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THE EUROPEAN SPACE AGENCY

8th ESA Advanced Course on Radar Polarimetry 2026

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SAR polarimetry basics

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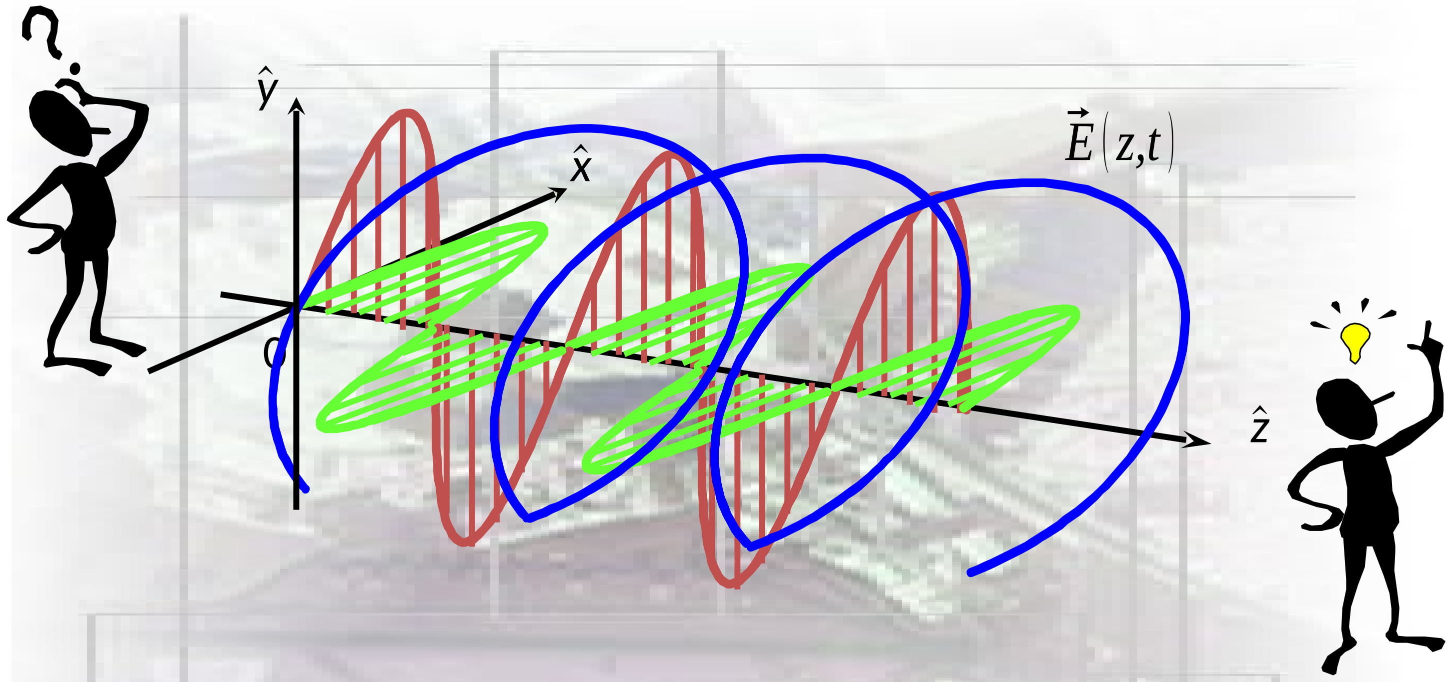
Objective

To provide the minimum, but necessary, amount of knowledge required to understand scientific works on :

SAR Polarimetry (PolSAR)

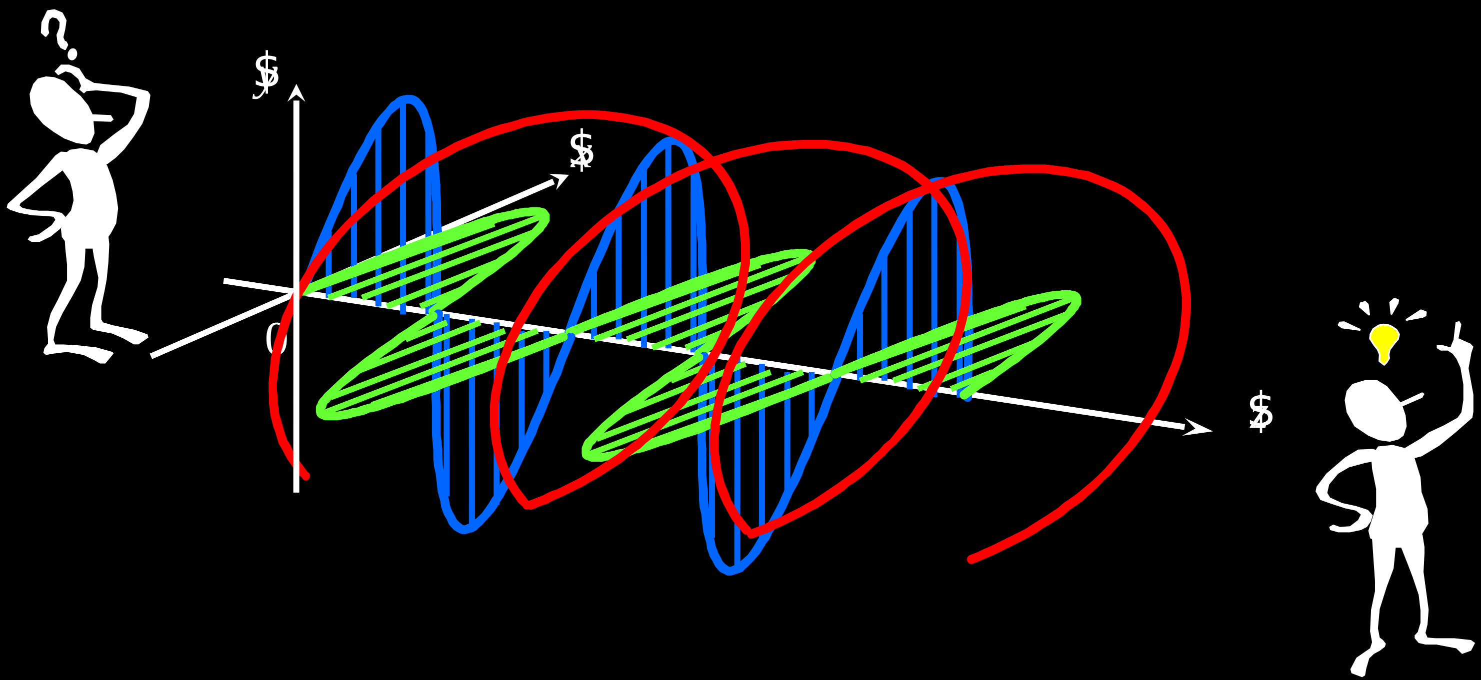
SAR Polarimetry + Interferometry (Pol-InSAR)

SAR Polarimetry + Tomography (Pol-TomSAR)



GENERAL INTRODUCTION

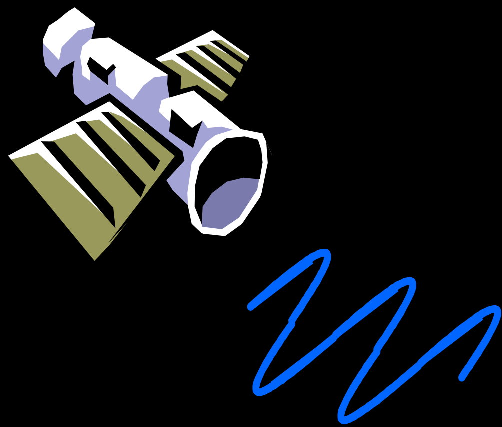
Radar Polarimetry



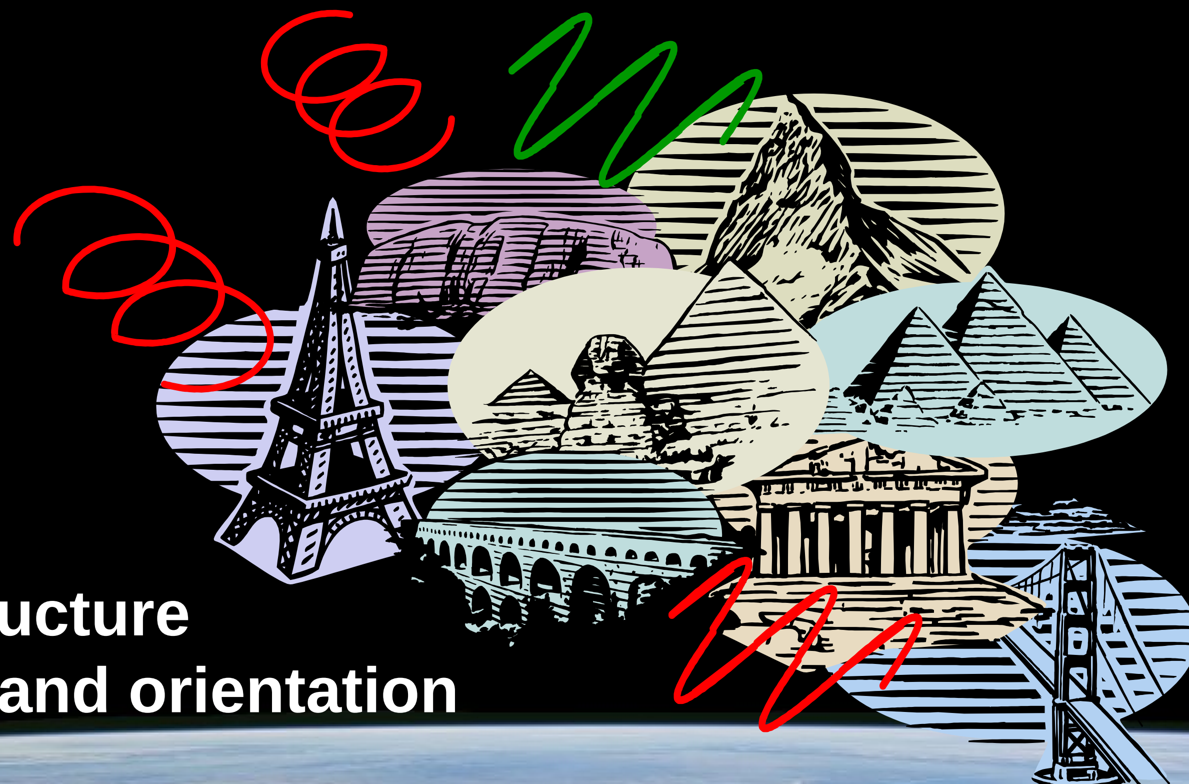
Radar Polarimetry (Polar : polarisation Metry: measure) is the science of acquiring, processing and analysing the polarization state of an electromagnetic field

Radar Polarimetry deals with the full vector nature of polarized electromagnetic waves

Radar Polarimetry



The POLARISATION information
Contained in the waves backscattered
from a given medium is highly related to:



its geometrical structure
reflectivity, shape and orientation

its geophysical properties such as humidity, roughness, …

SAR Polarimetry Applications

Forest Vegetation

- Forest Height
- Forest Biomass
- Forest Structure
- Canopy Extinction
- Underlying Topography

- Forest Ecology
- Forest Management
- Ecosystem Change
- Carbon Cycle

Agriculture

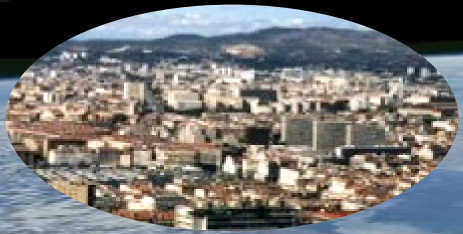
- Soil Moisture Content
- Soil roughness
- Height of Vegetation Layer
- Extinction of Vegetation Layer
- Moisture of Vegetation Layer

- Farming Management
- Water Cycle
- Desretification

Snow and Ice

- Topography
- Penetration Depth / Density
- Snow Ice Layer
- Snow Ice Extinction
- Water Equivalent

- Ecosystem Change
- Water Cycle
- Water Management



Urban Areas

- Geometric Properties
- Dielectric Properties

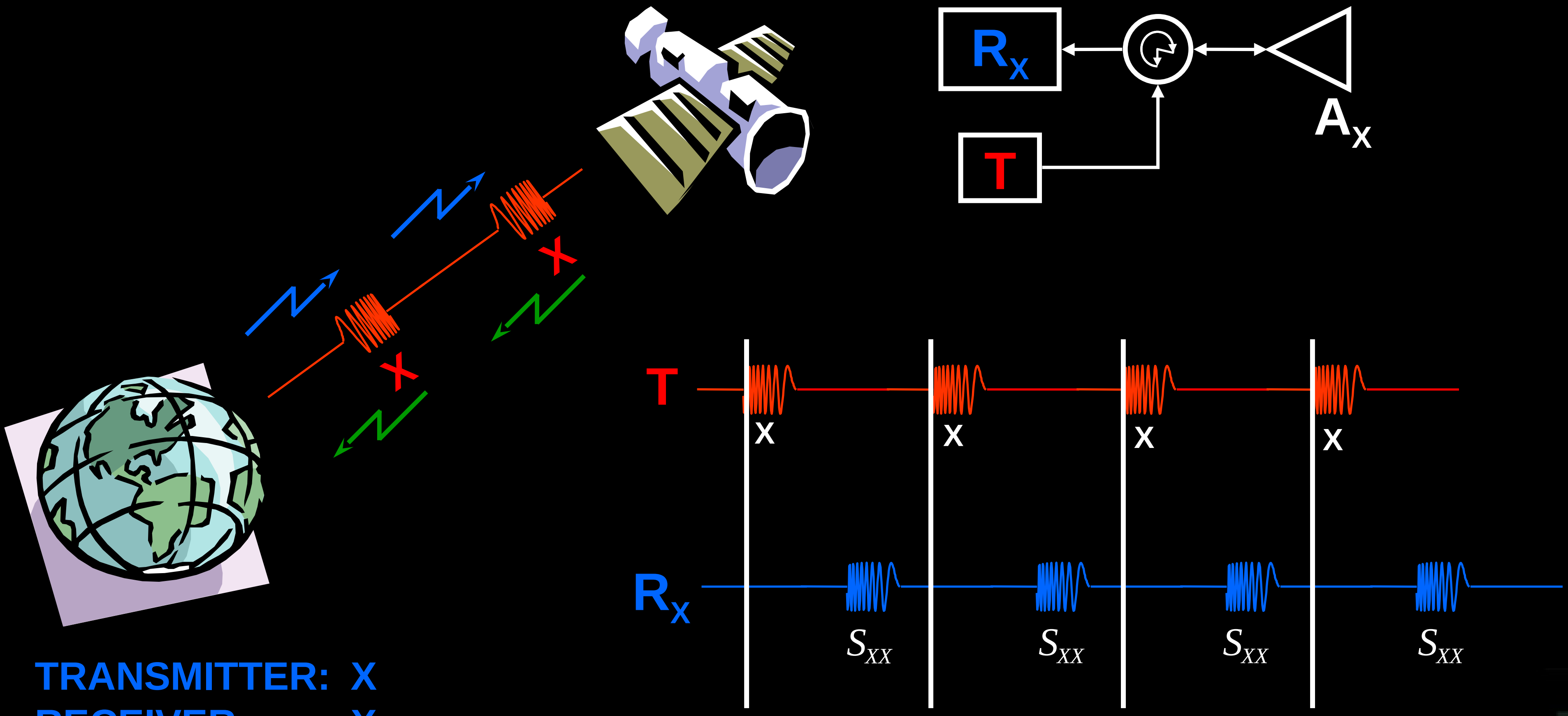
- Urban Monitoring

DLR Courtesy of Dr. I. Hajnsek

Polarimetric Radar (SAR)

