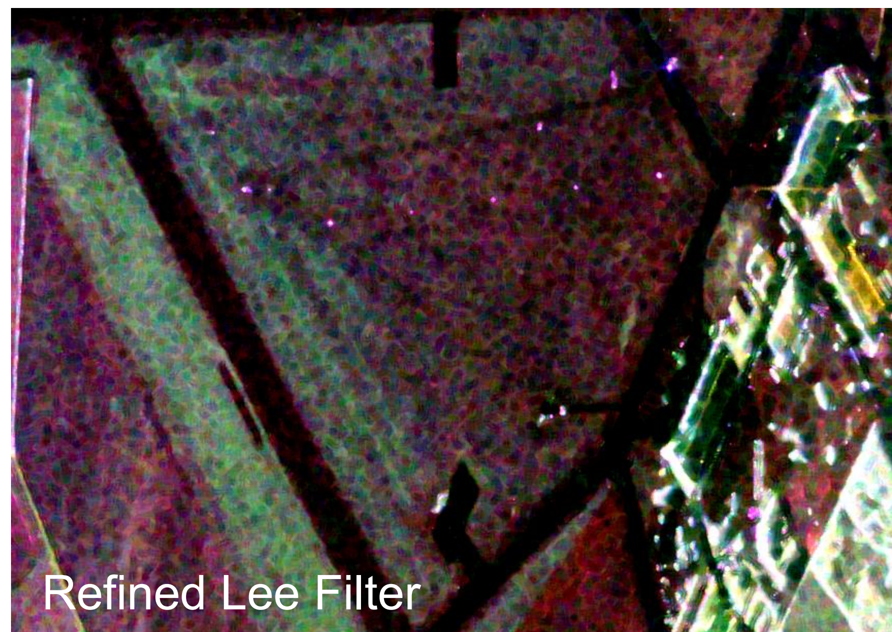
Local Statistics Linear Filter





|Shh| **|Shv|** **|Svv|**

L-band (1.3 GHz) fully PolSAR data
E-SAR system. Oberpfaffenhofen test area (D)



Outline

- Polarimetric Target Decomposition Theorems
- Coherent Decompositions
- Incoherent Decompositions
- Examples

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory
and Communications

Polarimetric Decompositions

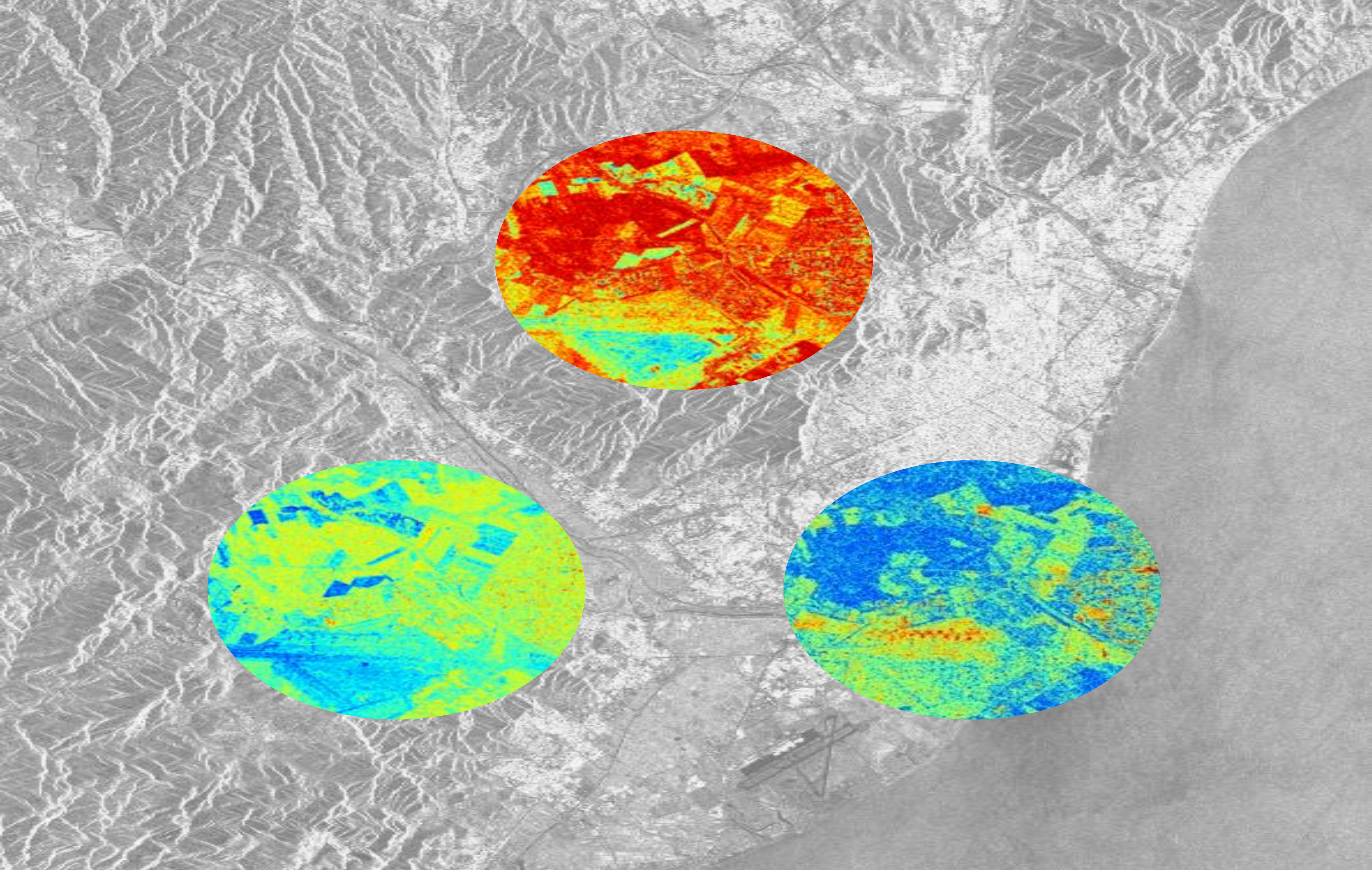
Polarimetric Decompositions allow the interpretation of measured PolSAR data by decomposing them into more explainable terms

- Coherent Decompositions
  - Applied to the first order descriptors, i.e., the Scattering Matrix
  - Valid for the interpretation of Pure or Deterministic scatters
  - Decomposition of the data into canonical scattering mechanisms

$$\mathbf{S} = \sum_{i=1}^{k} c_i \mathbf{S}_i$$

- Incoherent Decompositions
  - Applied to second order descriptors, i.e., the Covariance and Coherency matrices
  - Valid for the interpretation of Deterministic and Distributed scatters
  - Decomposition of the data into simple scattering mechanisms that may, or not, admit an equivalent scattering matrix

$$\langle \mathbf{C} \rangle = \sum_{i=1}^{k} c_i \mathbf{C}_i \qquad \langle \mathbf{T} \rangle = \sum_{i=1}^{k} c_i \mathbf{T}_i$$



Coherent Decompositions

Pauli Decomposition

Based on the decomposition of the measured scattering matrix into the orthogonal Pauli decompositions basis

- Basis components

$$\mathbf{S}_a = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{S}_b = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \mathbf{S}_c = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \mathbf{S}_d = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}$$

Single bounce　　Double bounce　　Volume

  - Admit a physical interpretation into simple scattering mechanisms
  - Orthogonal scattering mechanisms

- Decomposition into three orthogonal scattering mechanisms (monostatic case)

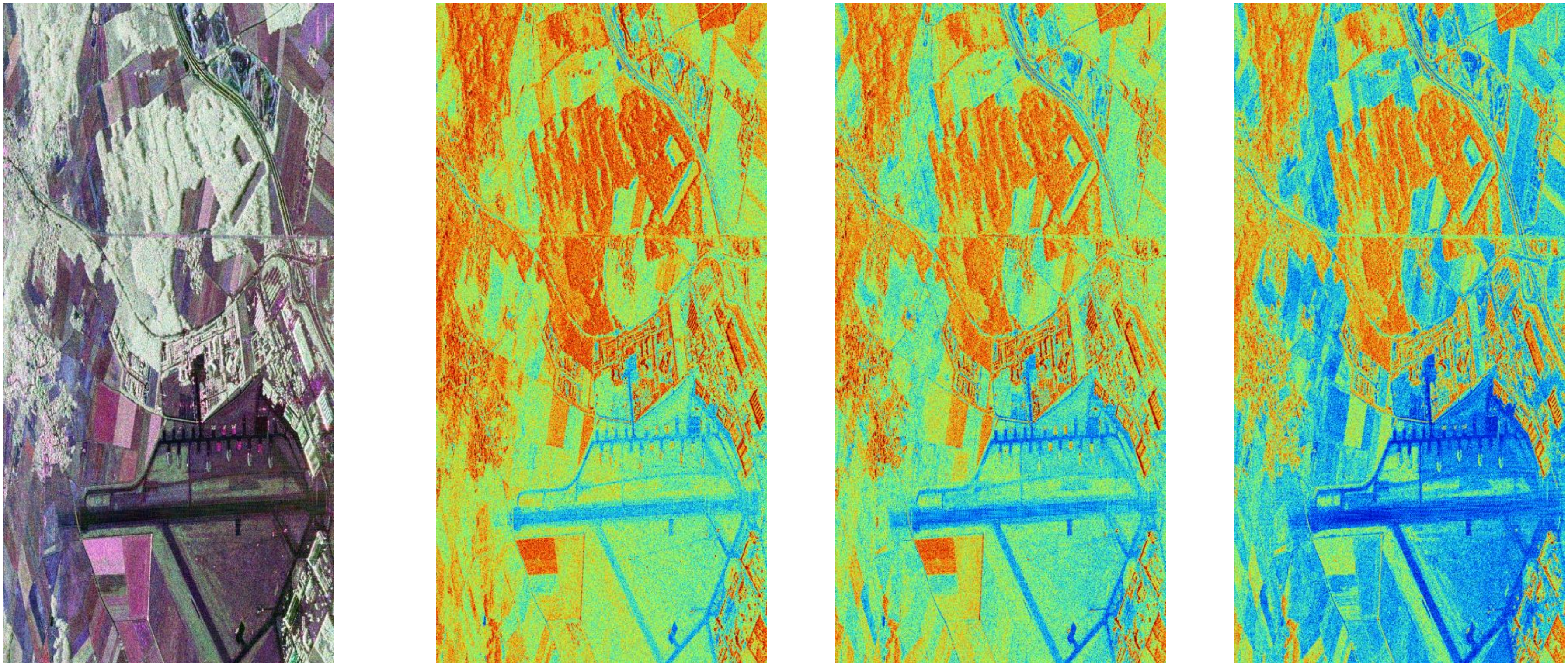
$$\mathbf{S} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{hv} & S_{vv} \end{bmatrix} = \alpha\mathbf{S}_a + \beta\mathbf{S}_b + \gamma\mathbf{S}_c$$