Spaceborne Sensors

Scattering Coefficient

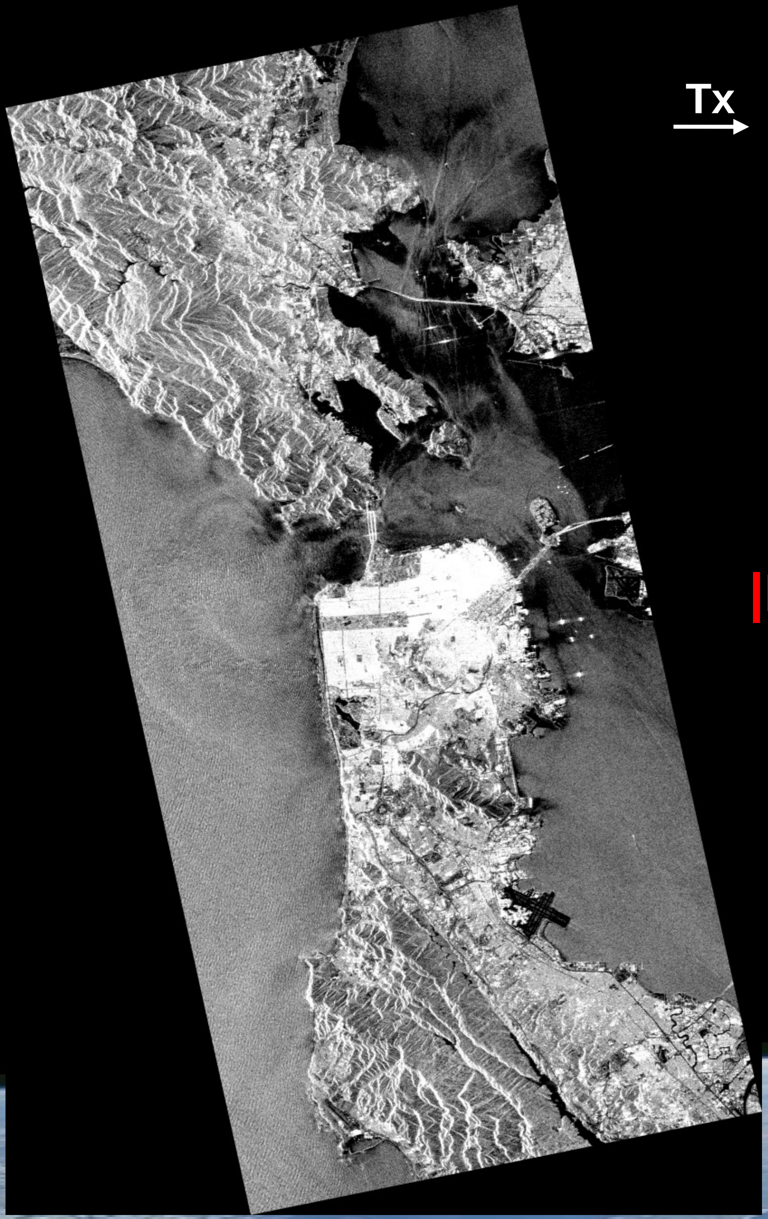


TRANSMITTER: X
RECEIVER: X

BACKSCATTERING COEFFICIENT $\{ S_{XX} \}$

NO POLARIMETRY

Space-borne Sensors

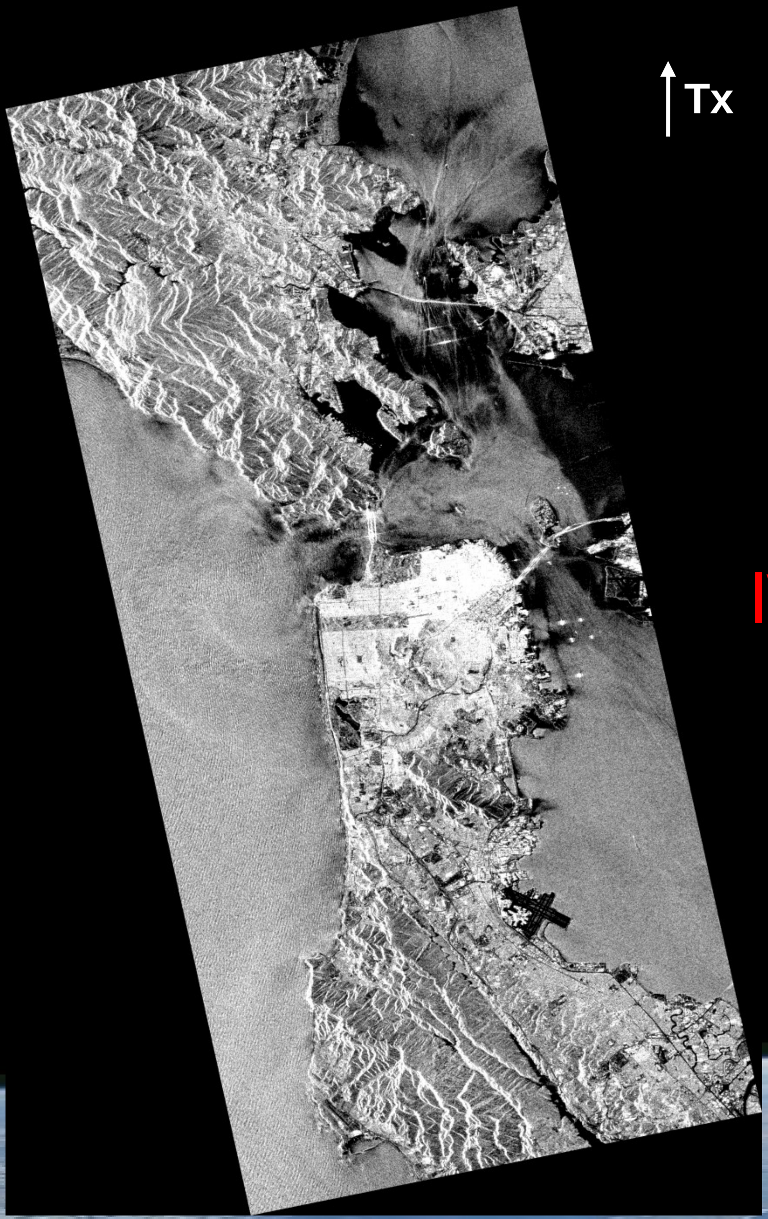


Tx → Rx →

$|HH|_{dB}$

San Francisco Bay – (L-Band)

Space-borne Sensors

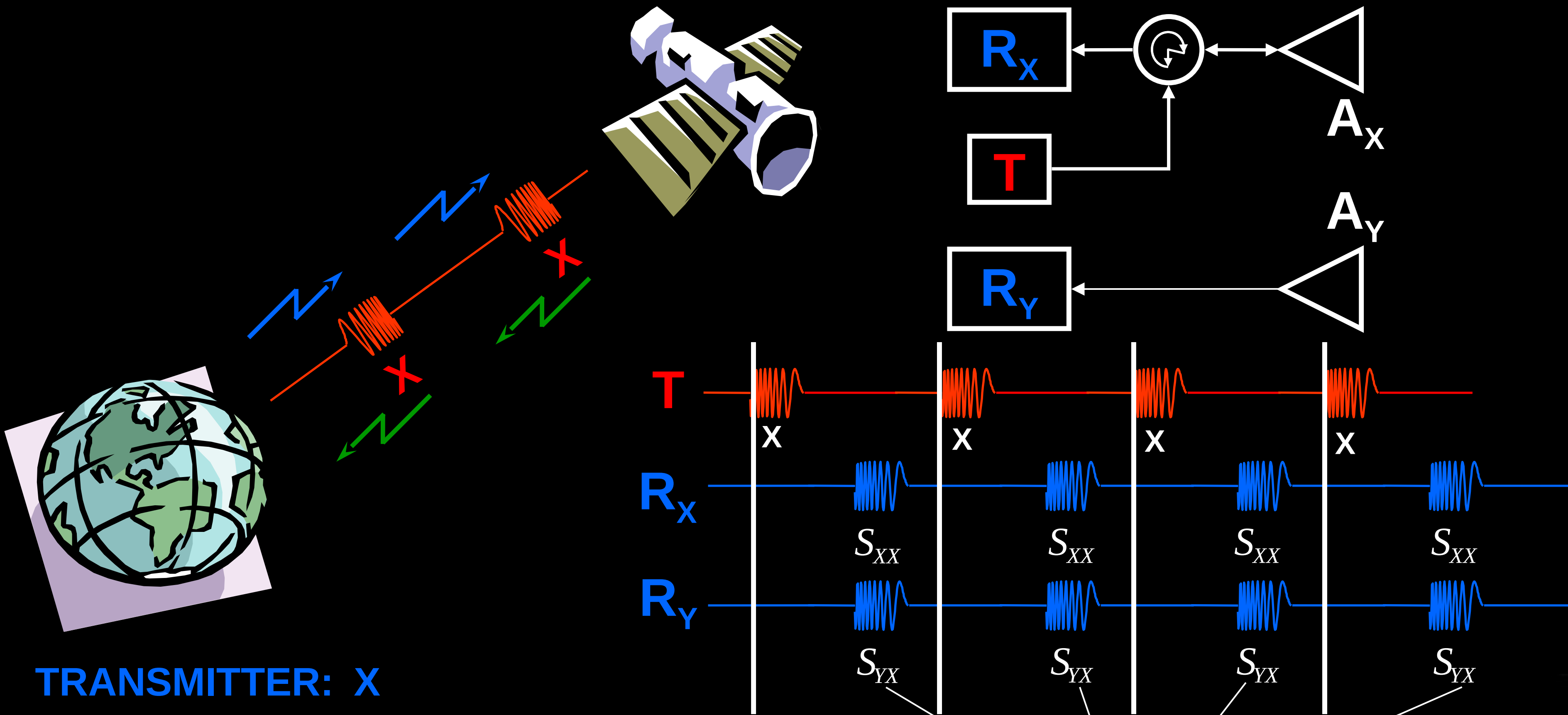


Tx Rx

$|VV|_{dB}$

San Francisco Bay – (L-Band)

Wave Polarimetry

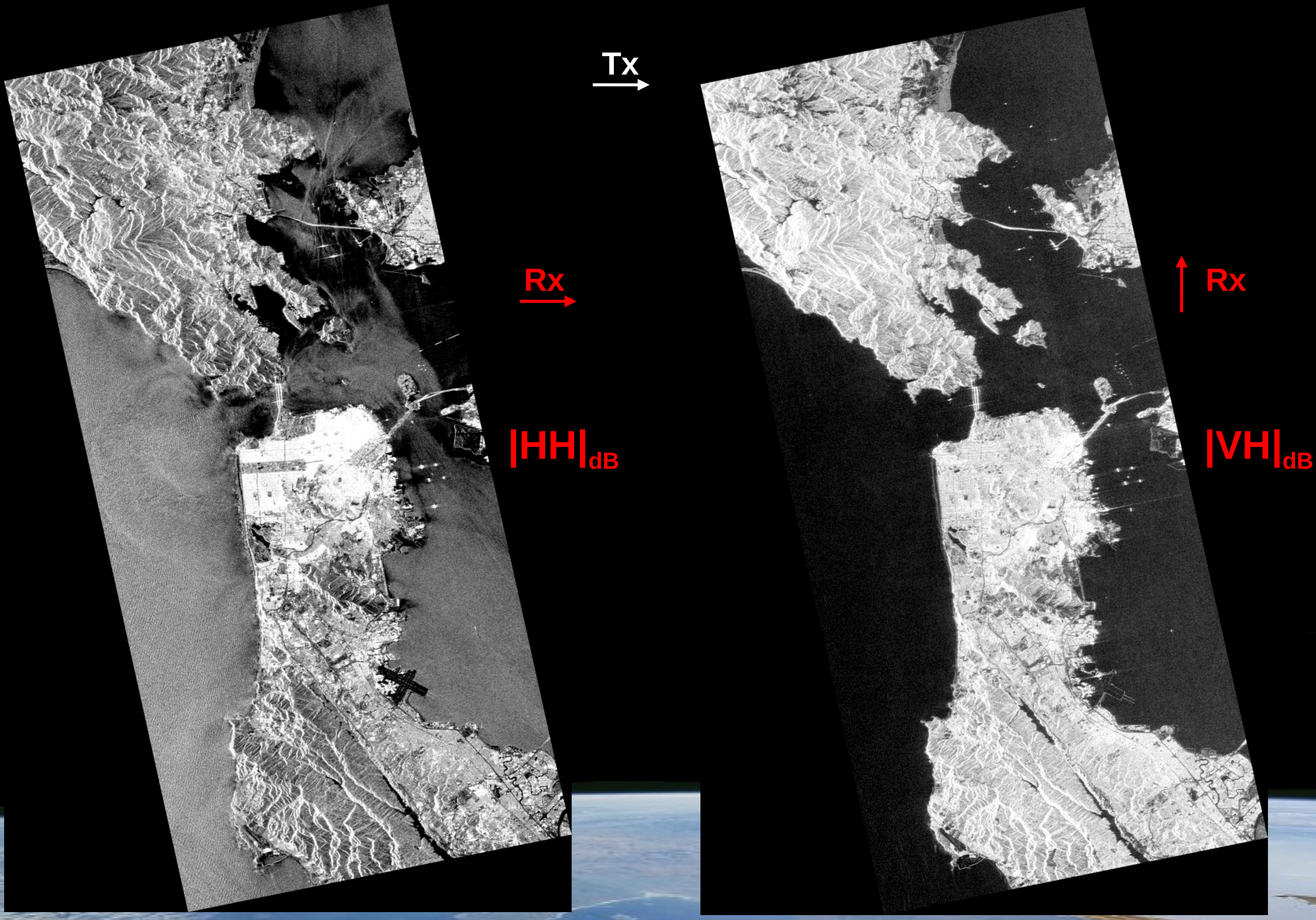


TRANSMITTER: X
RECEIVERS: X & Y

JONES VECTORS $\left\{ \underline{E}_S = \begin{bmatrix} S_{XX} \\ S_{YX} \end{bmatrix} \right\}$

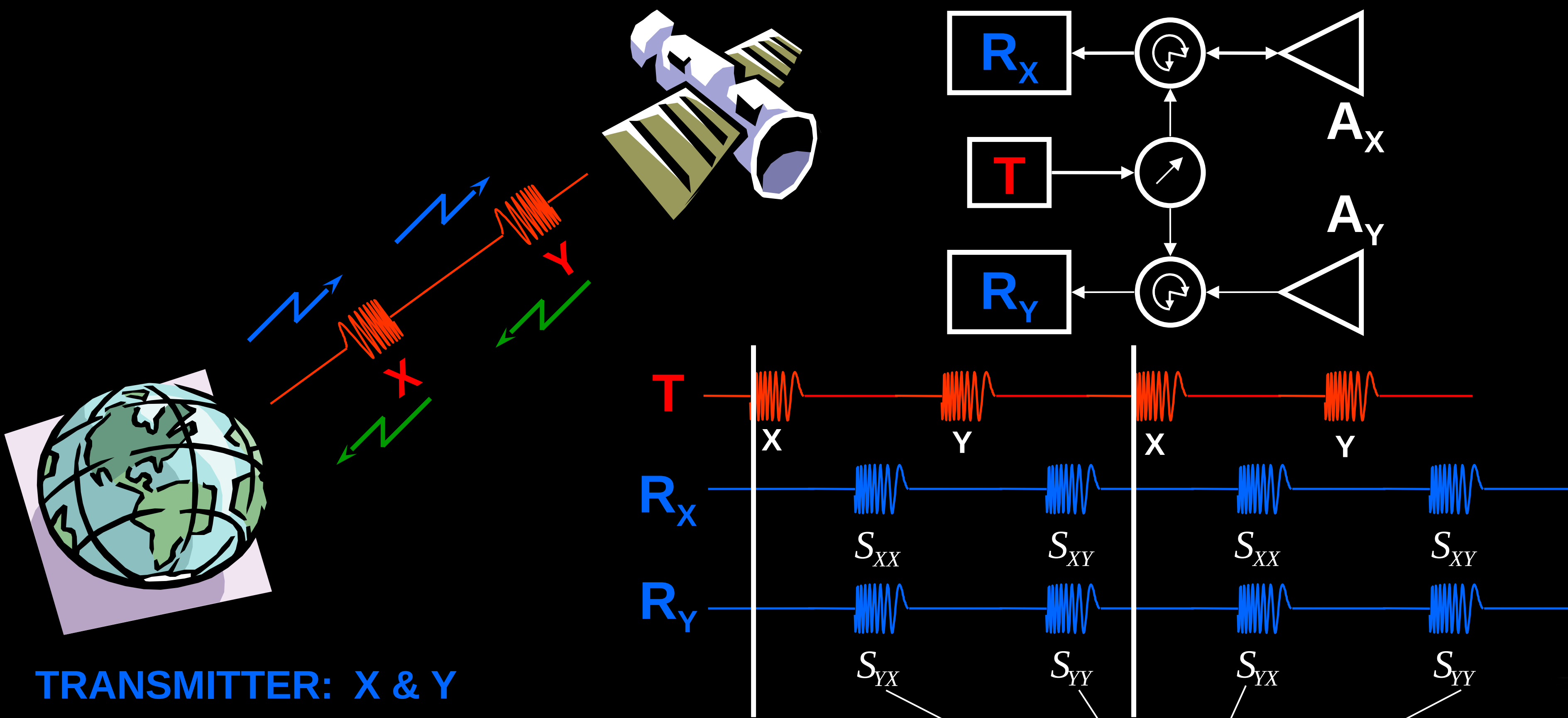
WAVE POLARIMETRY

Space-borne Sensors



San Francisco Bay – (L-Band)

Scattering Polarimetry



TRANSMITTER: X & Y

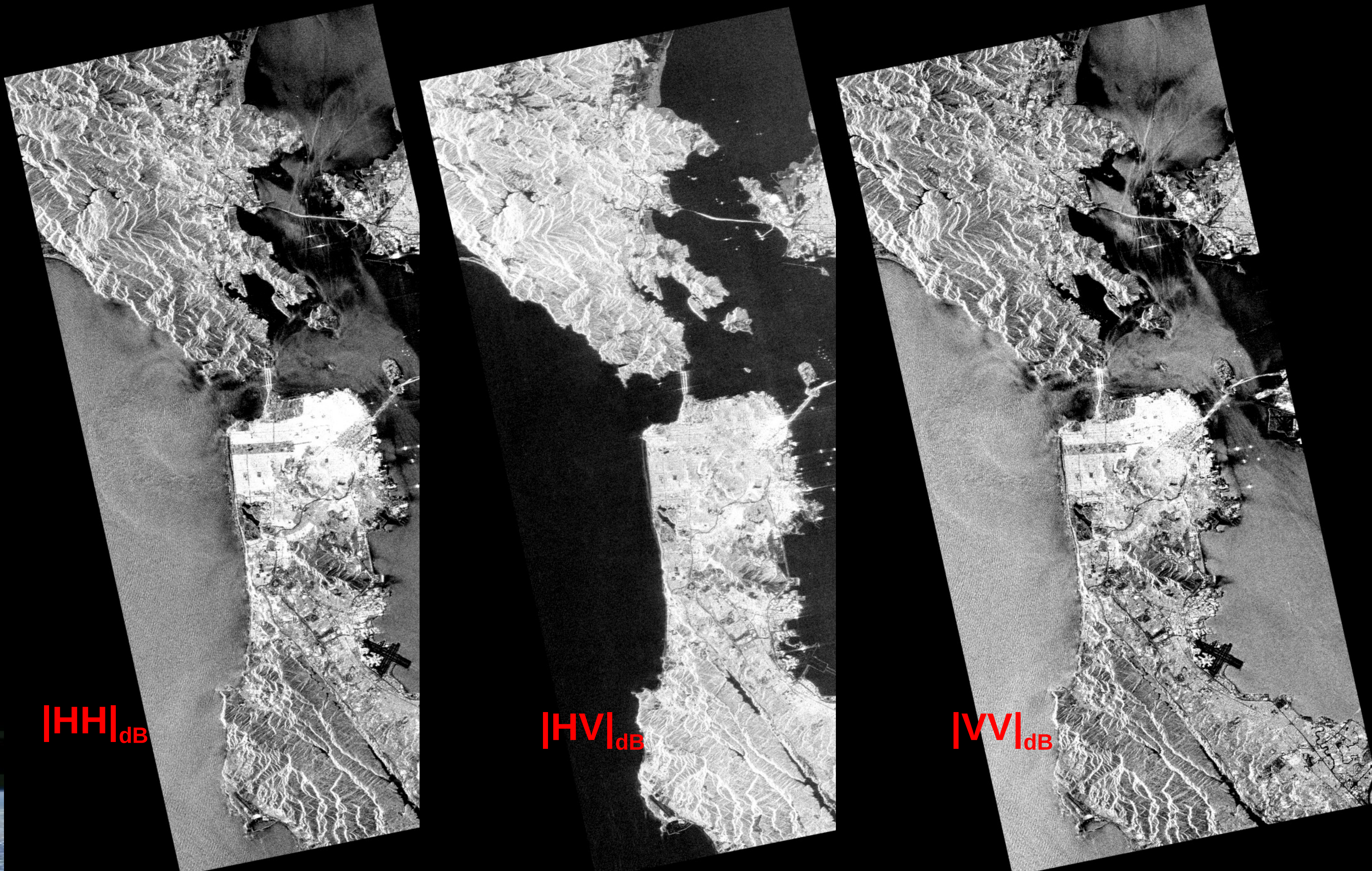
RECEIVERS: X & Y

SINCLAIR MATRICES

$$\left\{ [S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix} \right\}$$

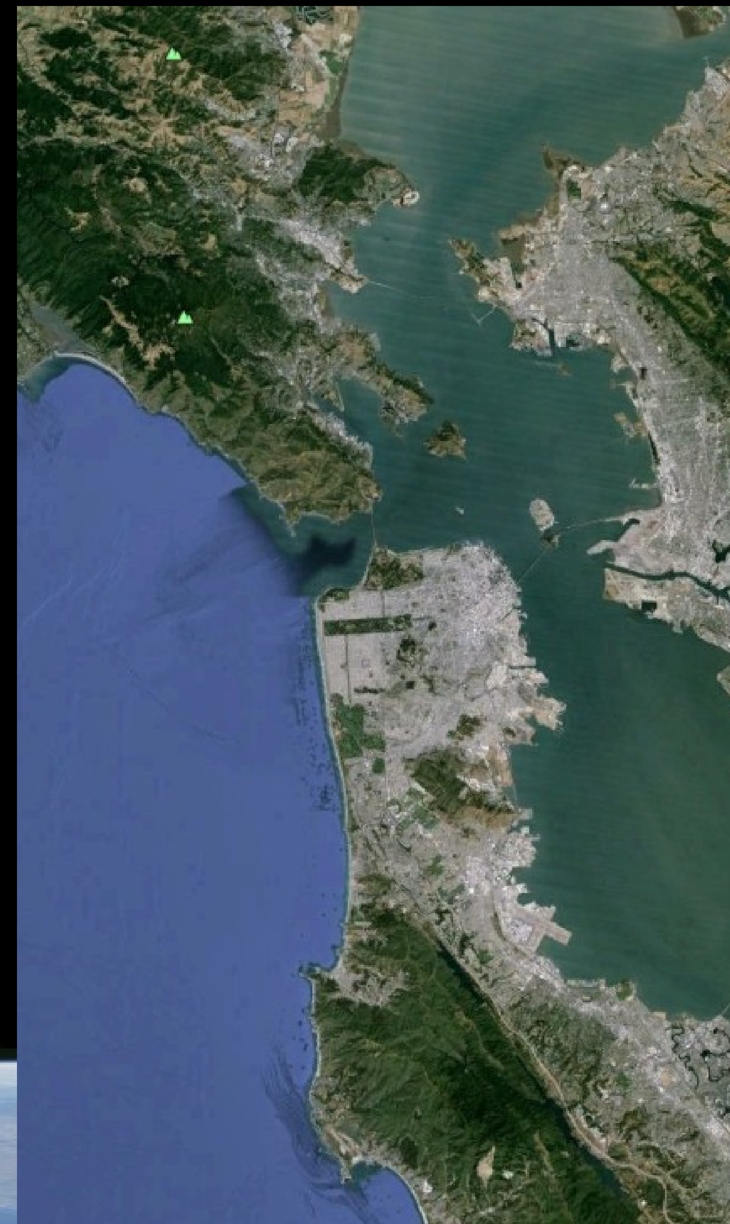
SCATTERING POLARIMETRY

Space-borne Sensors



San Francisco Bay – (L-Band)

Space-borne Sensors

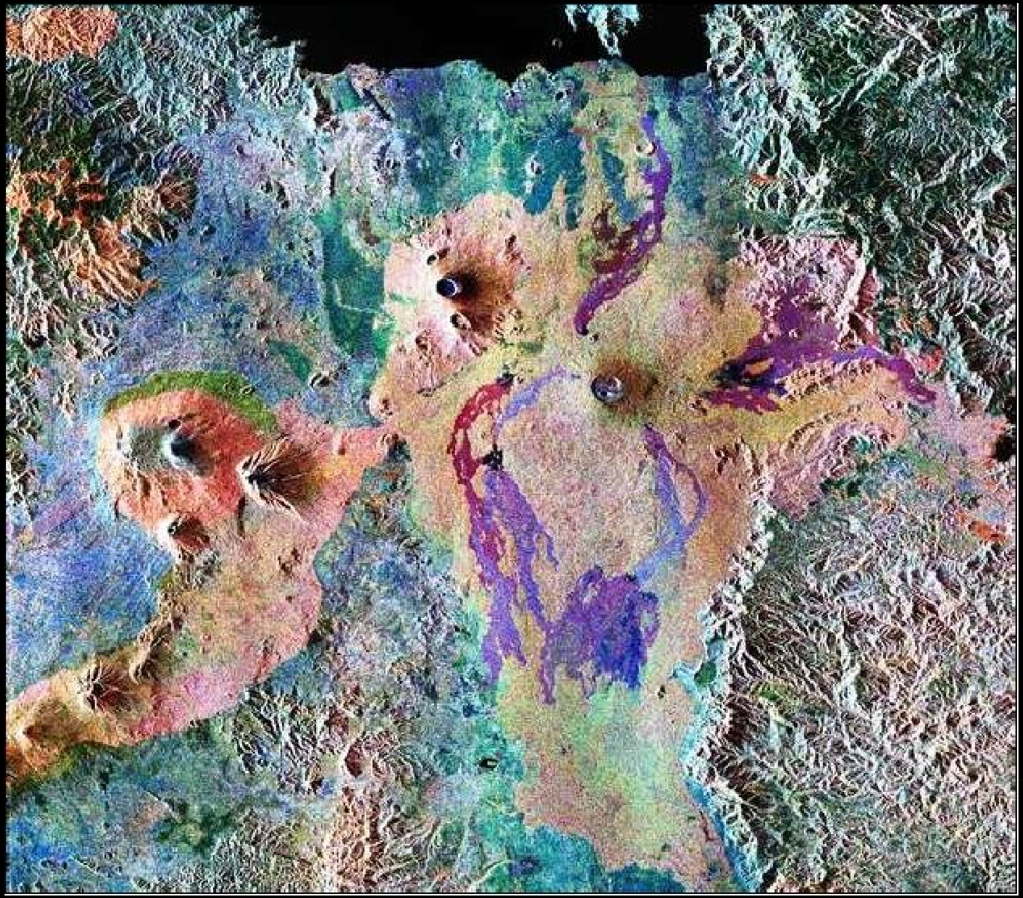


San Francisco Bay – (L-Band)

Space-borne PolSAR Sensors

SIR-C / X-SAR

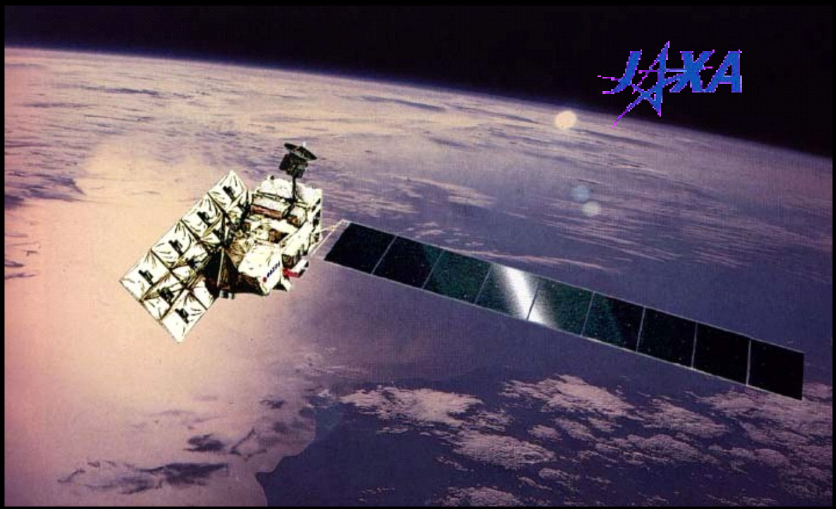
April 1994
L- and C-Band (Quad)
X-Band (Sngl)



Rwanda, Zaire, Uganda

Space-borne PolSAR Sensors

ALOS - PALSAR



January 2006

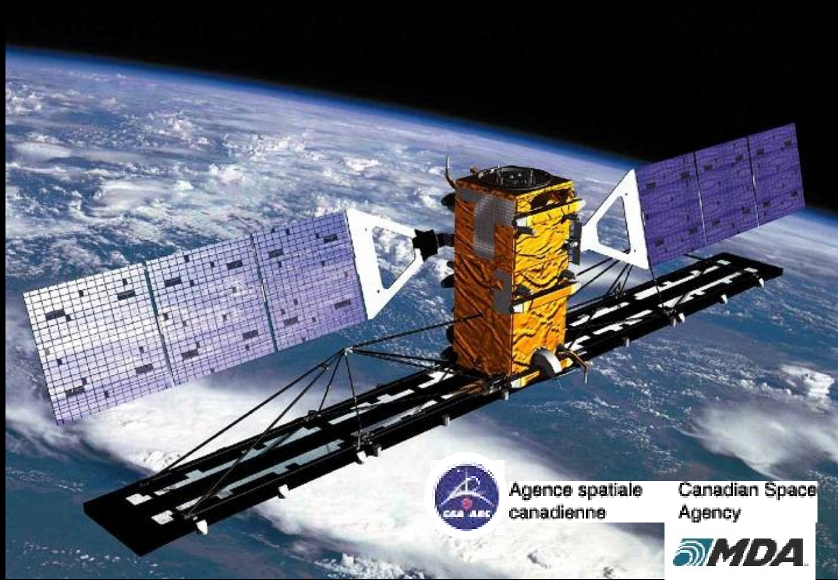
L-Band (Sngl / Twin / Quad)

ALOS : Advanced Land Observing Satellite

PALSAR : Phase Array L-Band SAR

Space-borne PolSAR Sensors

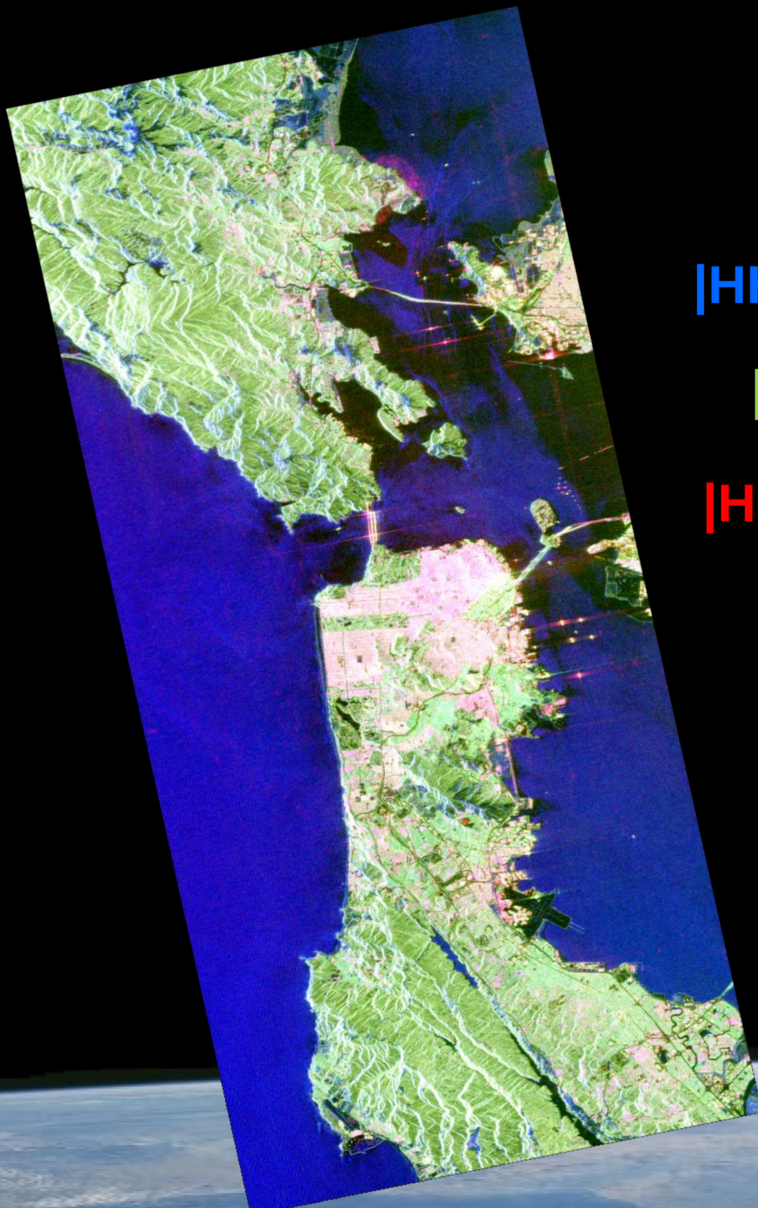
RADARSAT - 2



December 2007

C-Band (Quad)

Space-borne Sensors



$|HH+VV|_{dB}$

$|HV|_{dB}$

$|HH-VV|_{dB}$

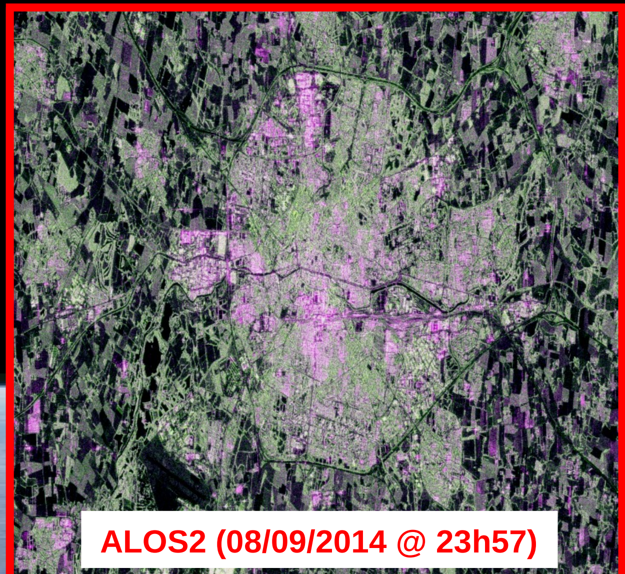
San Francisco Bay – (L-Band and C-Band)

Space-borne PolSAR Sensors

ALOS - 2

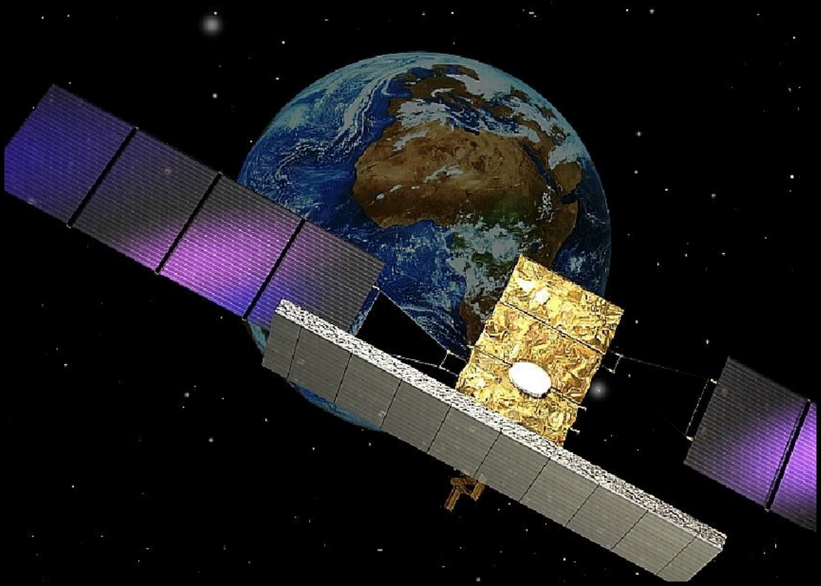
May 2014

L-Band (Quad)



Space-borne PolSAR Sensors

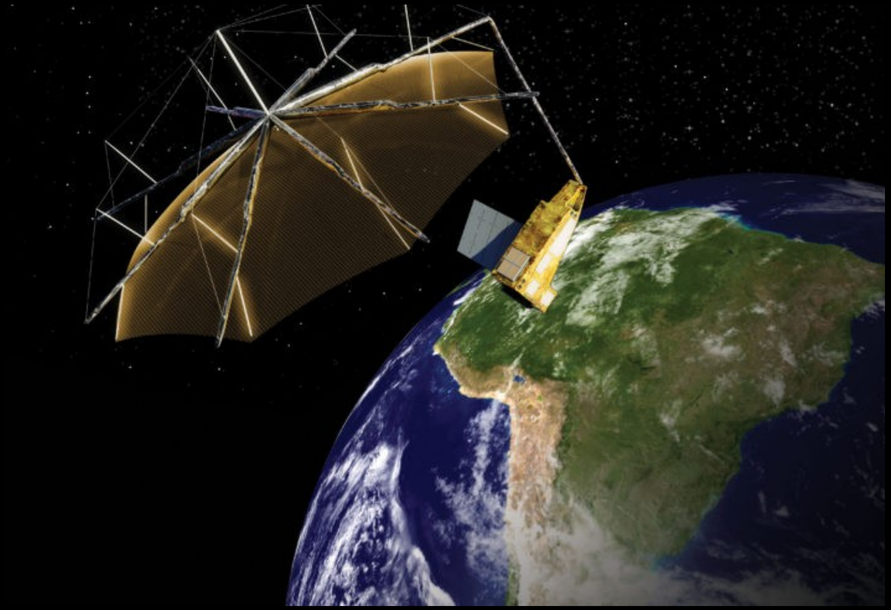
COSMO - SkyMed - CSG



2A : 2018 2B : 2019

X-Band (Sngl / Dual / Quad Exp.)