- Decomposition coefficients

$$\alpha = \frac{S_{hh} + S_{vv}}{\sqrt{2}} \qquad \beta = \frac{S_{hh} - S_{vv}}{\sqrt{2}} \qquad \gamma = \sqrt{2}S_{hv}$$

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

Pauli Decomposition

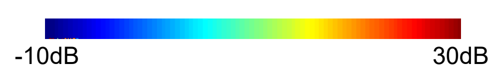


|Shh-Svv| 2|Shv| | Shh +Svv|

$$\alpha = \frac{S_{hh} + S_{vv}}{\sqrt{2}}$$

$$\beta = \frac{S_{hh} - S_{vv}}{\sqrt{2}}$$

$$\gamma = \sqrt{2} S_{hv}$$



UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory
and Communications

Incoherent Decompositions

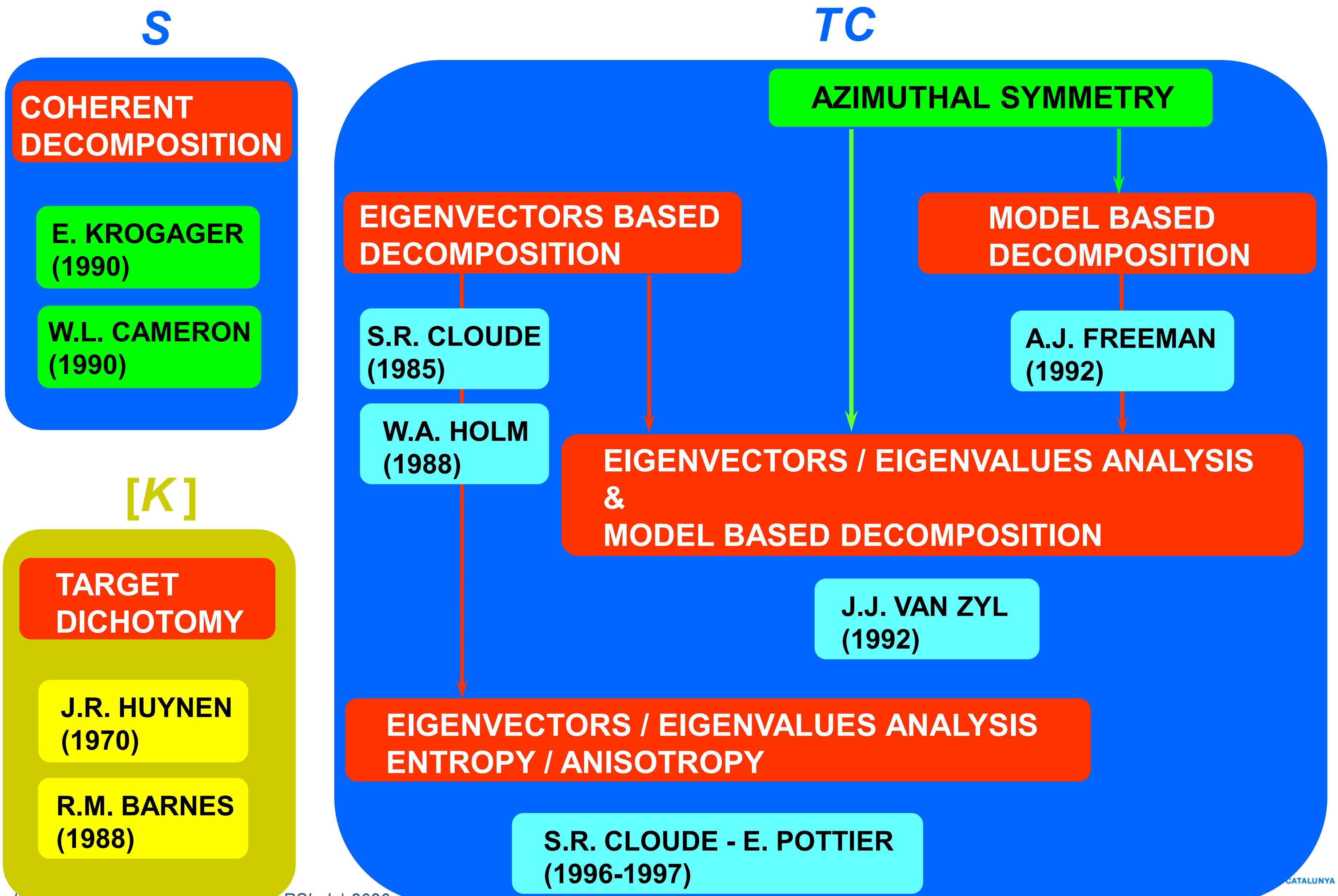
Two type of decompositions

- Model-based decompositions: based on components that model physical scattering mechanisms
  - Simple physical interpretation
  - Assumptions concerning the type of scattering are necessary

- Mathematical-based decompositions: based on mathematical or spectral decompositions, as the eigenvalues/eigenvectors
  - Simple computation of the decomposition
  - No assumptions about the type of scattering are necessary
  - Physical interpretation of the different decomposition components are necessary

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH
Department of Signal Theory and Communications

Target Decomposition Theorems



Courtesy of Prof. E. Pottier

Freeman Decomposition

Incoherent decomposition proposed by A. Freeman (1992)

- Decomposition

$\langle \mathbf{T} \rangle$ $\langle \mathbf{C} \rangle$ → Three components scattering mechanism model

$\mathbf{T}_S$ $\mathbf{C}_S$ Single bounce scattering

$\mathbf{T}_D$ $\mathbf{C}_D$ Double bounce scattering

$\mathbf{T}_V$ $\mathbf{C}_V$ Volume scattering

$$\langle \mathbf{T} \rangle = f_S \mathbf{T}_S + f_D \mathbf{T}_D + f_V \mathbf{T}_V$$

$$\langle \mathbf{C} \rangle = f_S \mathbf{C}_S + f_D \mathbf{C}_D + f_V \mathbf{C}_V$$

Decomposition coefficients

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH
Department of Signal Theory and Communications

Freeman Decomposition

Incoherent decomposition proposed by A. Freeman (1992)

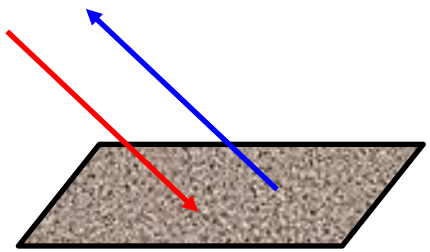
- Decomposition

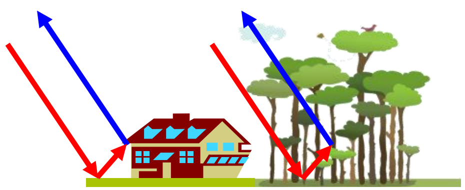
$\langle \mathbf{T} \rangle$ $\langle \mathbf{C} \rangle$ ➡ Three components scattering mechanism model

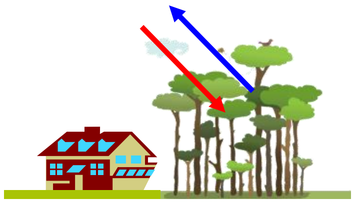
$\mathbf{T}_S$ $\mathbf{C}_S$ Single bounce scattering

$\mathbf{T}_D$ $\mathbf{C}_D$ Double bounce scattering

$\mathbf{T}_V$ $\mathbf{C}_V$ Volume scattering







$$\mathbf{C}_S = f_S \begin{bmatrix} \beta^2 & 0 & \beta \\ 0 & 0 & 0 \\ \beta & 0 & 1 \end{bmatrix}$$

$$\mathbf{C}_D = f_D \begin{bmatrix} \alpha^2 & 0 & -\alpha \\ 0 & 0 & 0 \\ -\alpha & 0 & 1 \end{bmatrix}$$

$$\mathbf{C}_V = f_V \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & \frac{2}{3} & 0 \\ \frac{1}{3} & 0 & 1 \end{bmatrix}$$

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH
Department of Signal Theory and Communications

Freeman Decomposition

Incoherent decomposition proposed by A. Freeman (1992)