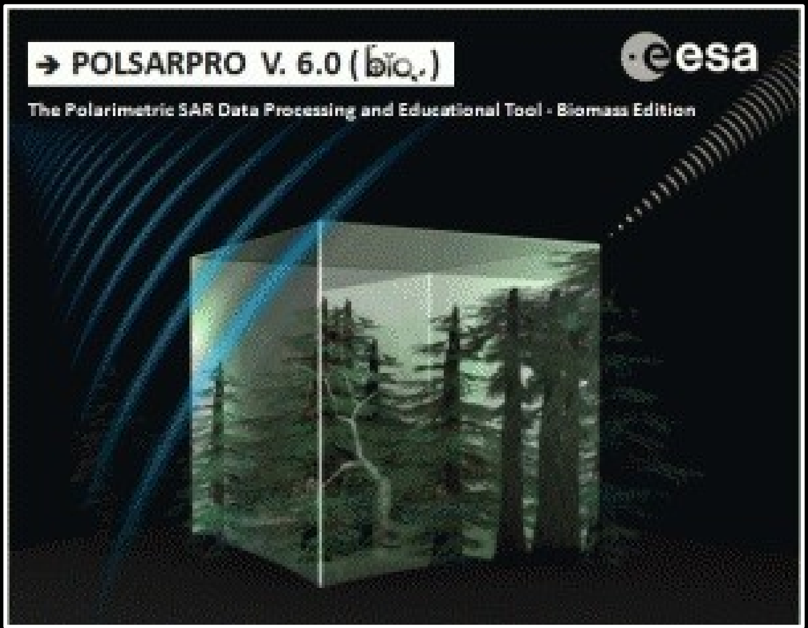
Earth Explorer - BIOMASS



2021

P-Band (Quad)

ESA PolSARpro Toolbox



Polarimetric SAR data Processing and educational tool

esa since 2003

- +3000 registered users
- +70 foreign countries

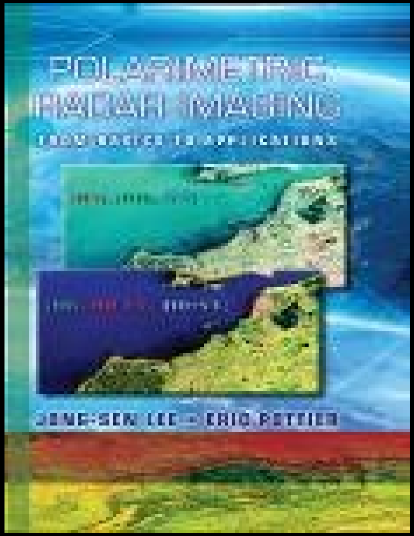
International Collaborative Project

(Agencies, Research Centres, Universities)

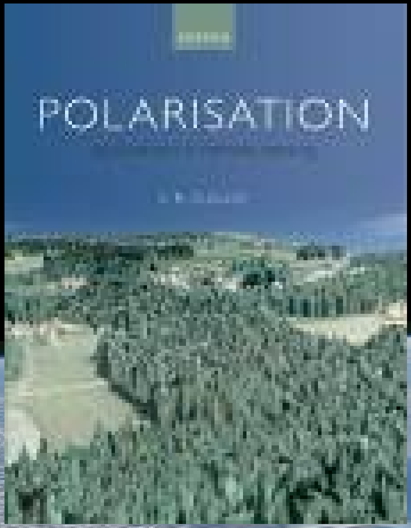
Teaching / Training

Next P.I Generations

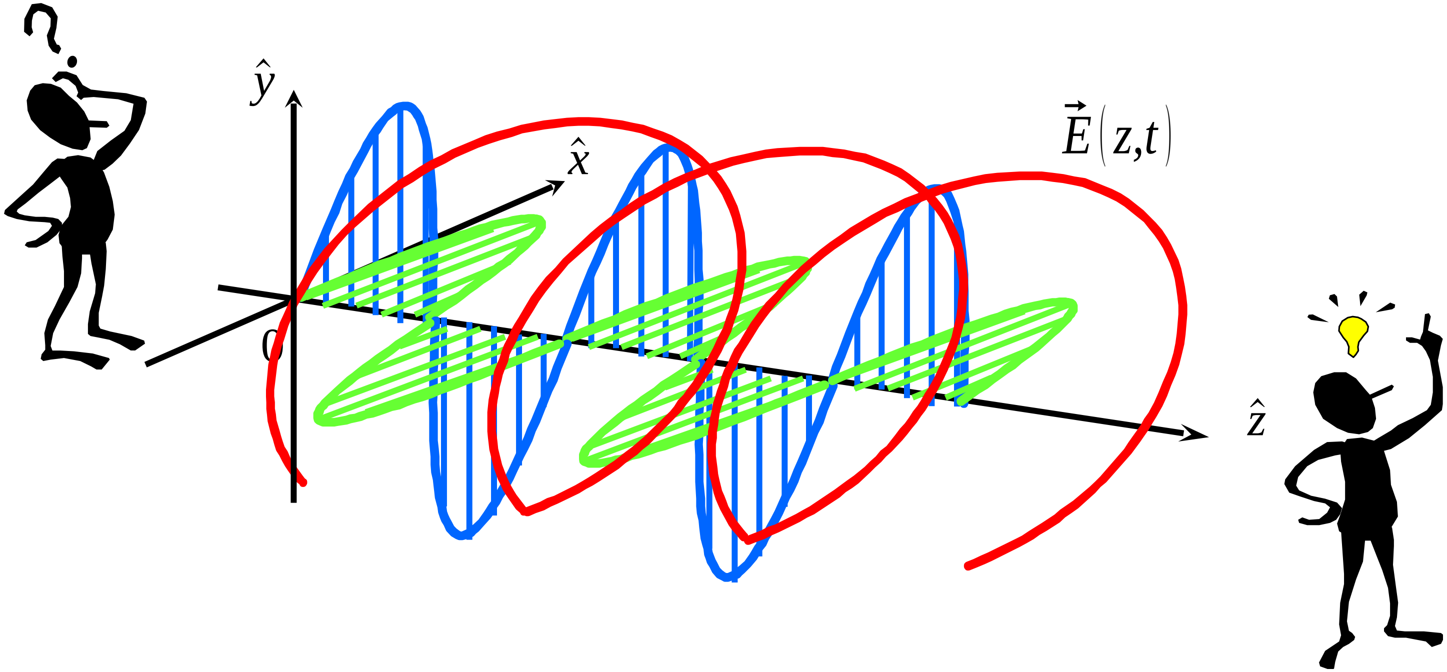
Books On Polarimetric Radar
SAR, Polarimetric Interferometry



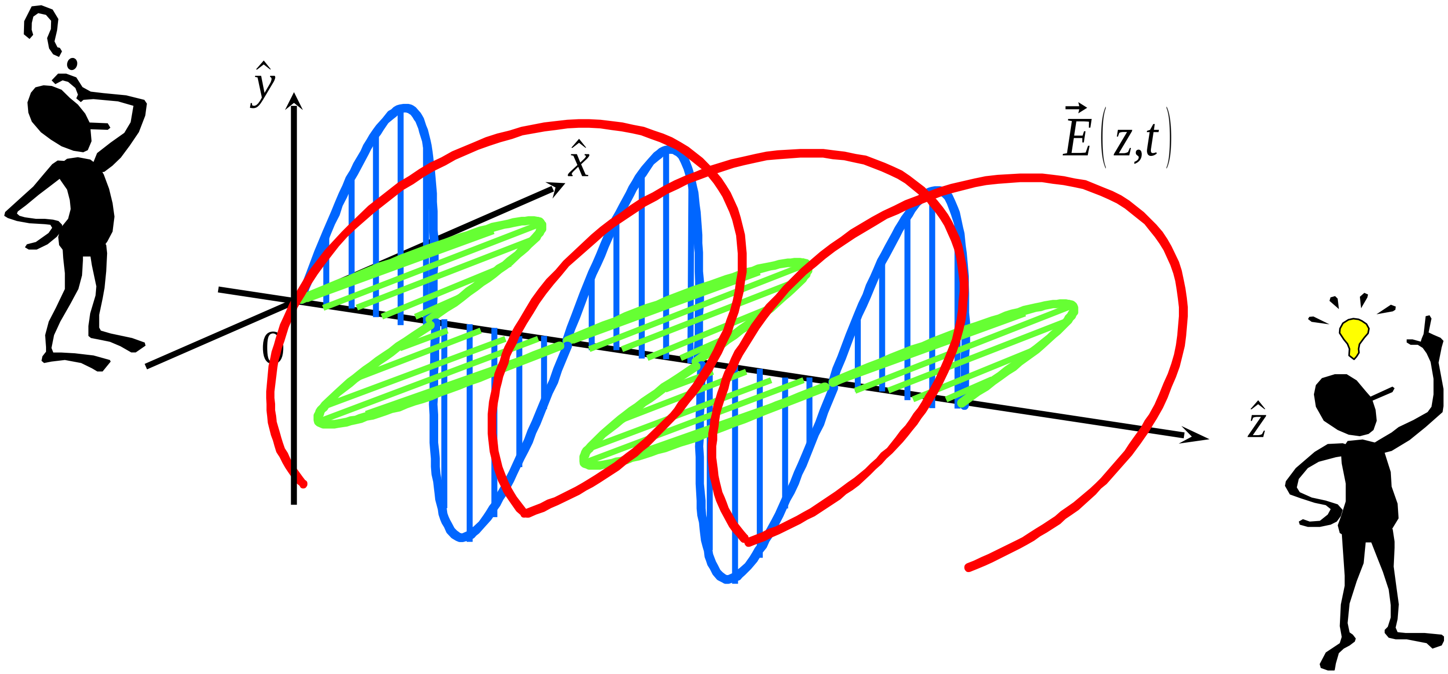
Polarimetric Radar Imaging: From basics to applications
Jong-Sen LEE – Eric POTTIER
CRC Press; 1st ed., February 2009, pp 422
ISBN: 978-1420054972



Polarisation: Applications in Remote Sensing
Shane R. CLOUDE
Oxford University Press, October 2009, pp 352
ISBN: 978-0199569731



BASIC CONCEPTS



WAVE POLARIMETRY

PROPAGATION EQUATION

REAL ELECTRIC FIELD VECTOR $\vec{E}(z,t)$

MAXWELL EQUATIONS

MAXWELL – FARADAY EQUATION $\nabla \wedge \vec{E}(z,t) = -\dfrac{\partial \vec{B}(z,t)}{\partial t}$

MAXWELL – AMPERE EQUATION $\nabla \wedge \vec{H}(z,t) = \vec{J}_T(z,t)$

GAUSS THEOREM $\nabla \cdot \vec{D}(z,t) = \rho(z,t)$

$$\nabla \cdot \vec{B}(z,t) = 0$$

σ *(Conductivity)*

μ *(Permeability)*

ε *(Permittivity)*

PROPAGATION EQUATION

$$\nabla \wedge \left(\nabla \wedge \vec{A}\right) = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla \cdot \left(\nabla \vec{A}\right)$$

PROPAGATION EQUATION

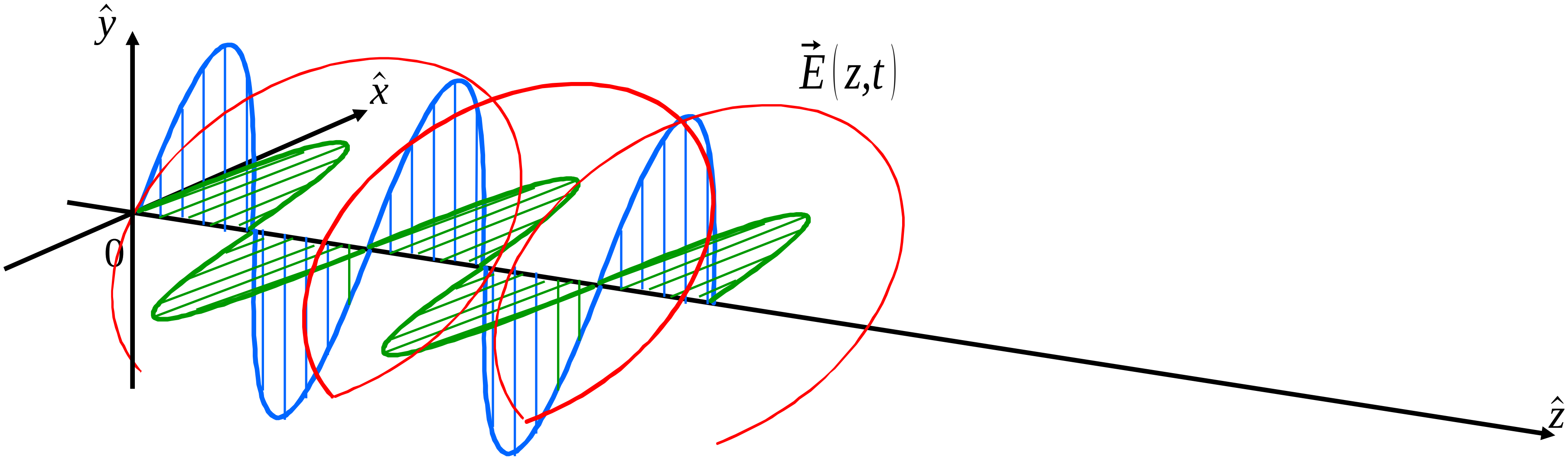
$$\nabla^2 \vec{E}\left(z,t\right) - \mu\varepsilon \frac{\partial^2 \vec{E}\left(z,t\right)}{\partial\ t^2} - \mu\sigma \frac{\partial\ \vec{E}\left(z,t\right)}{\partial\ t} = -\frac{1}{\varepsilon}\frac{\partial\ \rho\left(z,t\right)}{\partial\ t}$$

HELMHOLTZ PROPAGATION EQUATION

$$\nabla^2 \vec{E}\left(z,t\right) - \mu\varepsilon \frac{\partial^2 \vec{E}\left(z,t\right)}{\partial\ t^2} = 0$$

Source Free, Linear, Homogeneous, Isotropic, Dielectric and lossless Medium

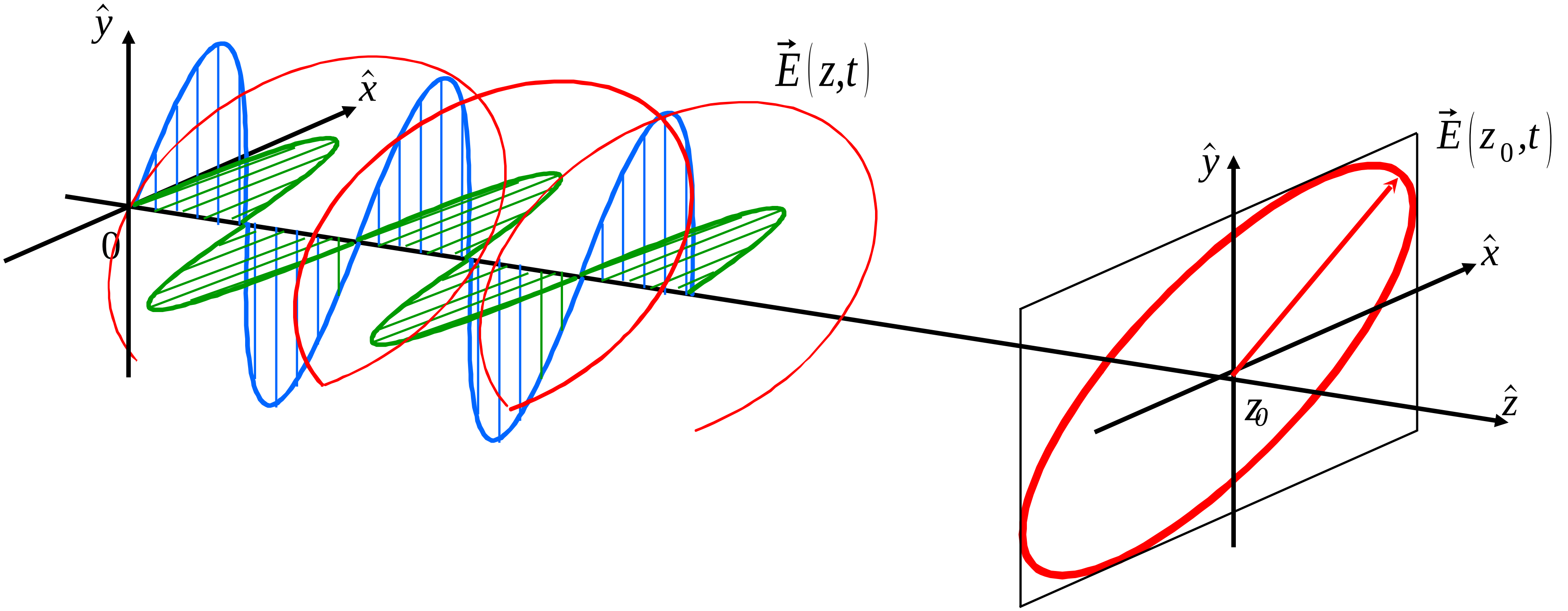
POLARISATION ELLIPSE



REAL ELECTRIC FIELD VECTOR

$$\vec{E}(z,t) = \begin{Bmatrix} E_x = E_{0x}\cos(\omega t - kz - \delta_x) \\ E_y = E_{0y}\cos(\omega t - kz - \delta_y) \\ E_z = 0 \end{Bmatrix}$$

POLARISATION ELLIPSE

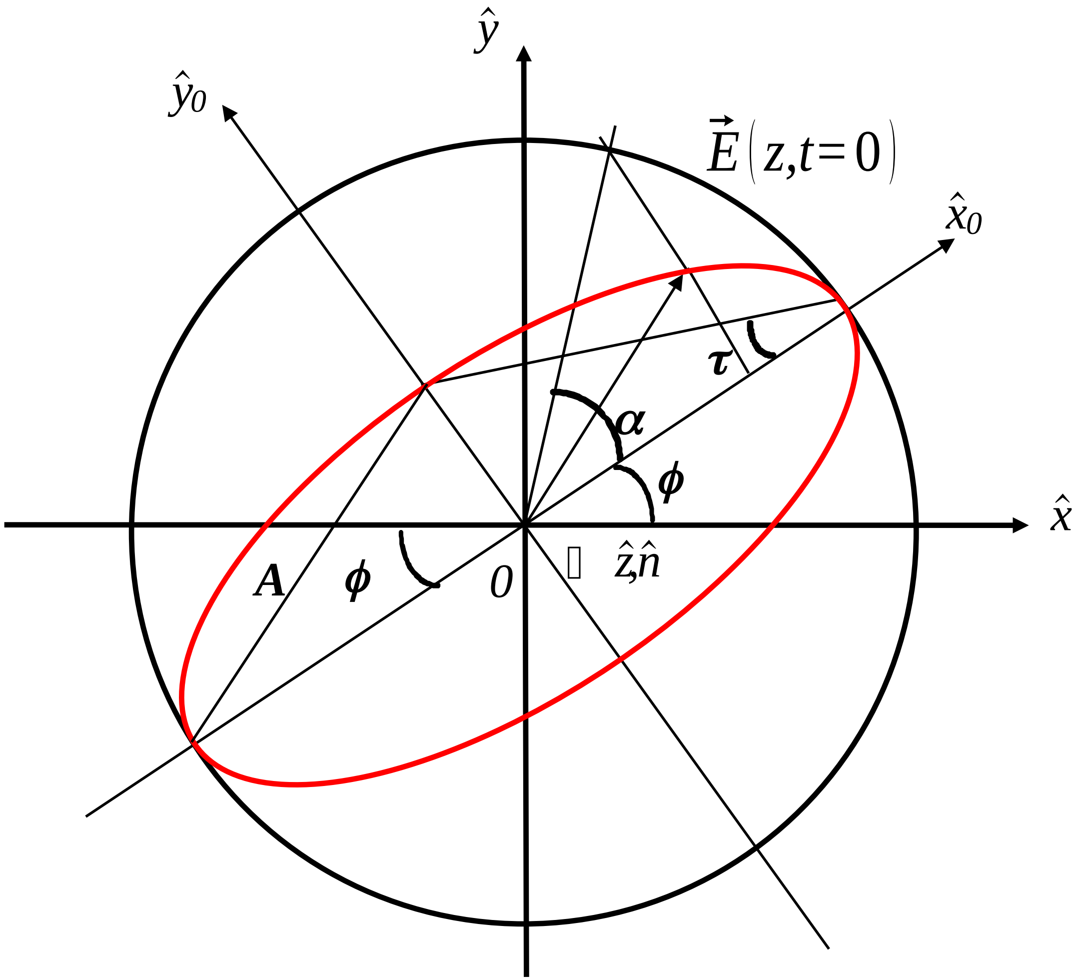


THE REAL ELECTRIC FIELD VECTOR MOVES IN TIME ALONG AN ELLIPSE

$$\left(\frac{E_x}{E_{0x}}\right)^2 - 2\frac{E_x E_y}{E_{0x} E_{0y}}\cos(\delta) + \left(\frac{E_y}{E_{0y}}\right)^2 = \sin^2(\delta)$$

With: $\delta = \delta_y - \delta_x$

POLARISATION ELLIPSE



A : **WAVE AMPLITUDE**

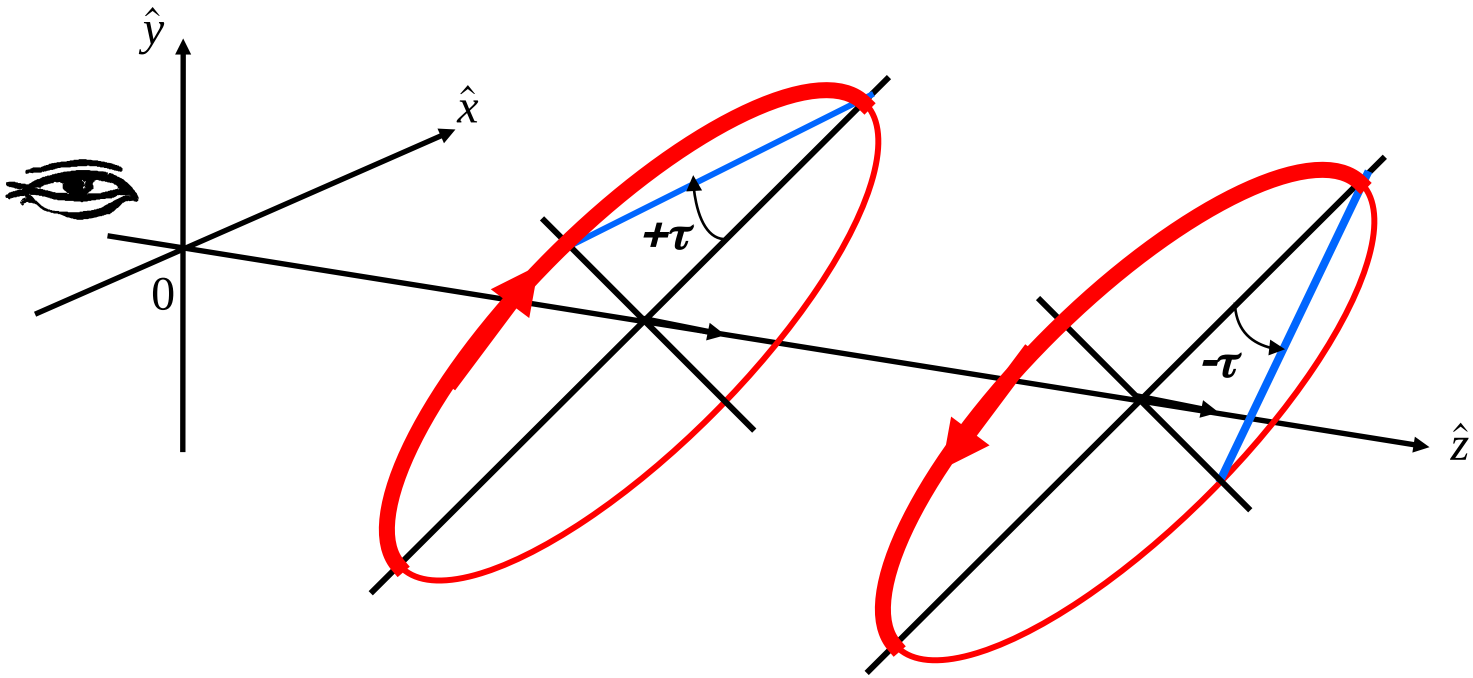
$\boldsymbol{\alpha}$: **ABSOLUTE PHASE**

$\boldsymbol{\phi}$: **ORIENTATION ANGLE** $-\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}$

$\boldsymbol{\tau}$: **ELLIPTICITY ANGLE** $0 \leq \tau \leq \frac{\pi}{4}$

POLARISATION HANDENESS

ROTATION SENSE: LOOKING INTO THE DIRECTION OF THE WAVE PROPAGATION



ANTI-CLOCKWISE ROTATION	CLOCKWISE ROTATION
LEFT HANDED POLARISATION	**RIGHT HANDED POLARISATION**
⬇	⬇
ELLIPTICITY ANGLE : $\tau > 0$	ELLIPTICITY ANGLE : $\tau < 0$

$$-\frac{\pi}{4} \leq \tau \leq \frac{\pi}{4}$$

JONES VECTOR

REAL ELECTRIC FIELD VECTOR

$$\vec{E}(z,t)=\begin{Bmatrix} E_x = E_{0x}\cos(\omega t - kz - \delta_x) \\ E_y = E_{0y}\cos(\omega t - kz - \delta_y) \\ E_z = 0 \end{Bmatrix}$$

PHASOR = JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x = E_{ox} e^{j\delta_x} \\ E_y = E_{oy} e^{j\delta_y} \end{bmatrix}$$

With: $\vec{E}(z,t) = \Re\left(\underline{E}\, e^{j(\omega t - kz)}\right)$

GEOMETRICAL PARAMETERS

ABSOLUTE PHASE

$$\alpha = \delta_x$$

AMPLITUDE

$$A = \sqrt{E_{0x}^2 + E_{0y}^2}$$

ORIENTATION ANGLE

$$\tan 2\varphi = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2}\cos\delta$$

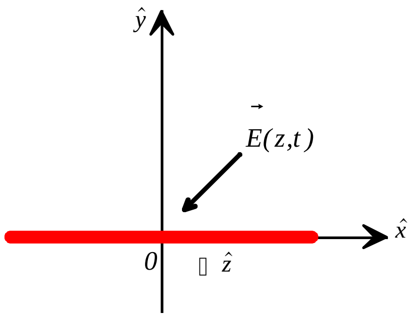
ELLIPTICITY ANGLE

$$\sin 2\tau = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 + E_{0y}^2}\sin\delta$$

POLARISATION HANDENESS: ***Sign(τ)***

JONES VECTOR

HORIZONTAL POLARISATION STATE

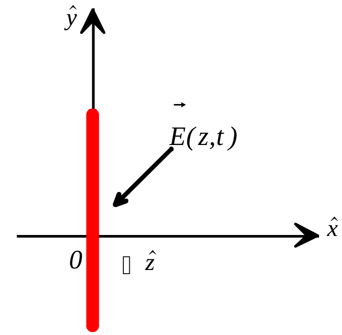


$$\underline{H}=\begin{bmatrix}1\\0\end{bmatrix}$$

$$\varphi=0$$

$$\tau=0$$

VERTICAL POLARISATION STATE

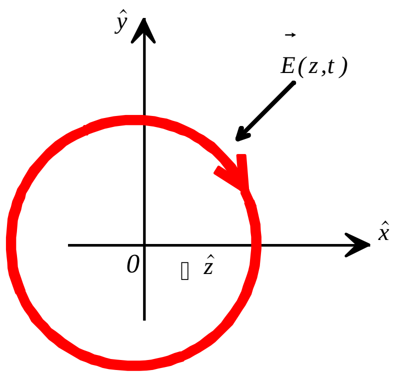


$$\underline{V}=\begin{bmatrix}0\\1\end{bmatrix}$$

$$\varphi=\frac{\pi}{2}$$

$$\tau=0$$

LEFT CIRCULAR POLARISATION STATE

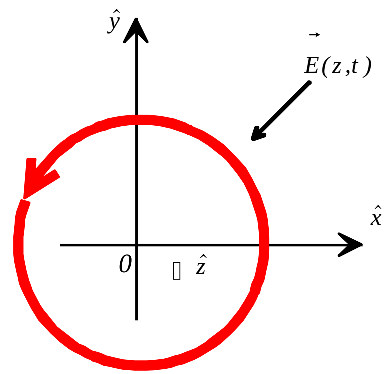


$$\underline{LC}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\j\end{bmatrix}$$

$$-\frac{\pi}{2}\leq\varphi\leq+\frac{\pi}{2}$$

$$\tau=+\frac{\pi}{4}$$

RIGHT CIRCULAR POLARISATION STATE

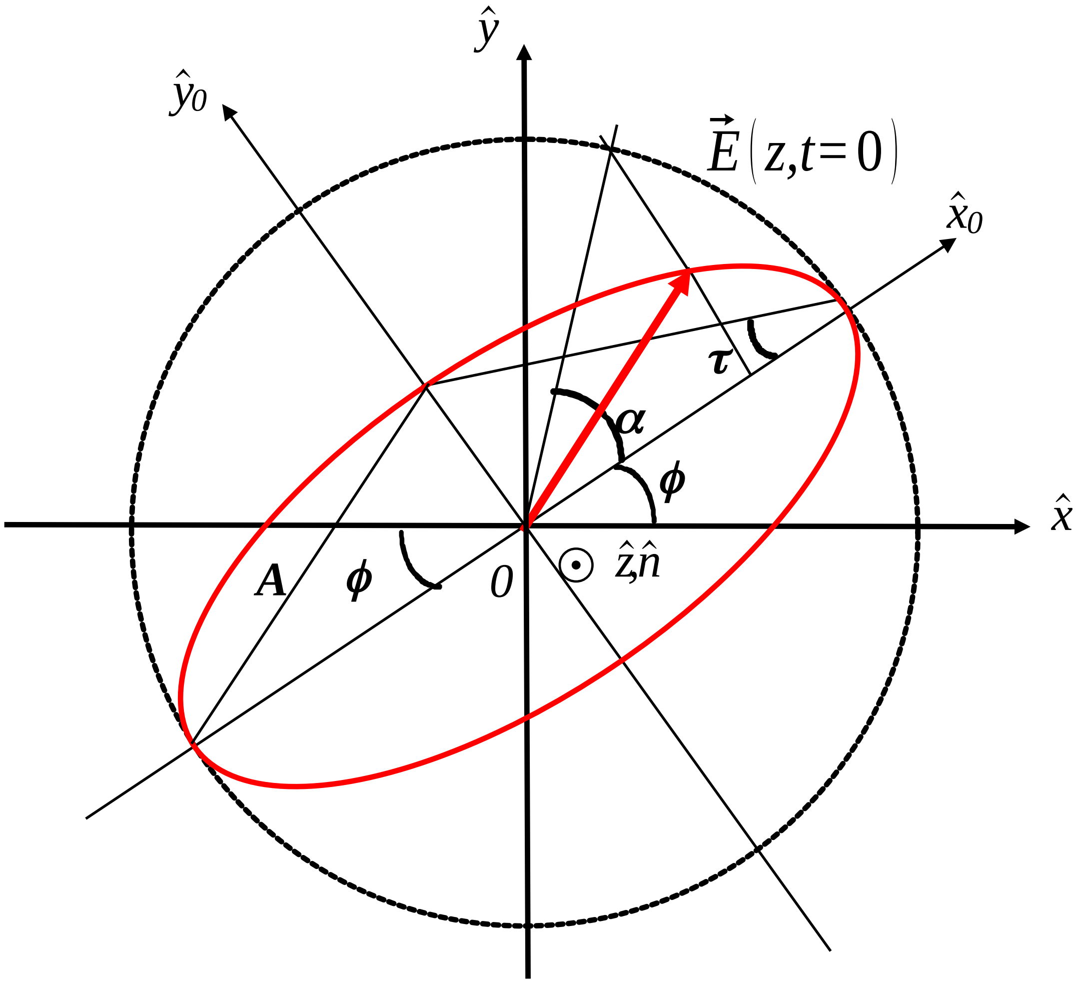


$$\underline{RC}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-j\end{bmatrix}$$

$$-\frac{\pi}{2}\leq\varphi\leq+\frac{\pi}{2}$$

$$\tau=-\frac{\pi}{4}$$

JONES VECTOR



$$\underline{E} = A \begin{bmatrix} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix} \hat{u}_x$$

ORTHOGONAL JONES VECTOR

JONES VECTOR

$$\underline{E} \qquad \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_{ox}\, e^{j\delta_x} \\ E_{oy}\, e^{j\delta_y} \end{bmatrix}$$

$$= A \begin{bmatrix} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix} \hat{u}_x$$

POLARISATION ALGEBRA

NORM OF A JONES VECTOR $\quad \|\underline{E}\| = \sqrt{E_{0x}^2 + E_{0y}^2}$

SCALAR PRODUCT $\quad \langle \underline{AB} \rangle = \underline{A}^{*T} \underline{B}$

ORTHOGONALITY $\quad \langle \underline{AA_\perp} \rangle = 0$

ELLIPTICAL BASIS TRANSFORMATION

JONES VECTOR

$$\underline{E} = A\begin{bmatrix}\cos(\varphi) & -\sin(\varphi)\\ \sin(\varphi) & \cos(\varphi)\end{bmatrix}\begin{bmatrix}\cos(\tau) & j\sin(\tau)\\ j\sin(\tau) & \cos(\tau)\end{bmatrix}\begin{bmatrix}e^{-j\alpha} & 0\\ 0 & e^{j\alpha}\end{bmatrix}\hat{u}_x$$