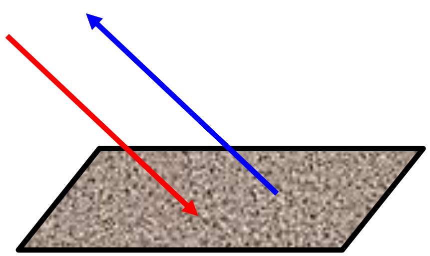
- Decomposition

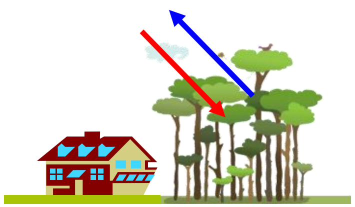
$\langle \mathbf{T} \rangle \quad \langle \mathbf{C} \rangle$ ➡ Three components scattering mechanism model

$\mathbf{T}_S$ $\mathbf{C}_S$ Single bounce scattering

$\mathbf{T}_D$ $\mathbf{C}_D$ Double bounce scattering

$\mathbf{T}_V$ $\mathbf{C}_V$ Volume scattering





$$\mathbf{C}_S = f_S \begin{bmatrix} \beta^2 & 0 & \beta \\ 0 & 0 & 0 \\ \beta & 0 & 1 \end{bmatrix}$$

$$\mathbf{C}_D = f_D \begin{bmatrix} \alpha^2 & 0 & -\alpha \\ 0 & 0 & 0 \\ -\alpha & 0 & 1 \end{bmatrix}$$

$$\mathbf{C}_V = f_V \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & \frac{2}{3} & 0 \\ \frac{1}{3} & 0 & 1 \end{bmatrix}$$

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

Freeman Decomposition

3 Components Scattering Mechanism Model

$$\langle \mathbf{C} \rangle = \begin{bmatrix} f_S\beta^2 + f_D\alpha^2 + f_V & 0 & f_S\beta - f_D\alpha + \frac{f_V}{3} \\ 0 & \frac{2f_V}{3} & 0 \\ f_S\beta - f_D\alpha + \frac{f_V}{3} & 0 & f_S + f_D + f_V \end{bmatrix}$$

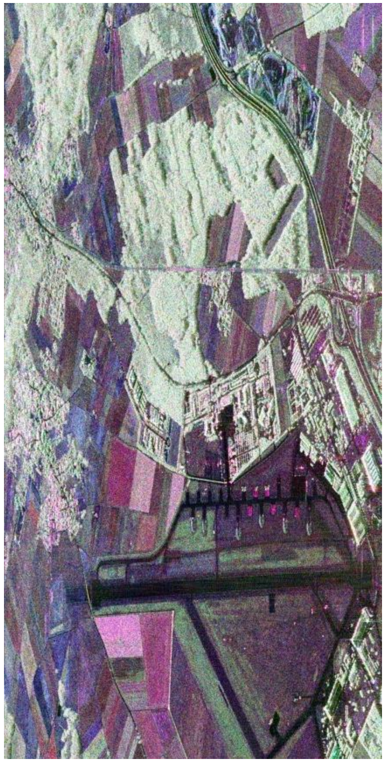
- 5 Unknown real coefficients
- 4 Observed equations
- Negative power is observed

Assumptions must be considered !!!

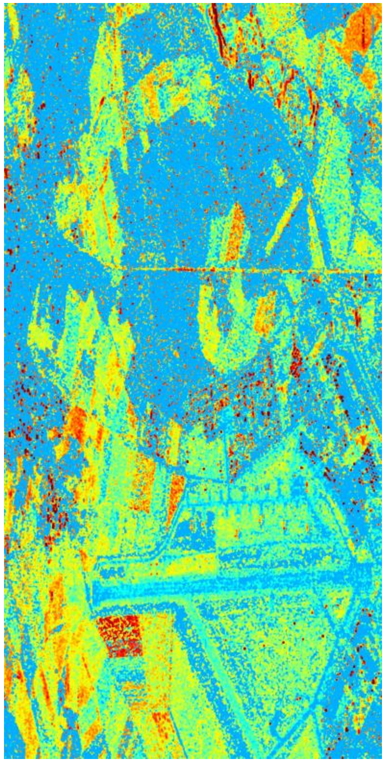
UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

Freeman Decomposition

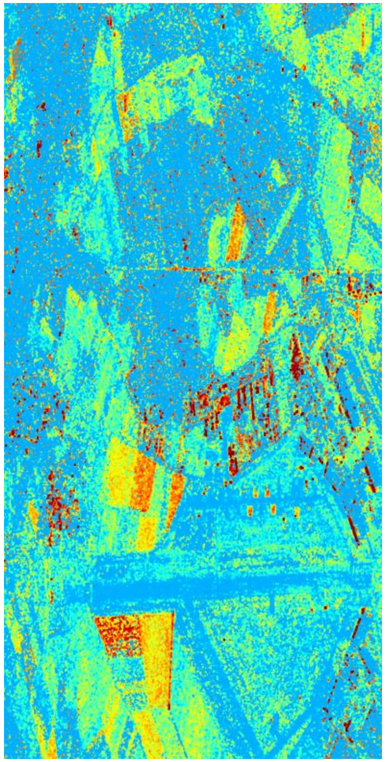
7x7 Multilook



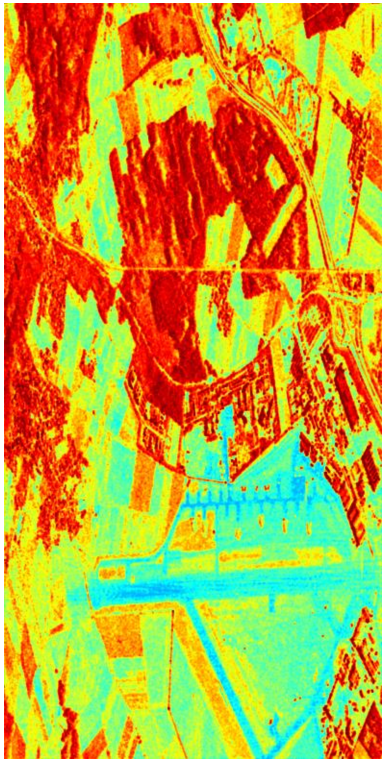
|Shh-Svv| 2|Shv| | Shh +Svv|



f_S



f_D

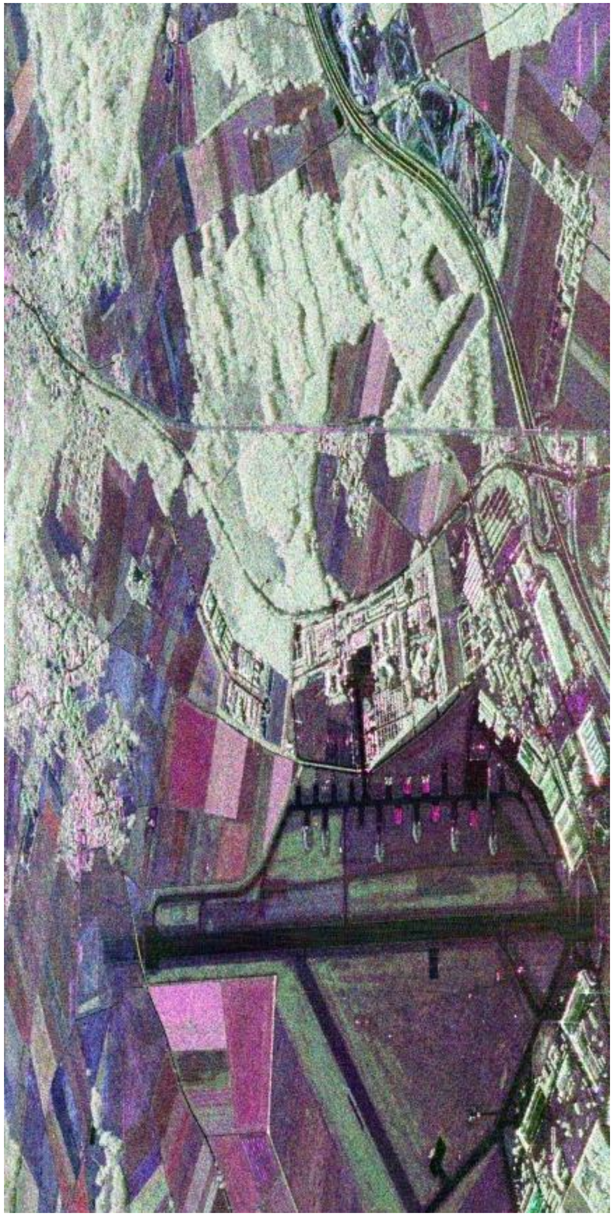


f_V

-10dB 30dB

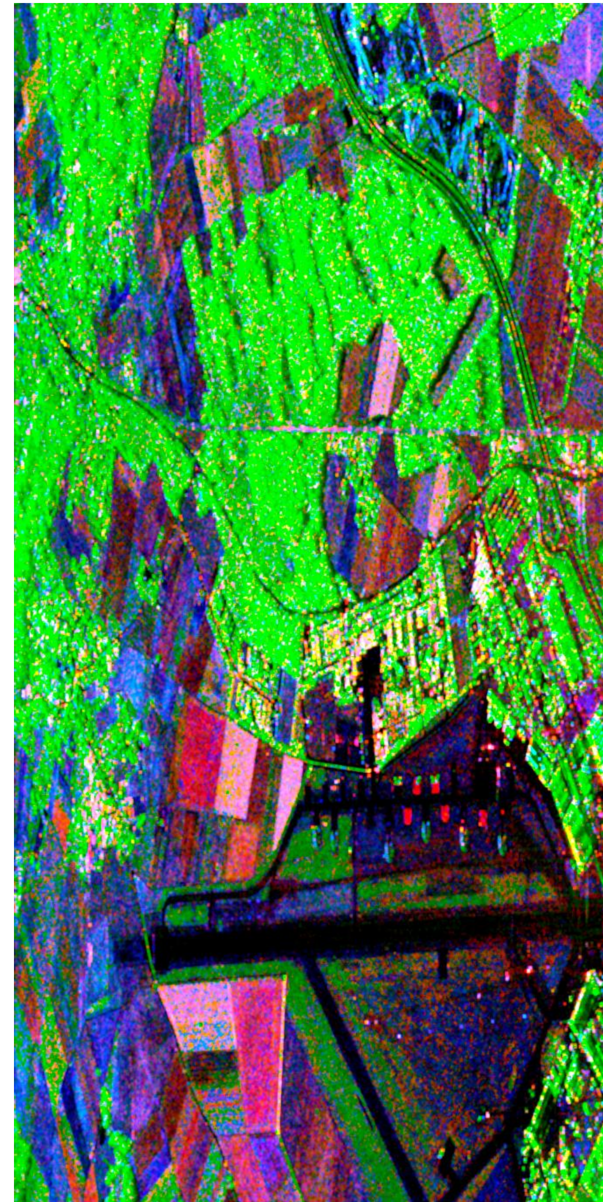
UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

Freeman Decomposition



|Shh-Svv| 2|Shv| | Shh +Svv|

$|f_D|$ $|f_V|$ $|f_S|$



UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory
and Communications

Eigenvalue/vectors Decomposition

Decomposition proposed by Shane Cloude, based on the mathematical decomposition of the coherency matrix on its eigenvalue and eigenvectors

$$\mathbf{k}_p = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{hh} + S_{vv} \\ S_{hh} - S_{vv} \\ 2S_{hv} \end{bmatrix}$$

Sample

$$\langle \mathbf{T} \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathbf{k}_i \mathbf{k}_i^H = \frac{1}{N} \sum_{i=1}^{N} \mathbf{T}_i$$

Estimation of the covariance matrix

- Decomposition (i)

$$\langle \mathbf{T} \rangle = \mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^{-1} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}^H$$

  - The eigenvectors $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$ are orthonormal
  - The eigenvalues are real $\lambda_1 > \lambda_2 > \lambda_3 \geq 0$

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

Eigenvalue/vectors Decomposition

- Decomposition components
  - The eigenvectors represent rank 1 scattering mechanisms that are related with a scattering matrix
  - Parametrization of the eigenvectors matrix

$$\mathbf{U}=\begin{bmatrix}\cos(\alpha_1) & \cos(\alpha_2) & \cos(\alpha_3)\\ \sin(\alpha_1)\cos(\beta_1)e^{j\delta_1} & \sin(\alpha_2)\cos(\beta_2)e^{j\delta_2} & \sin(\alpha_3)\cos(\beta_3)e^{j\delta_3}\\ \sin(\alpha_1)\sin(\beta_1)e^{j\gamma_1} & \sin(\alpha_2)\sin(\beta_2)e^{j\gamma_2} & \sin(\alpha_3)\sin(\beta_3)e^{j\gamma_3}\end{bmatrix}$$

Target 1 Target 2 Target 3

Decomposition basis

  - The eigenvalues represent the power associated to every target

$$\lambda_1 \qquad \lambda_2 \qquad \lambda_3$$

Decomposition coefficients

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

Eigenvalue/vectors Decomposition

- Decomposition (ii)

$$\langle \mathbf{T} \rangle = \mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^{-1} = \lambda_1 \mathbf{u}_1 \mathbf{u}_1^H + \lambda_2 \mathbf{u}_2 \mathbf{u}_2^H + \lambda_3 \mathbf{u}_3 \mathbf{u}_3^H = \lambda_1 \mathbf{T}_1 + \lambda_2 \mathbf{T}_2 + \lambda_3 \mathbf{T}_3$$

  - The coherency matrices $\mathbf{T}_k$ are rank 1 matrices associated with a single scattering matrix
  - It is possible to associate a probability to every scattering mechanism

$$P_i = \frac{\lambda_i}{\sum_{k=1}^{3} \lambda_k} \qquad Span = \sum_{k=1}^{3} \lambda_k$$

  - Definition of the mean dominant scattering mechanism
    - Mean parameters

$$\bar{\alpha} = P_1 \alpha_1 + P_2 \alpha_2 + P_3 \alpha_3$$

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

Eigenvalue/vectors Decomposition

- Analysis of the eigenvalues. These parameters classify the spectrum of scattering mechanisms
  - Entropy: Degree of randomness or statistical disorder

$$H = -\sum_{i=1}^{3} P_i \log_3 \left(P_i \right)$$

    - Pure target (H=0). One scattering mechanism present

$$\lambda_1 = Span \quad \lambda_2 = 0 \quad \lambda_3 = 0$$

    - Random target (H=1). All scattering mechanisms equally probable

$$\lambda_1 = \lambda_2 = \lambda_3 = Span / 3$$

  - Anisotropy: Relative importance of the 2nd and 3rd scattering mechanisms

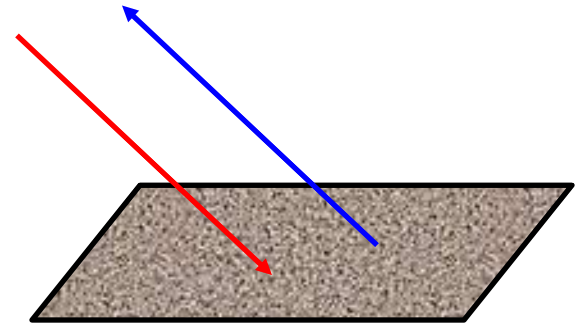
$$A = \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3}$$

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory
and Communications

Eigenvalue/vectors Decomposition

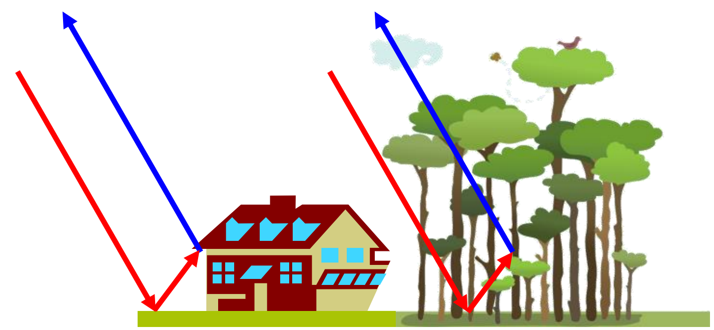
- Physical interpretation of the average alpha angle parameter

  - Single bounce scattering, for instance, a surface



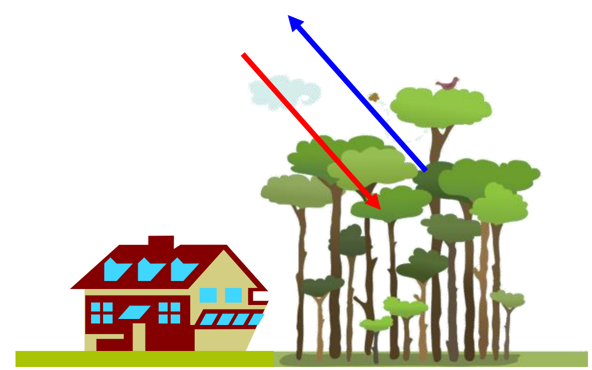
$$\underline{\alpha} \approx 0$$

  - Double bounce scattering



$$\underline{\alpha} \approx \frac{\pi}{2}$$

  - Volume scattering

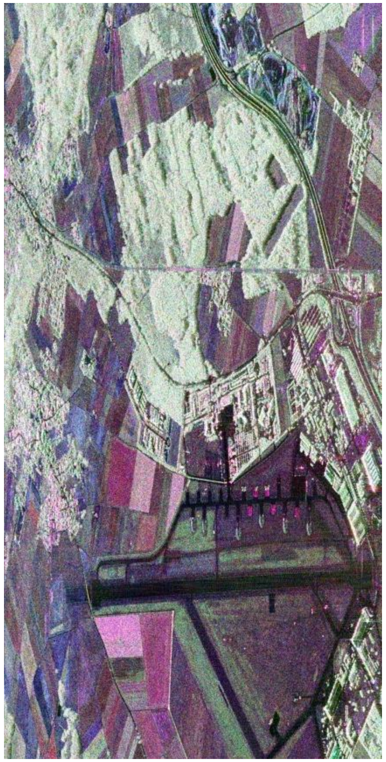


$$\underline{\alpha} \approx \frac{\pi}{4}$$

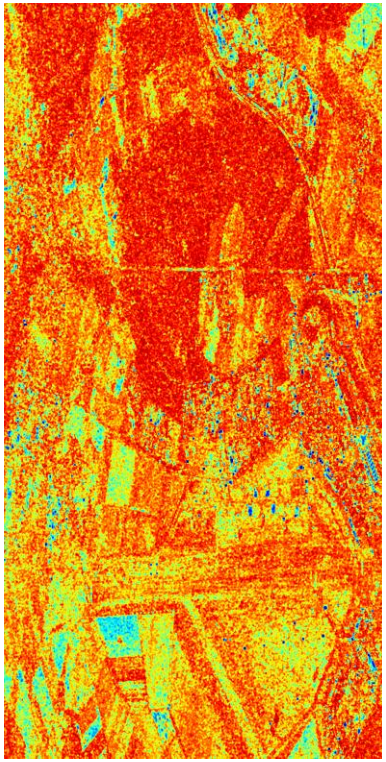
UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