ORTHOGONAL JONES VECTOR

$$\underline{E} = A\begin{bmatrix}\cos(\varphi) & -\sin(\varphi)\\ \sin(\varphi) & \cos(\varphi)\end{bmatrix}\begin{bmatrix}\cos(\tau) & j\sin(\tau)\\ j\sin(\tau) & \cos(\tau)\end{bmatrix}\begin{bmatrix}e^{-j\alpha} & 0\\ 0 & e^{j\alpha}\end{bmatrix}\hat{u}_y$$

↓

$$\left[\underline{E}, \underline{E}\right] = A\begin{bmatrix}\cos(\varphi) & -\sin(\varphi)\\ \sin(\varphi) & \cos(\varphi)\end{bmatrix}\begin{bmatrix}\cos(\tau) & j\sin(\tau)\\ j\sin(\tau) & \cos(\tau)\end{bmatrix}\begin{bmatrix}e^{-j\alpha} & 0\\ 0 & e^{j\alpha}\end{bmatrix}\left[\hat{u}_x, \hat{u}_y\right]$$

↓

ELLIPTICAL BASIS TRANSFORMATION

ELLIPTICAL BASIS TRANSFORMATION

SU(2) : SPECIAL UNITARY TRANSFORMATION MATRIX

$$\left[U\left(\varphi,\tau,\alpha\right)\right]=\begin{bmatrix}\cos\left(\varphi\right) & -\sin\left(\varphi\right)\\ \sin\left(\varphi\right) & \cos\left(\varphi\right)\end{bmatrix}\begin{bmatrix}\cos\left(\tau\right) & j\sin\left(\tau\right)\\ j\sin\left(\tau\right) & \cos\left(\tau\right)\end{bmatrix}\begin{bmatrix}e^{-j\alpha} & 0\\ 0 & e^{j\alpha}\end{bmatrix}$$

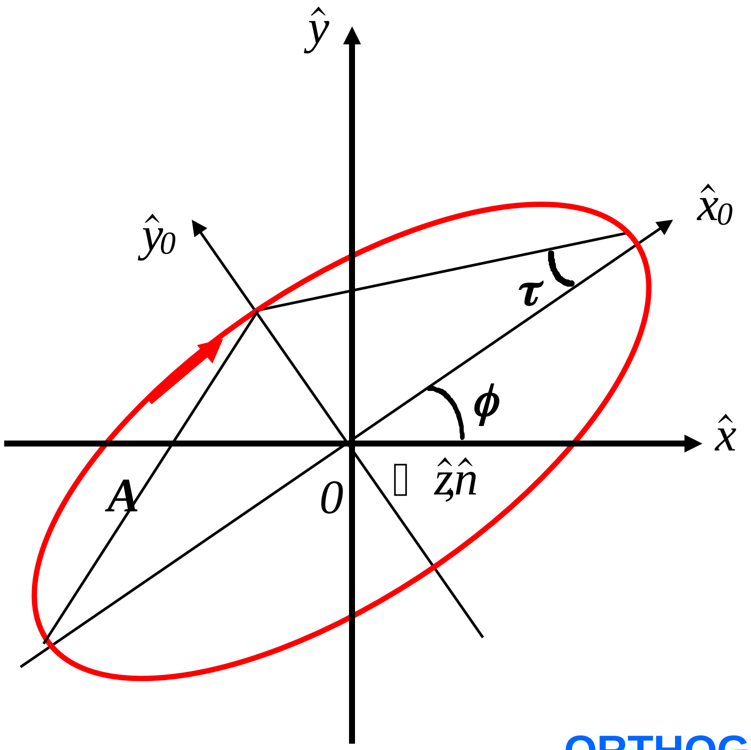
ELLIPTICAL BASIS TRANSFORMATION MATRIX

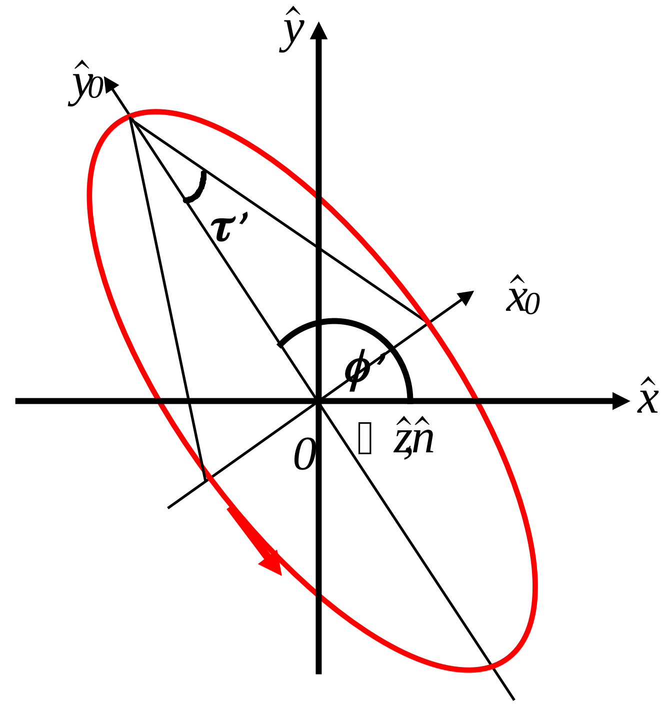
$$\left[U_{\left(A,A_{\perp}\right)\mapsto\left(B,B_{\perp}\right)}\right]$$

$$\left[U\left(\varphi,\tau,\alpha\right)\right]^{-1}$$

$$\begin{bmatrix}e^{j\alpha} & 0\\ 0 & e^{-j\alpha}\end{bmatrix}\begin{bmatrix}\cos\left(\tau\right) & -j\sin\left(\tau\right)\\ -j\sin\left(\tau\right) & \cos\left(\tau\right)\end{bmatrix}\begin{bmatrix}\cos\left(\varphi\right) & \sin\left(\varphi\right)\\ -\sin\left(\varphi\right) & \cos\left(\varphi\right)\end{bmatrix}$$

ORTHOGONAL JONES VECTOR

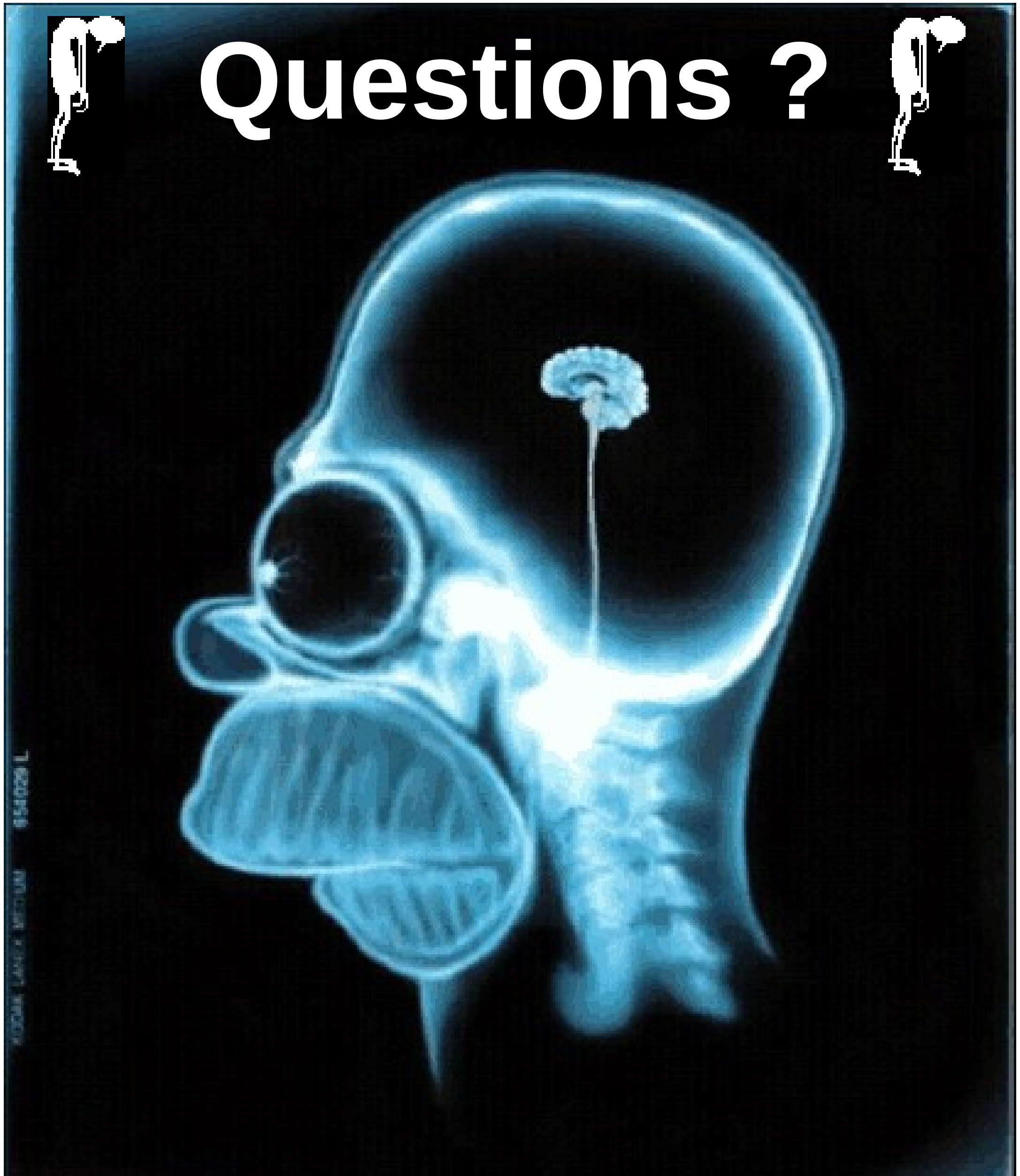


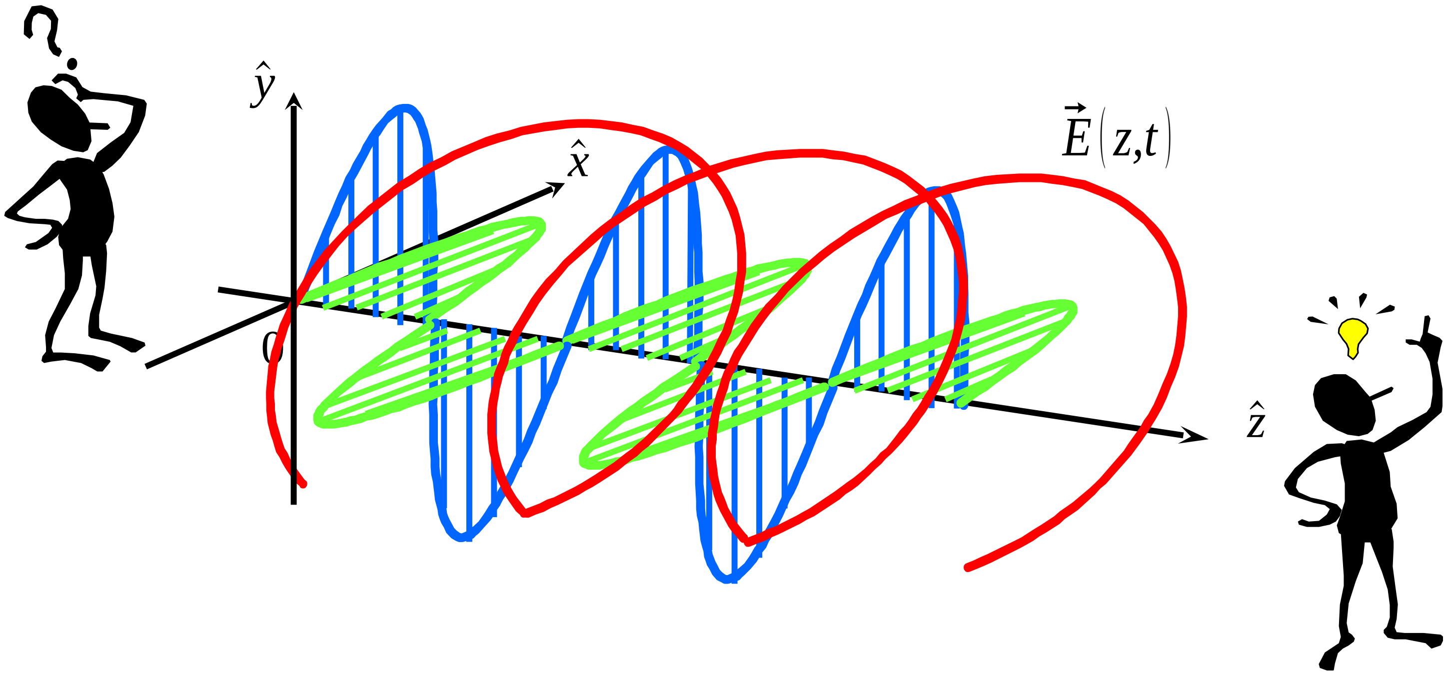


ORTHOGONALITY CONDITIONS

$$(\varphi,\tau) \mapsto \begin{Bmatrix} \varphi' = \varphi + \dfrac{\pi}{2} \\ \tau' = -\tau \end{Bmatrix}$$

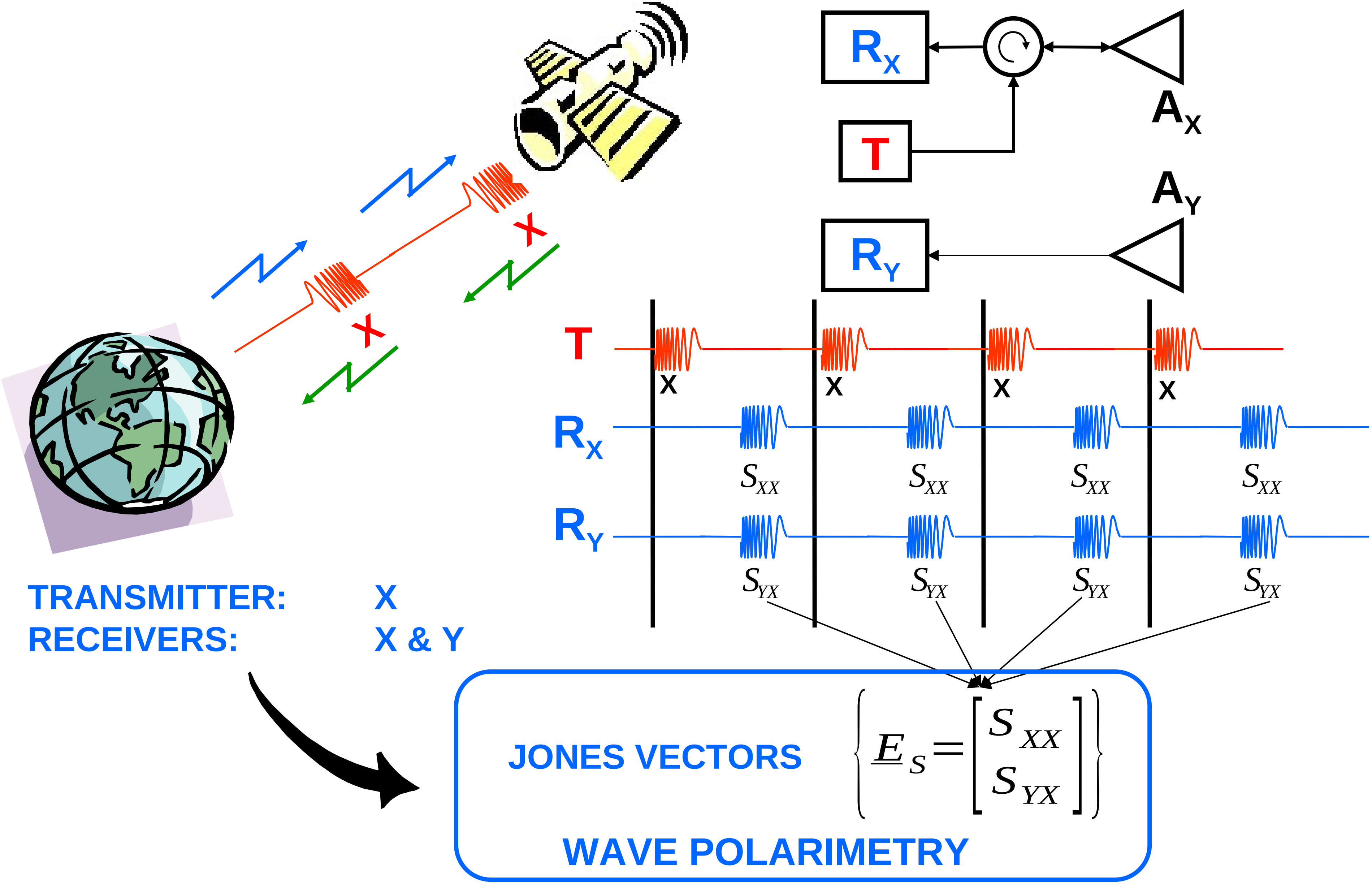
➡ CHANGE OF POLARISATION HANDENESS


Questions ?