H/A/α Decomposition

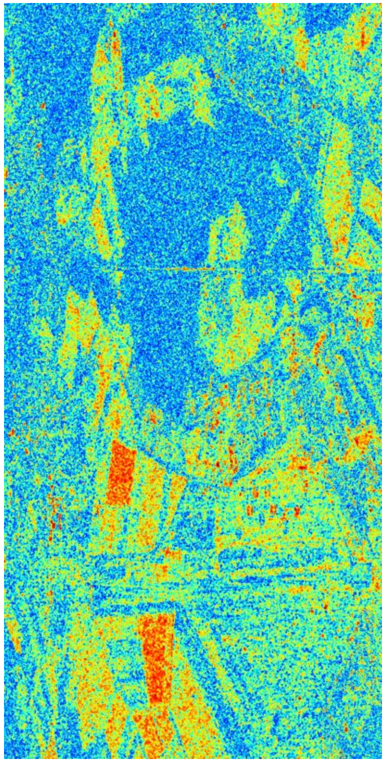
7x7 Multilook



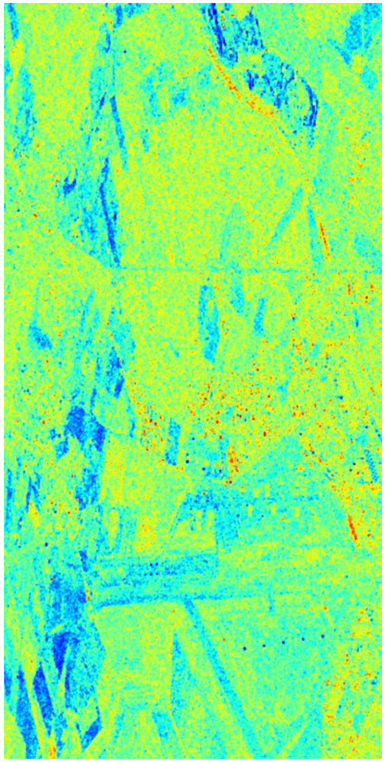
|Shh-Svv| 2|Shv| | Shh +Svv|



H



A



$\underline{\alpha}$

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory
and Communications

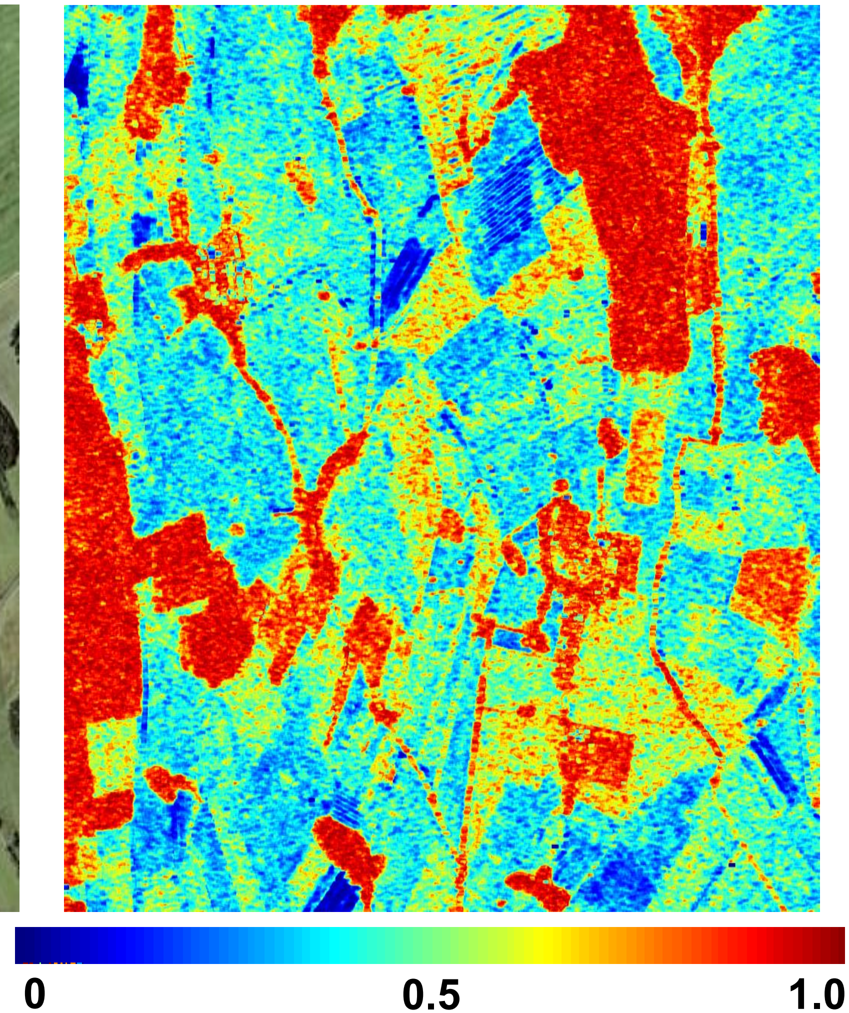
H/A/α Decomposition

|Shh| |Shv| |Svv|

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory
and Communications

H/A/α Decomposition

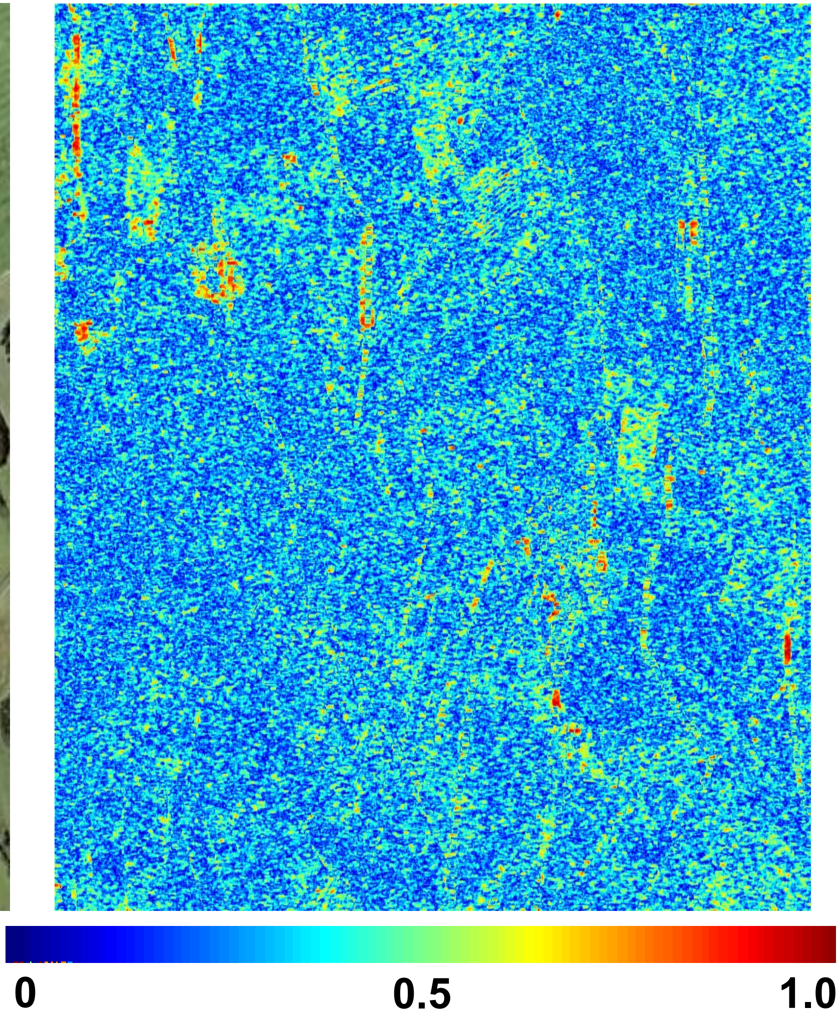
Entropy (H)



UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory
and Communications

H/A/α Decomposition

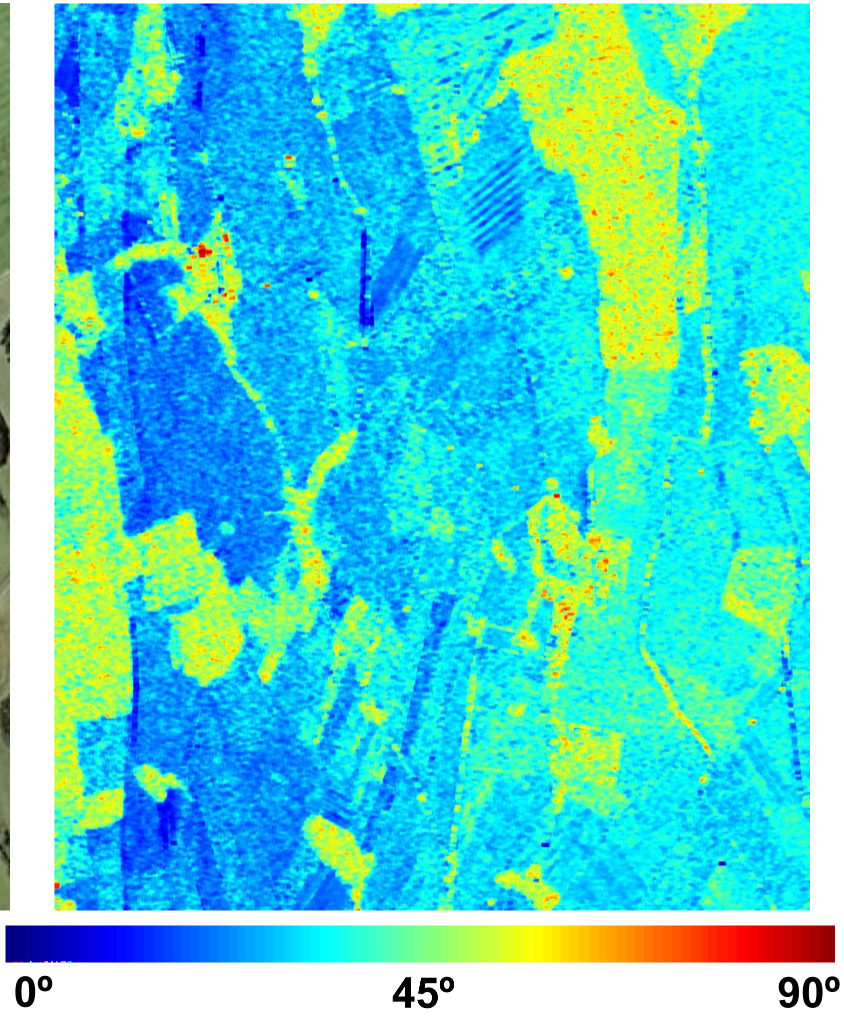
Anisotropy (A)



UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory
and Communications

H/A/α Decomposition

Alpha (α)

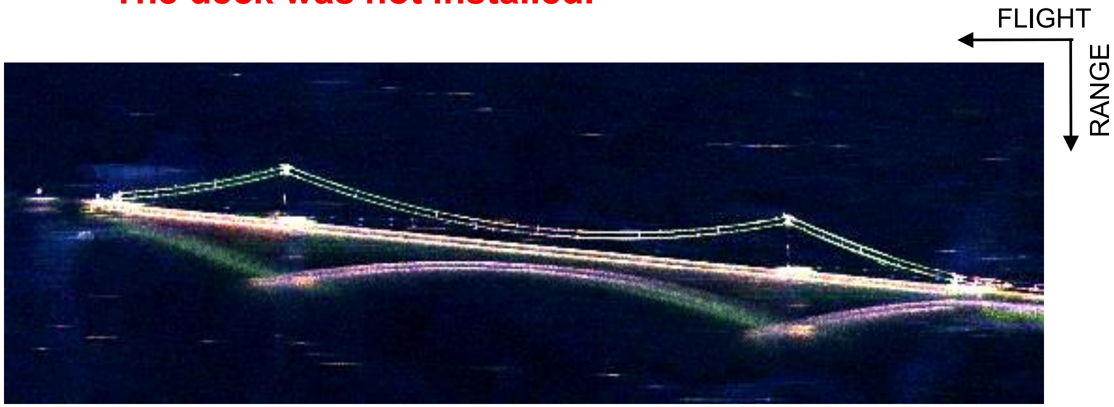


UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory
and Communications

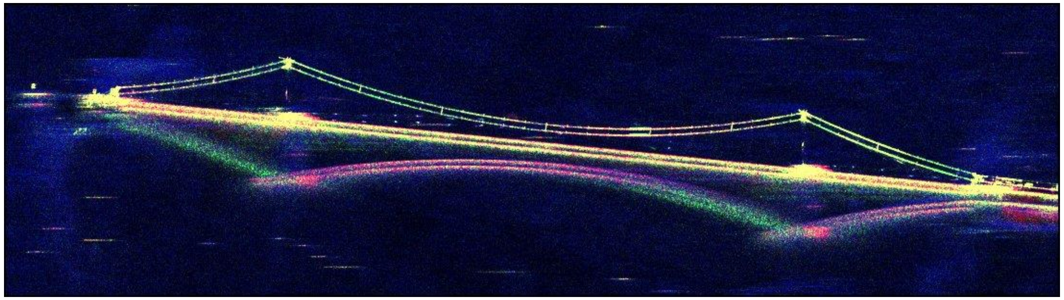
POLARIMETRIC TARGET SIGNATURES: An Example

High-resolution POLSAR signature of a suspension bridge under construction
The deck was not installed.

Aerial Photo



Pauli Decomposition, |HH-VV|, |HV|, |HH+VV|



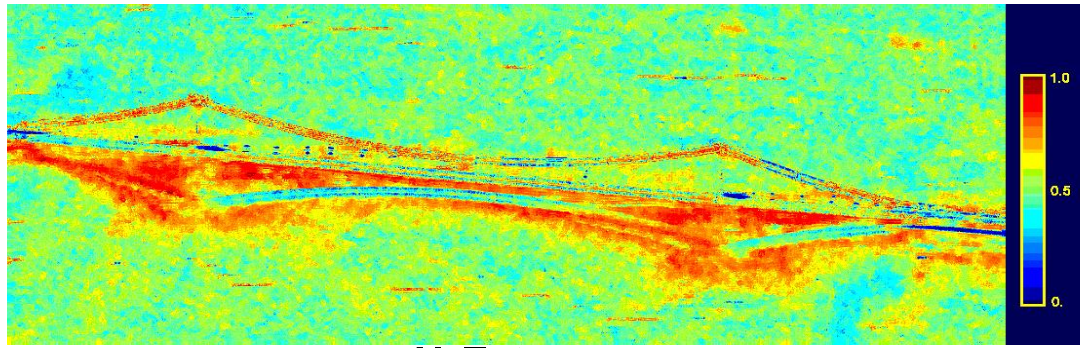
Krogager SDH Decomposition
Blue= Sphere Green = Helix Red = Diplane.

EMISAR C-Band Polarimetric SAR Image of StoreBelt Bridge

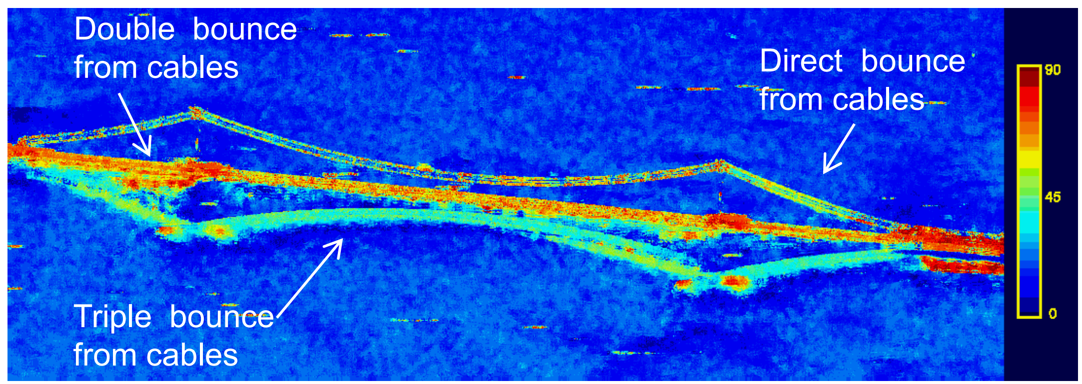
UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH
Department of Signal Theory and Communications

High-resolution POLSAR signature of a suspension bridge under construction
The deck was not installed.