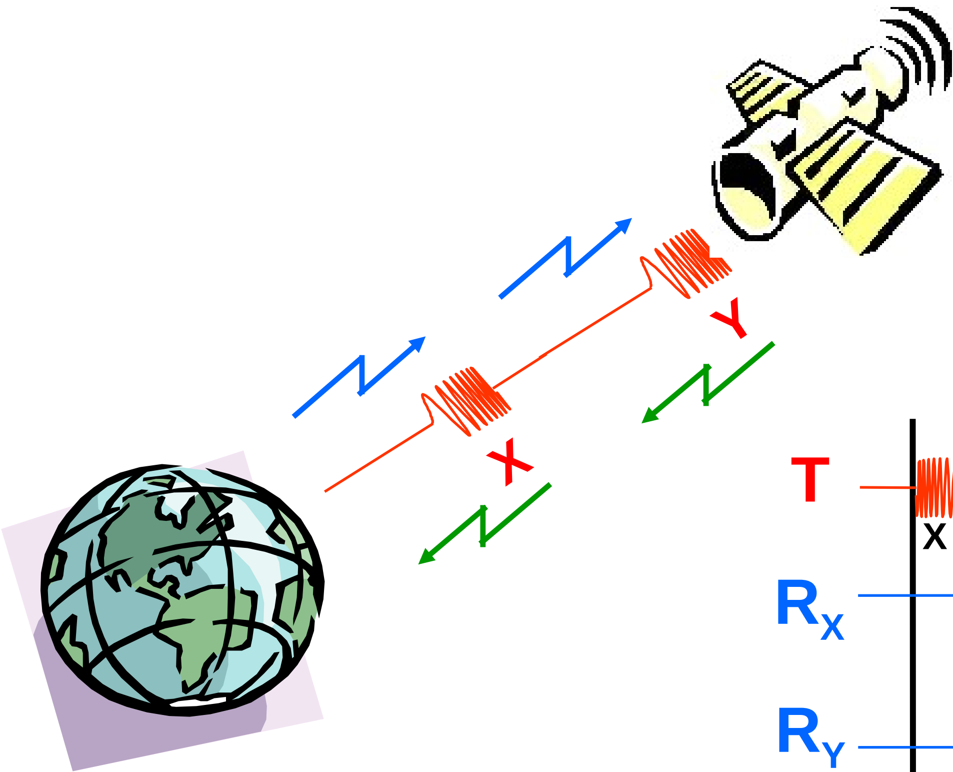


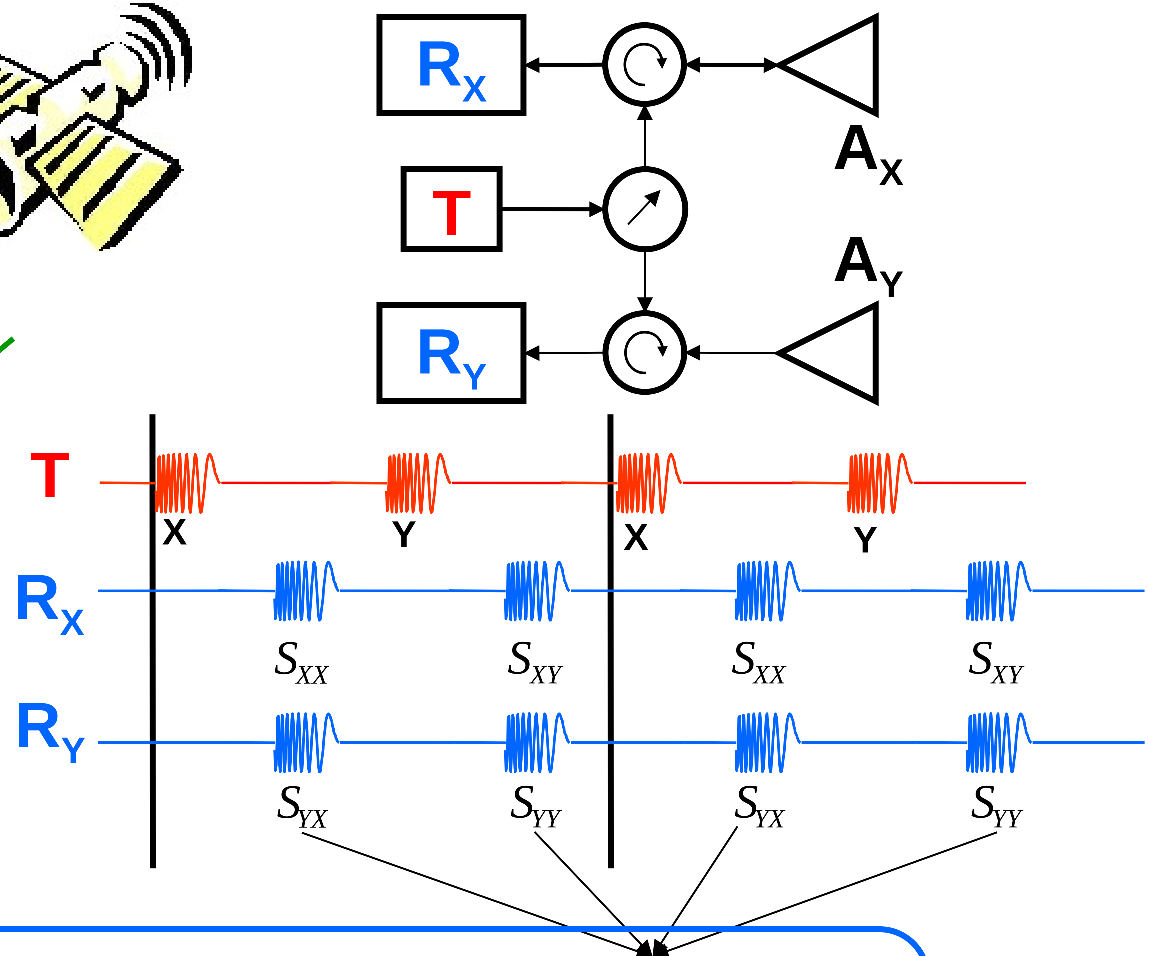
SCATTERING POLARIMETRY

WAVE POLARIMETRY



SCATTERING POLARIMETRY





TRANSMITTER: X & Y

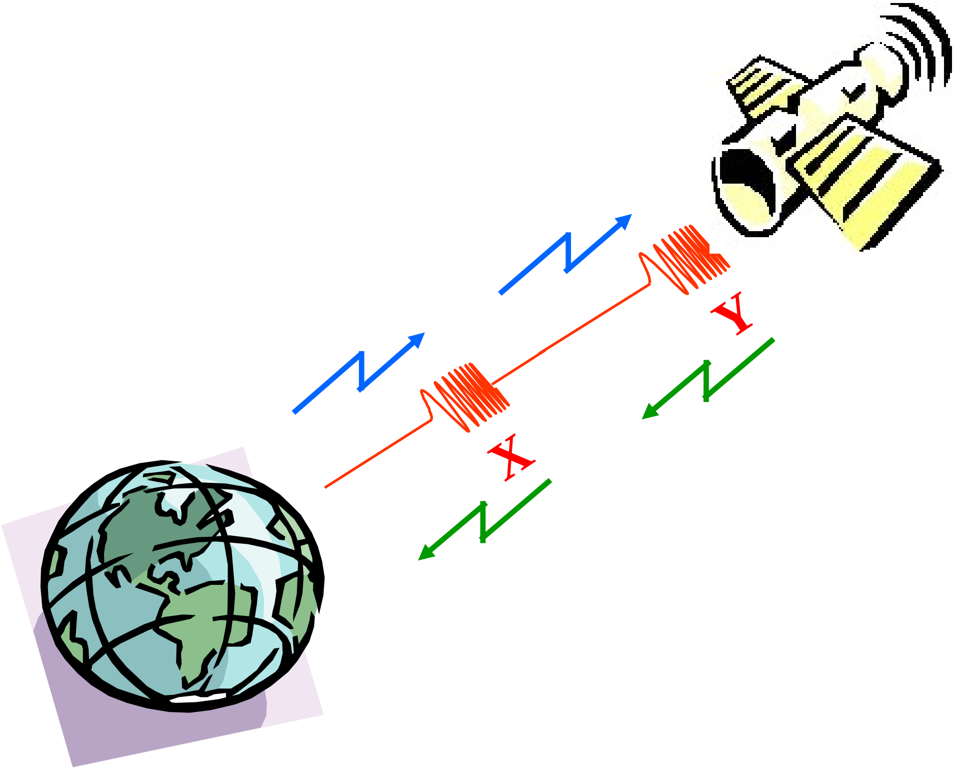
RECEIVERS: X & Y

SINCLAIR MATRICES

$$\left\{ [S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix} \right\}$$

SCATTERING POLARIMETRY

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
k, Ω	Target Vectors
[K]	KENNAUGH Matrix
[T]	Coherency Matrix
[C]	Covariance Matrix

BACKSCATTERING MATRIX

$$[S] = \frac{e^{jkr}}{r}\begin{bmatrix} S_{XX} & S_{XY} \\ S_{XY} & S_{YY} \end{bmatrix} = \frac{e^{jkr}}{r}\begin{bmatrix} |S_{XX}|e^{j\varphi_{XX}} & |S_{XY}|e^{j\varphi_{XY}} \\ |S_{XY}|e^{j\varphi_{XY}} & |S_{YY}|e^{j\varphi_{YY}} \end{bmatrix}$$

ABSOLUTE BACKSCATTERING MATRIX

$$[S] = \frac{e^{jkr}e^{j\varphi_{XX}}}{r}\begin{bmatrix} |S_{XX}| & |S_{XY}|e^{j(\varphi_{XY}-\varphi_{XX})} \\ |S_{XY}|e^{j(\varphi_{XY}-\varphi_{XX})} & |S_{YY}|e^{j(\varphi_{YY}-\varphi_{XX})} \end{bmatrix}$$

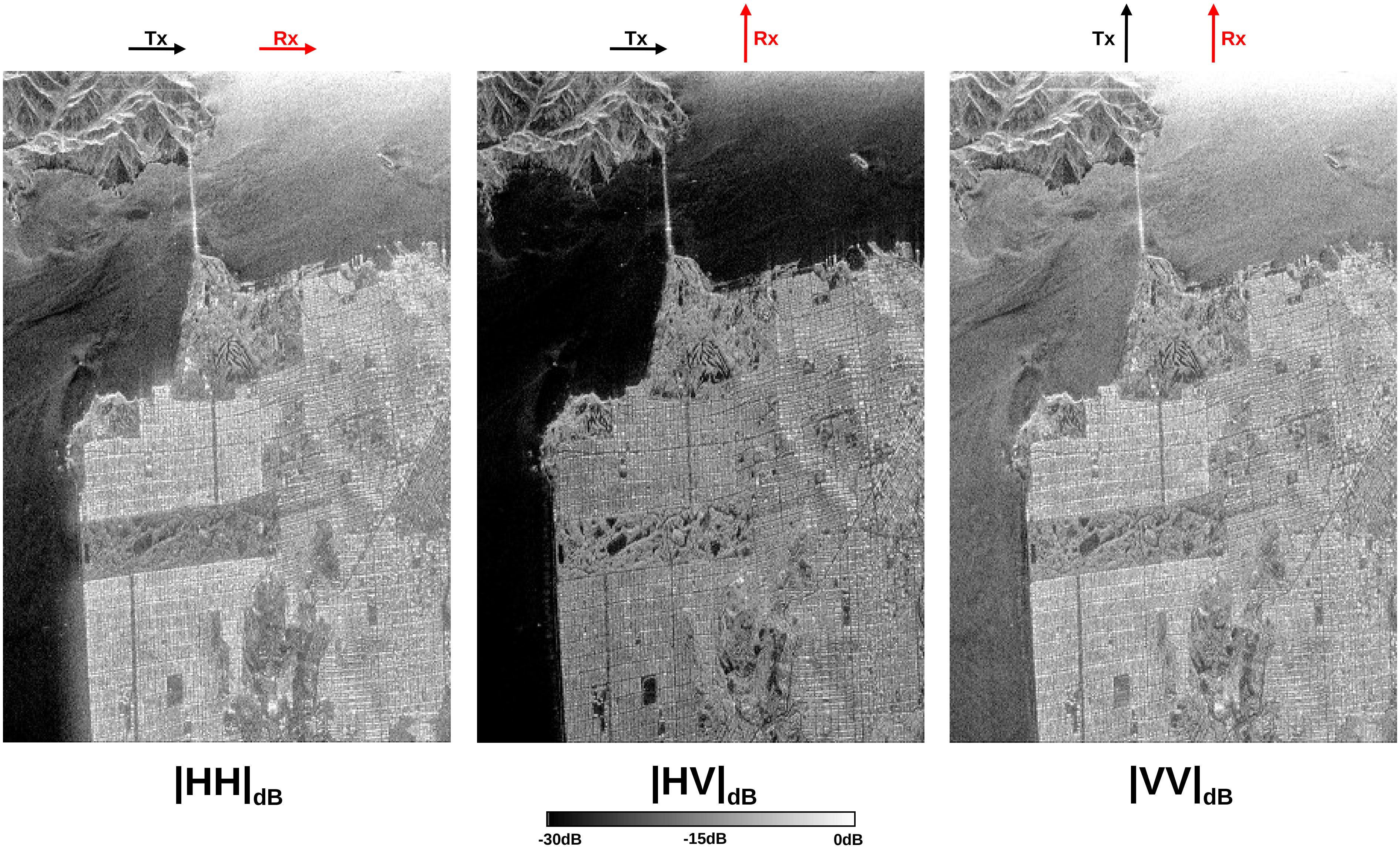
Absolute Phase Factor

RELATIVE BACKSCATTERING MATRIX

<u>Five Parameters:</u> 3 Amplitudes and 2 Phases

SCATTERER POLARIMETRIC DIMENSION = 5

SCATTERING POLARIMETRY



SCATTERING POLARIMETRY

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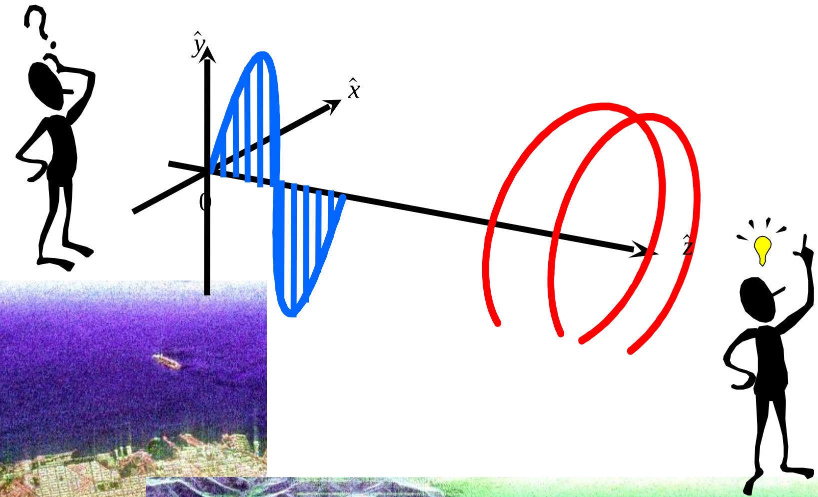
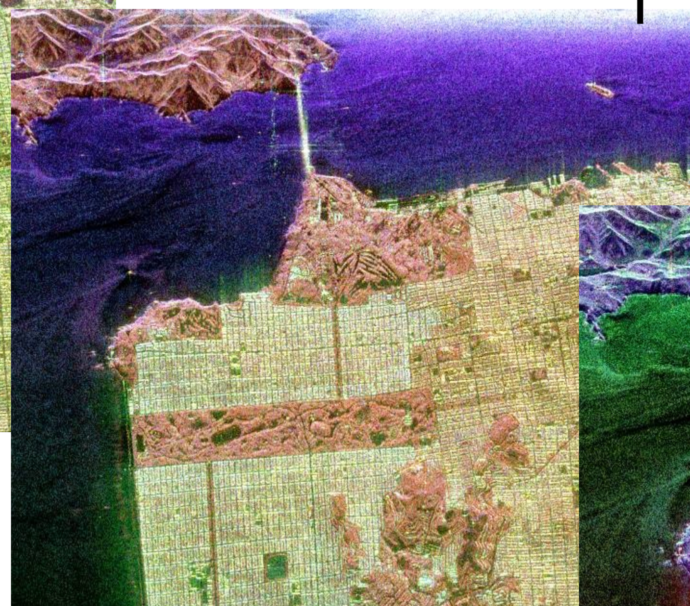
Sinclair Color Coding

|HH| **|HV|** **|VV|**

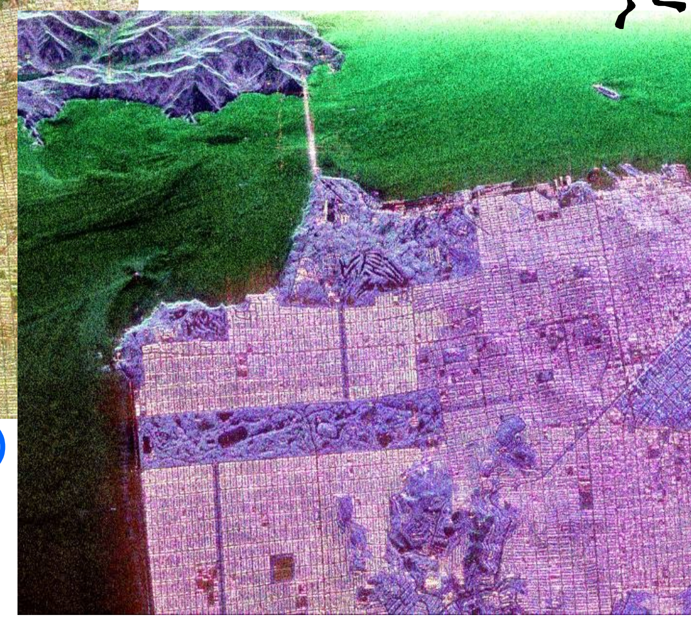
ELLIPTICAL BASIS TRANSFORMATION



Pauli Color Coding (H,V)



Pauli Color Coding (+45,-45)



Pauli Color Coding (L,R)