Aerial Photo



H *Entropy*



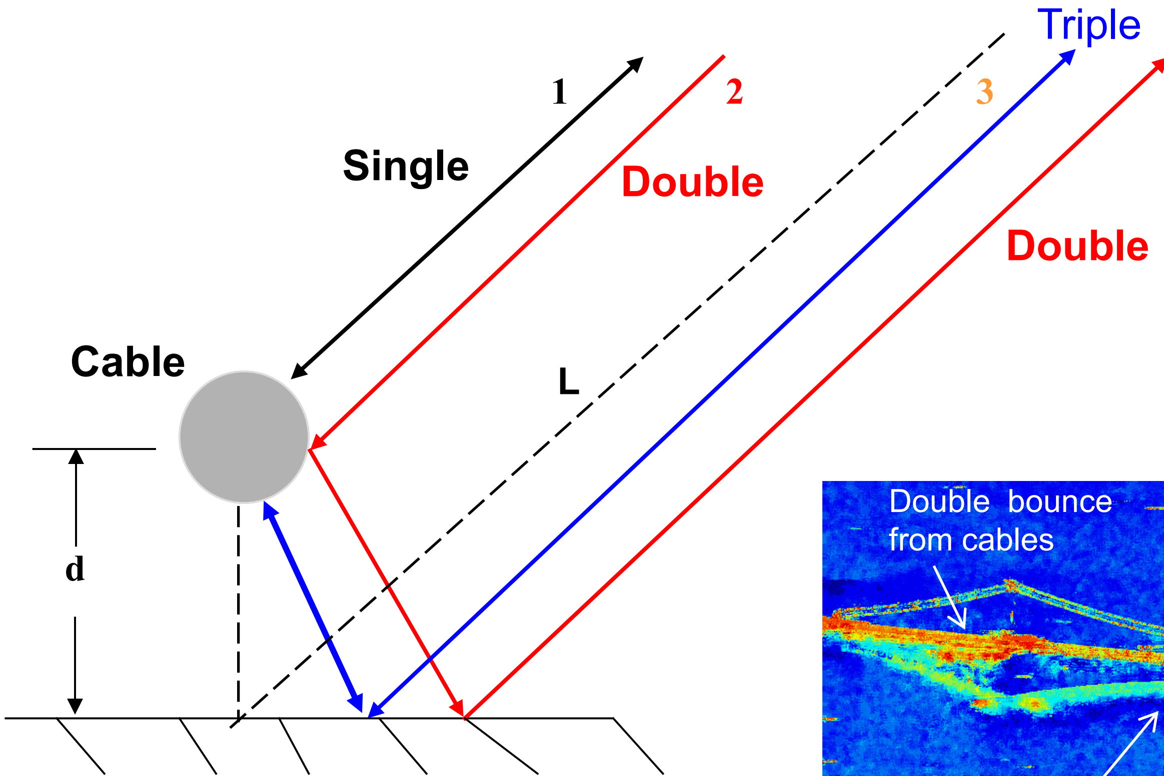
α *Angle*

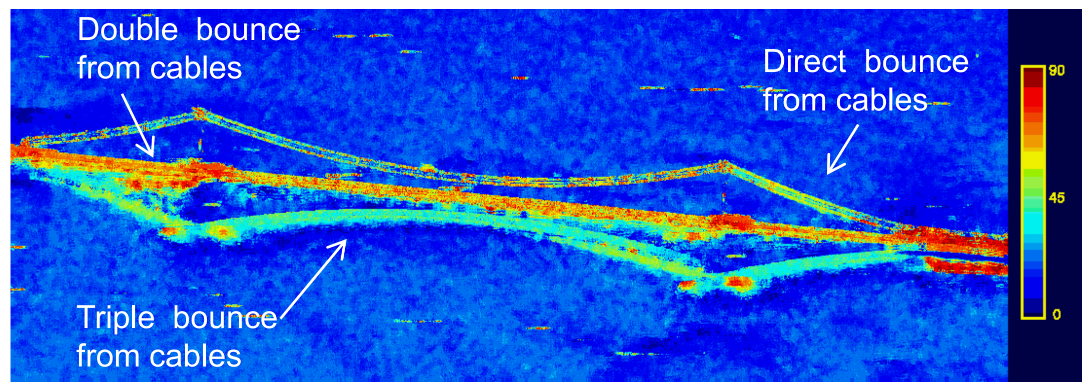
Dominant scattering mechanisms are extracted by applying target decomposition:
Blue= Surface Green = Dipole Red = Double Bounce.

EMISAR C-Band Polarimetric SAR Image of StoreBelt Bridge

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH
Department of Signal Theory and Communications

Multi – Bounce Scattering mechanisms





α *Angle*

Roundtrip distances:

Single bounce return: $2(L - d\cos\theta)$

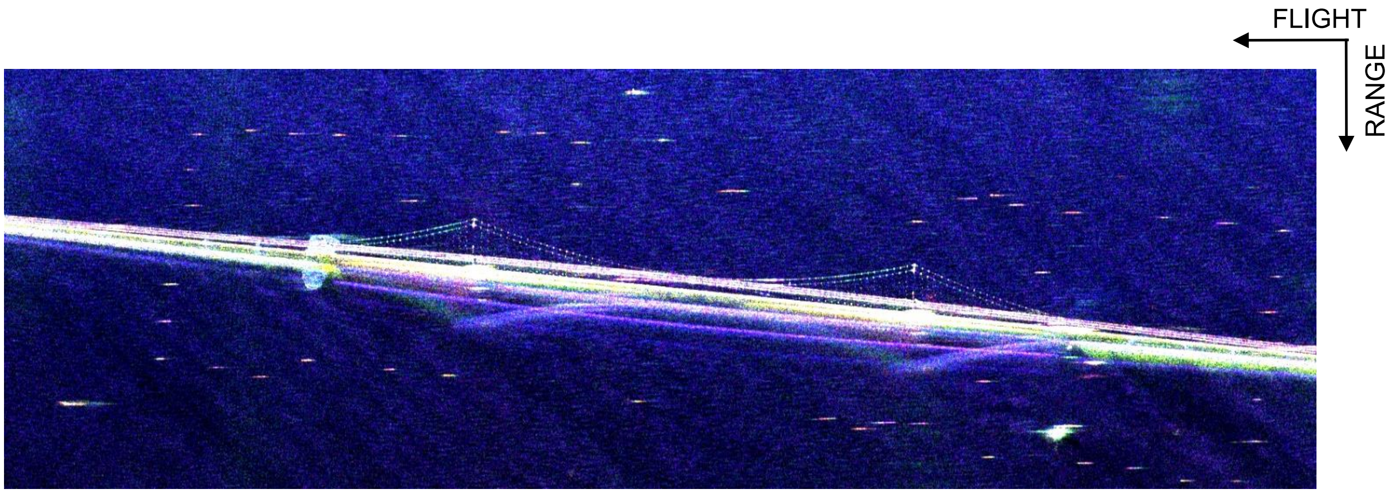
Double bounce return: $2L$ ⟸ ***Independent of height***

Triple bounce return: $2(L + d\cos\theta)$

θ is the local incidence angle.

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

High-resolution POLSAR signature of a suspension bridge after completion.
The deck is installed.



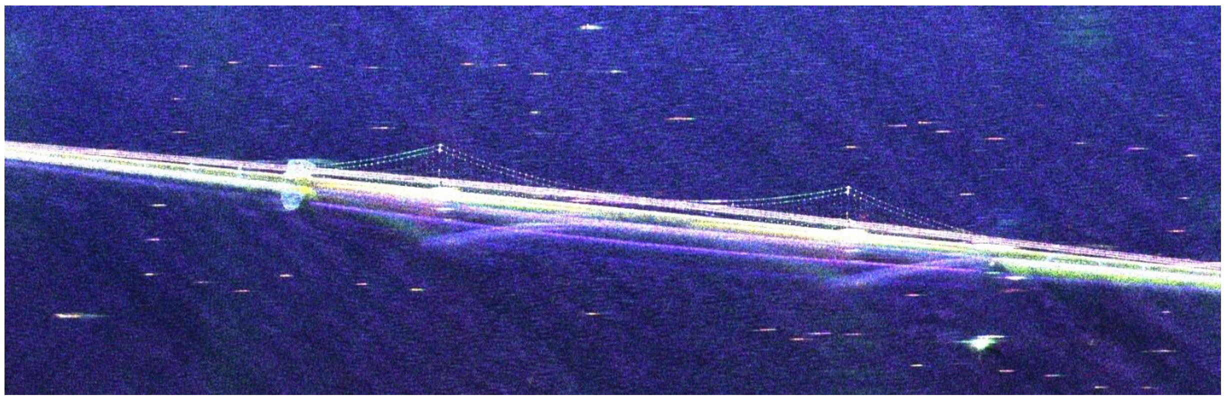
|HH-VV|, |HV|, |HH+VV|

EMISAR C-Band Polarimetric SAR Image of StoreBelt Bridge

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory
and Communications

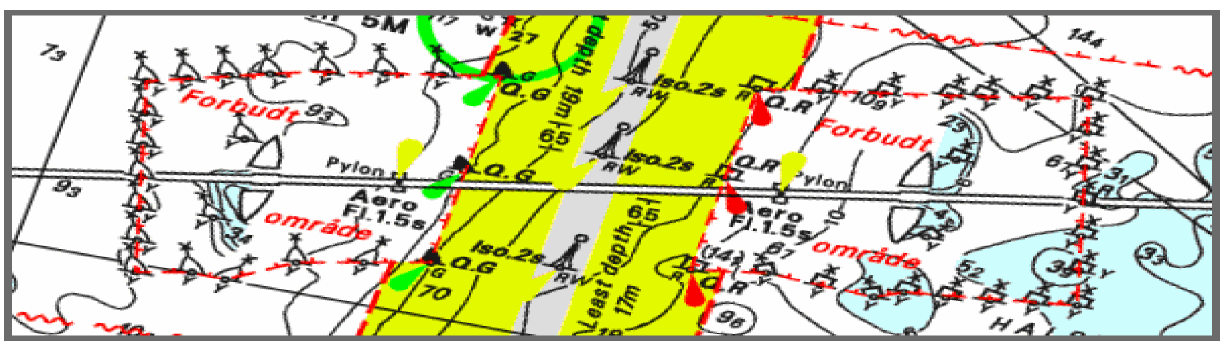
Polarimetric Scattering Characterization

Buoy



|HH-VV|, |HV|, |HH+VV|

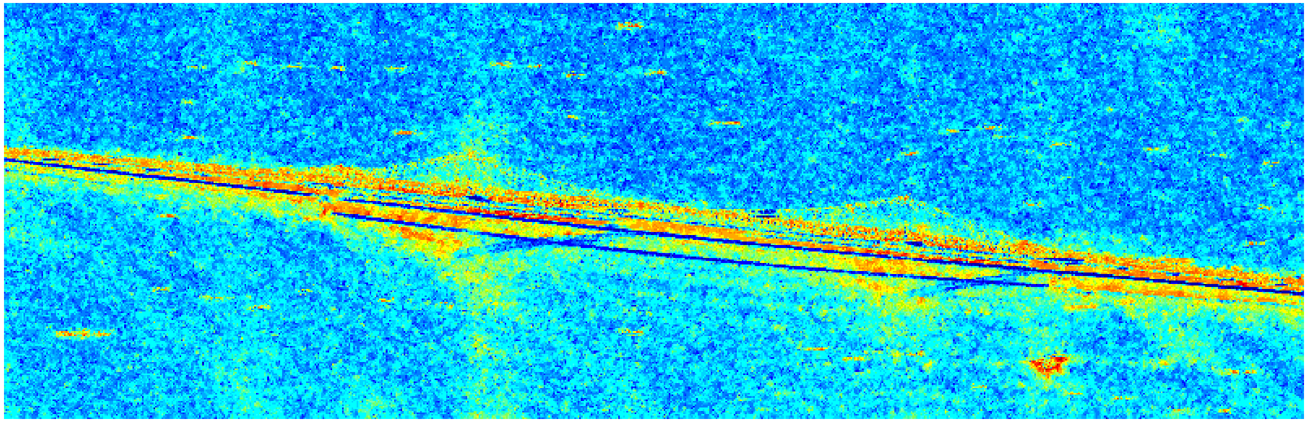
Buoy



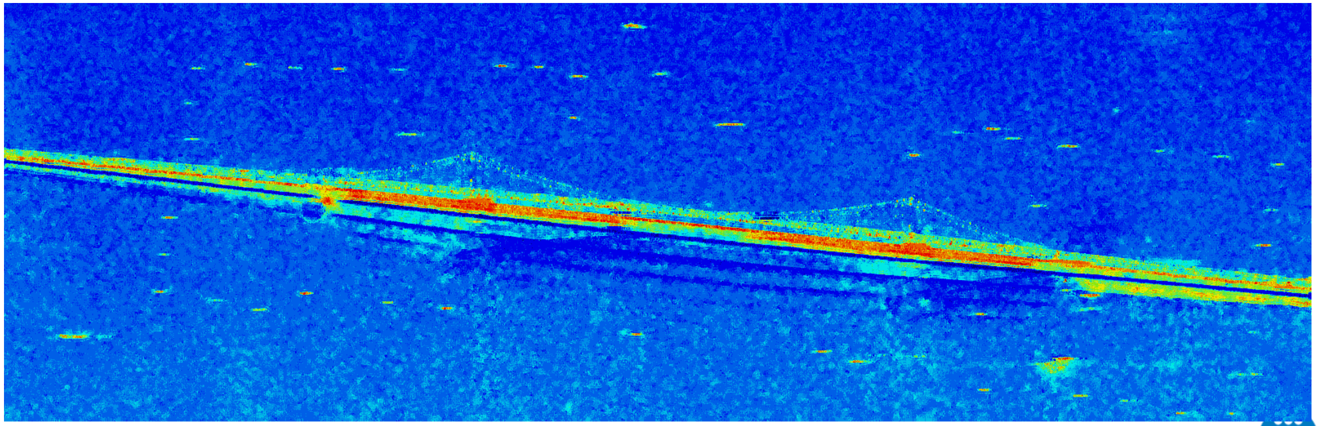
Navigation Map of Storebelt, Denmark

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory and Communications

Polarimetric Scattering Characterization

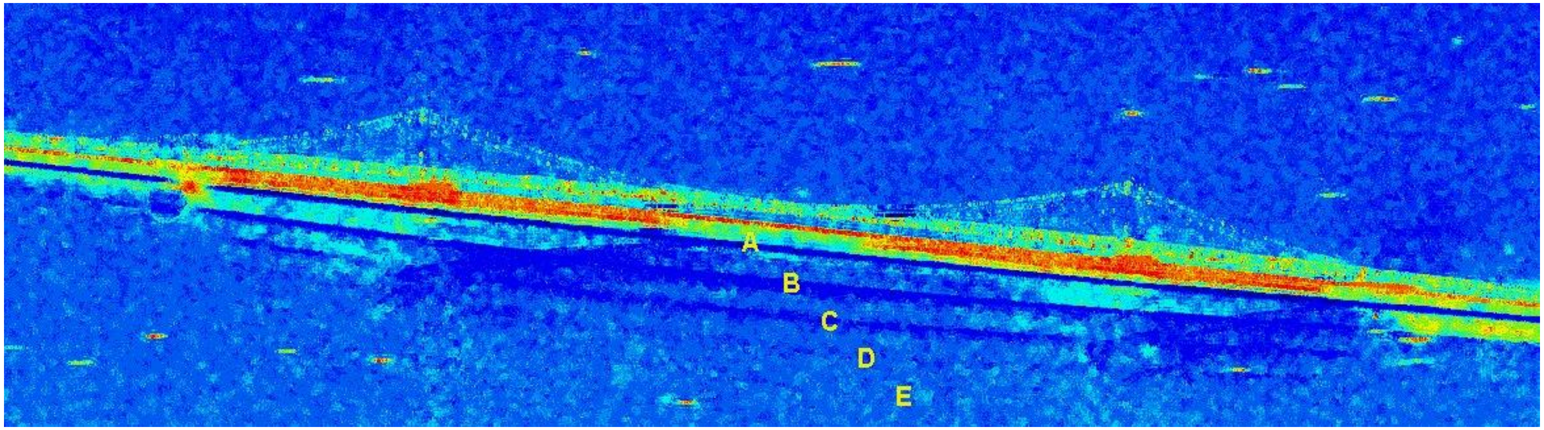


H *Entropy*

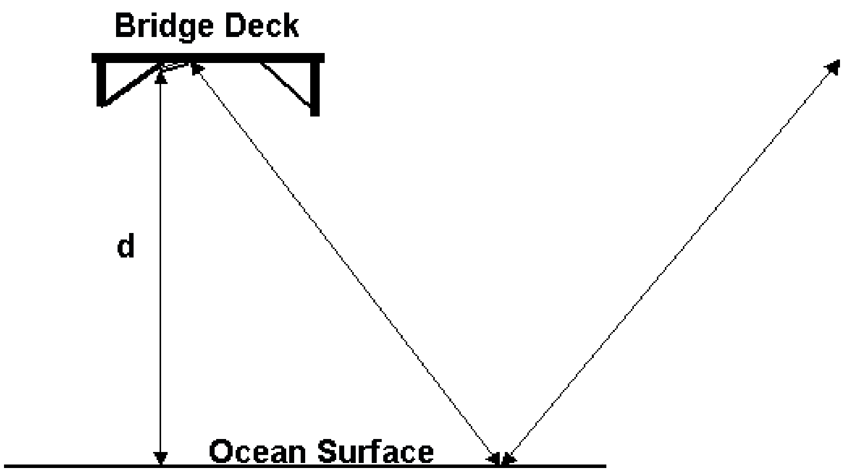


α *Angle*

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory
and Communications



α *Angle*



Roundtrip distances:

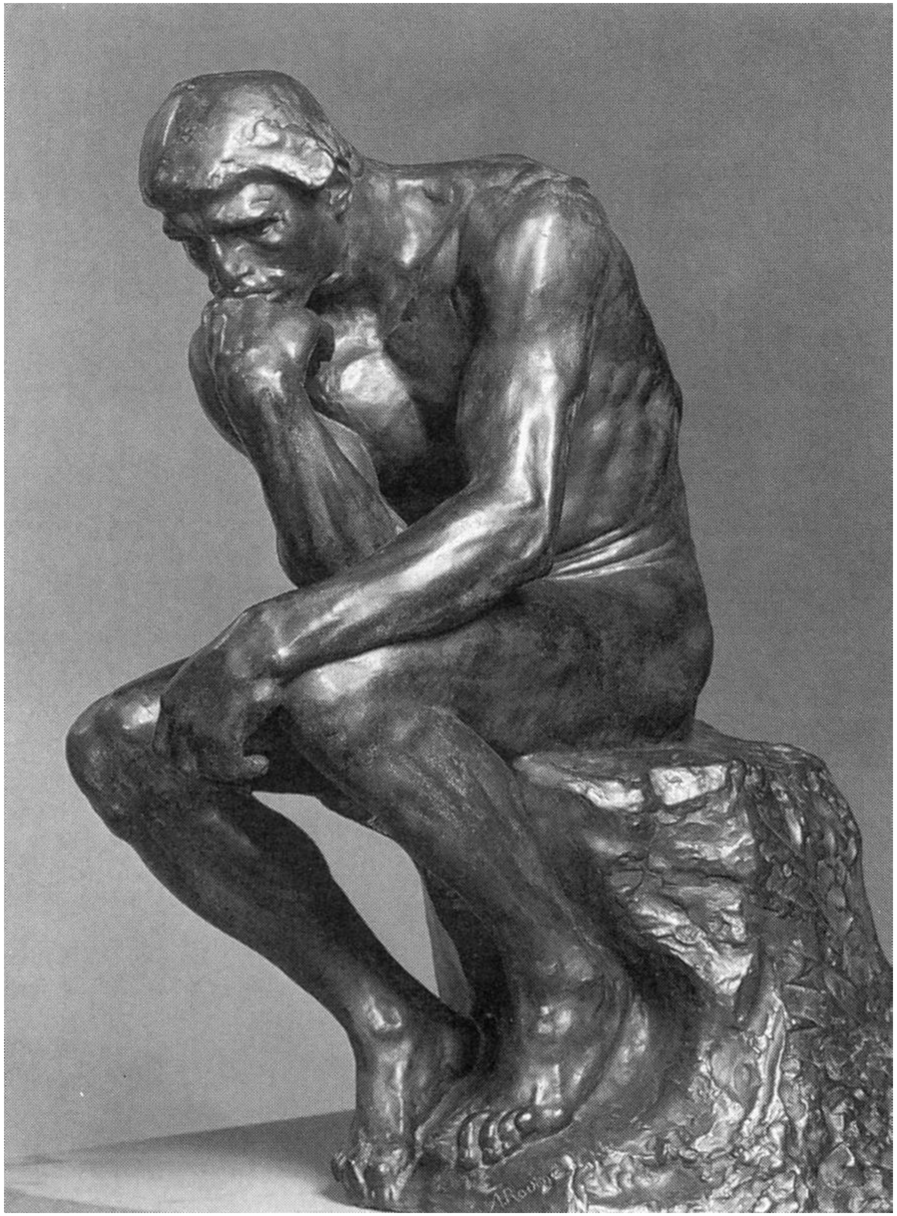
- **A: Triple bounces,** $2(L+d\cos\theta)$
- **B: 5 (or 7) bounces,** $2(L+d\cos\theta)+2d$
- **C: 7 (or 9) bounces,** $2(L+d\cos\theta)+4d$
- **D: 9 (or 11) bounces,** $2(L+d\cos\theta)+6d$

(…) indicates additional two bounces from the bottom of the deck.

RESONANT CAVITY

Dominant scattering mechanisms are extracted by applying target decomposition:
Blue= Surface Green = Dipole Red = Double Bounce

Questions



UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Department of Signal Theory
and Communications