Ernst LÜNEBURG
(PIERS95 - Pasadena)

ELLIPTICAL BASIS TRANSFORMATION

$$\left[S_{(B,B)}\right] = \left[U_{(A,A)\mapsto(B,B)}\right]^T \left[S_{(A,A)}\right] \left[U_{(A,A)\mapsto(B,B)}\right]$$

CON-SIMILARITY TRANSFORMATION

$$\left[U_{(A,A)\mapsto(B,B)}\right]$$

SU(2) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

$$\left[U_{(A,A)\mapsto(B,B)}\right] \quad \left[U(\varphi,\tau,\alpha)\right]^{-1}$$

$$\begin{bmatrix} e^{j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix} \begin{bmatrix} \cos(\tau) & -j\sin(\tau) \\ -j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{bmatrix}$$

$$\left[U_2(-\alpha)\right] \quad \left[U_2(-\tau)\right] \quad \left[U_2(-\varphi)\right]$$

ELLIPTICAL BASIS TRANSFORMATION

© Google Earth

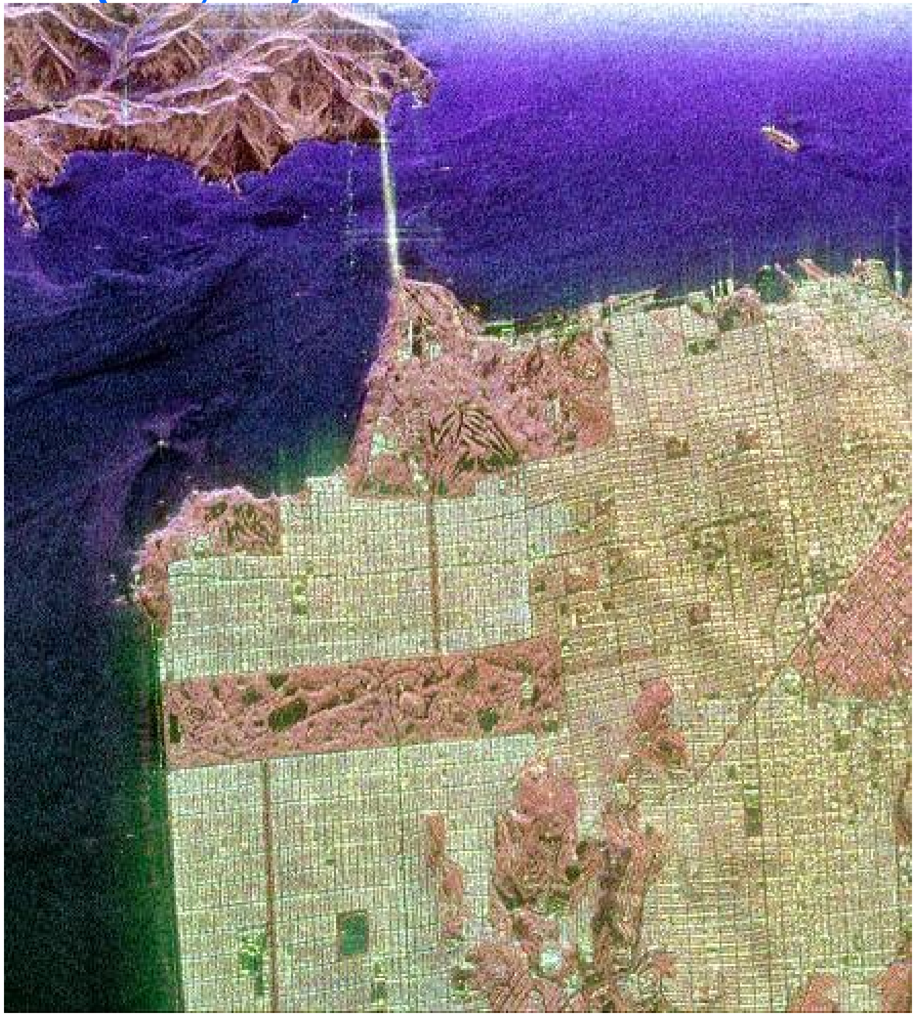
(H,V) POLARISATION BASIS

|HH+VV| **|HV |** **|HH-VV|**

ELLIPTICAL BASIS TRANSFORMATION

© Google Earth

(+45°,-45°) POLARISATION BASIS



|AA+BB| |AB | |AA-BB|

With: A=Linear +45°, B=Linear –45°

ELLIPTICAL BASIS TRANSFORMATION

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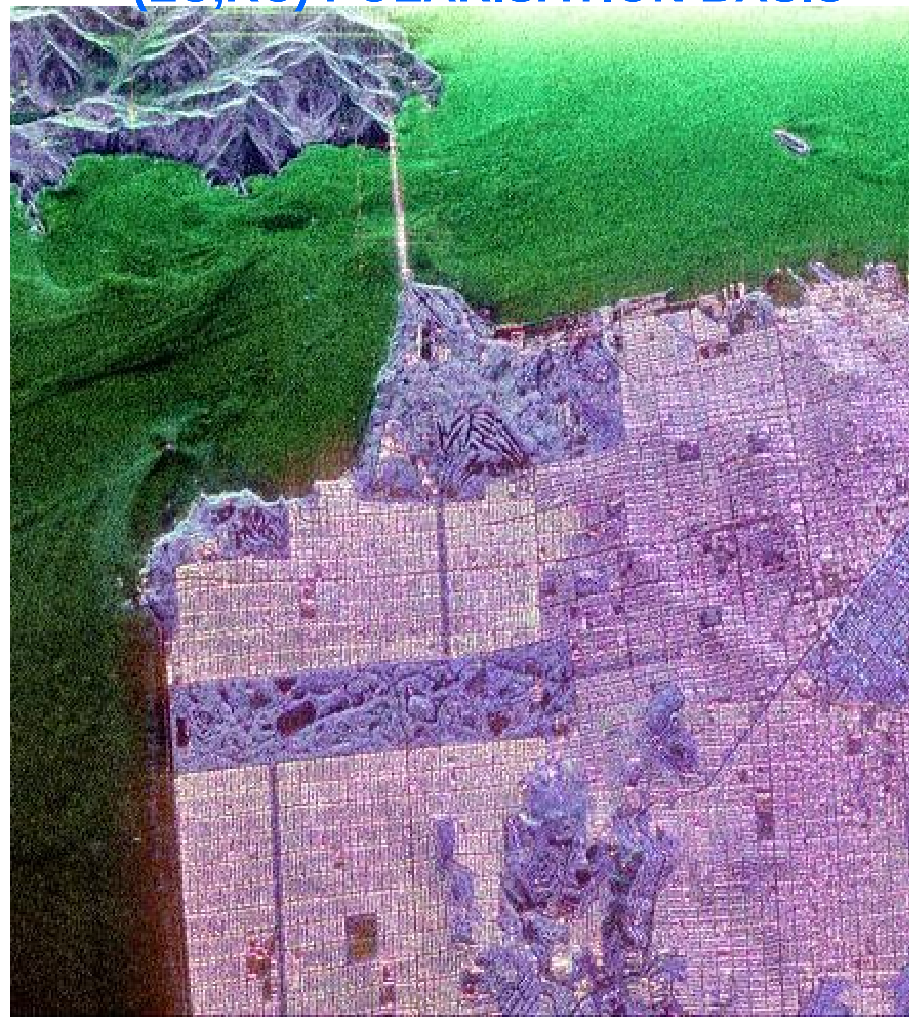
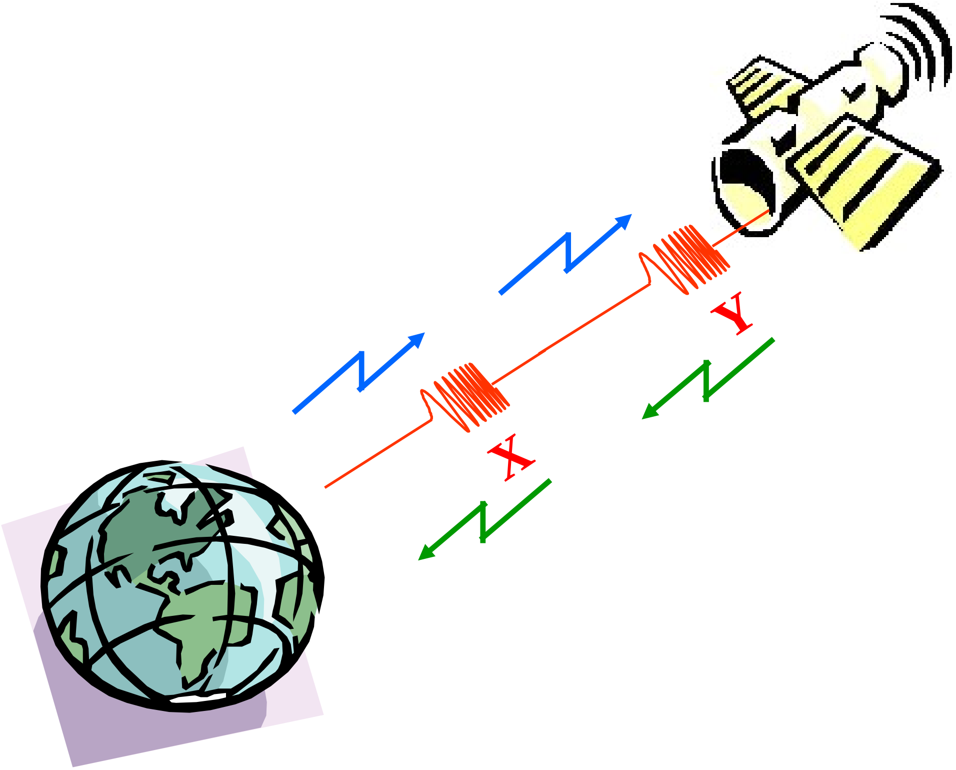
(LC,RC) POLARISATION BASIS



|LL+RR| |LR | |LL-RR|

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y

RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
k, Ω	**Target Vectors**
[K]	KENNAUGH Matrix
[T]	Coherency Matrix
[C]	Covariance Matrix

TARGET VECTORS

SCATTERING VECTOR TRANSFORMATIONS

Pauli Scattering Vector:

$$\underline{k} = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{XX} + S_{YY} \\ S_{XX} - S_{YY} \\ 2\, S_{XY} \end{bmatrix}$$

Lexicographic Scattering Vector:

$$\underline{\Omega} = \begin{bmatrix} S_{XX} \\ \sqrt{2}\, S_{XY} \\ S_{YY} \end{bmatrix}$$

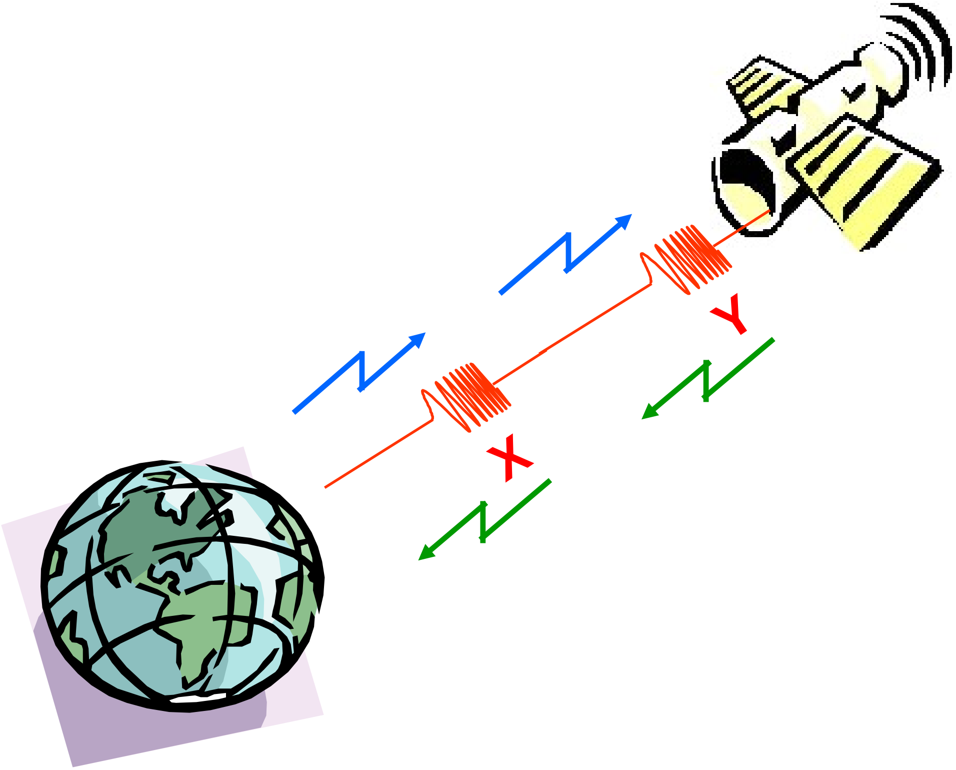
UNITARY TRANSFORMATION

$$\underline{k} = [D_3]\, \underline{\Omega} \quad and \quad \underline{\Omega} = [D_3]^{-1} \underline{k} = [D_3]^{T} \underline{k}$$

WHERE $[D_3]$ IS A SU(3) MATRIX
IN ORDER TO PRESERVE THE NORM
OF THE SCATTERING VECTOR

$$[D_3] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \end{bmatrix}$$

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
k, Ω	Target Vectors
[K]	KENNAUGH Matrix
[T]	**Coherency Matrix**
[C]	Covariance Matrix

STATISTICAL DESCRIPTION

PARTIAL SCATTERING POLARIMETRY

COHERENCY MATRIX

MONOSTATIC CASE

PAULI SCATTERING VECTOR $\underline{k}$

$$\underline{k} = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{XX} + S_{YY} & S_{XX} - S_{YY} & 2S_{XY} \end{bmatrix}^T$$

$\Downarrow$

COHERENCY MATRIX $[T]$

$$[T] = \underline{k} \cdot \underline{k}^{T} = \begin{bmatrix} 2A_0 & C - jD & H + jG \\ C + jD & B_0 + B & E + jF \\ H - jG & E - jF & B_0 - B \end{bmatrix}$$

HERMITIAN MATRIX - RANK 1

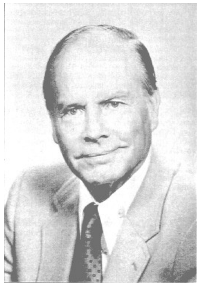
A0, B0+B, B0-B : HUYNEN TARGET GENERATORS

HUYNEN PARAMETERS

PHYSICAL INTERPRETATION

MAN-MADE TARGET DECOMPOSITION
IDENTIFICATION and ANALYSIS

« *PHENOMENOLOGICAL THEORY OF RADAR TARGETS* » (1970)



A0 : GENERATOR OF TARGET SYMMETRY

B0+B : GENERATOR OF TARGET NON-SYMMETRY

B0-B : GENERATOR OF TARGET IRREGULARITY

C : GENERATOR OF TARGET GLOBAL SHAPE (LINEAR)

D : GENERATOR OF TARGET LOCAL SHAPE (CURVATURE)

E : GENERATOR OF TARGET LOCAL TWIST (TORSION)

F : GENERATOR OF TARGET GLOBAL TWIST (HELICITY)

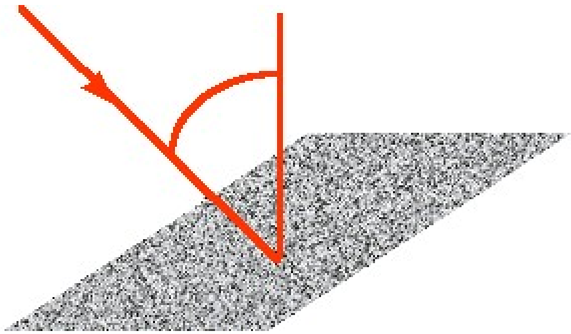
G : GENERATOR OF TARGET LOCAL COUPLING (GLUE)

H : GENERATOR OF TARGET GLOBAL COUPLING (ORIENTATION)

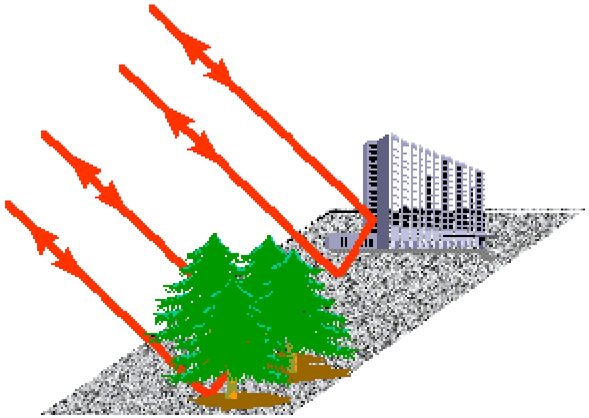
TARGET GENERATORS

PHYSICAL INTERPRETATION

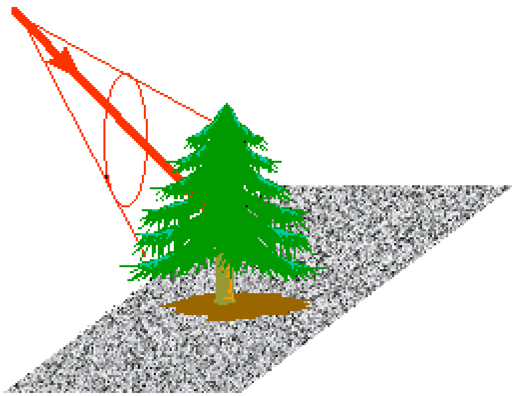
SINGLE BOUNCE SCATTERING
(ROUGH SURFACE)



DOUBLE BOUNCE SCATTERING



VOLUME SCATTERING

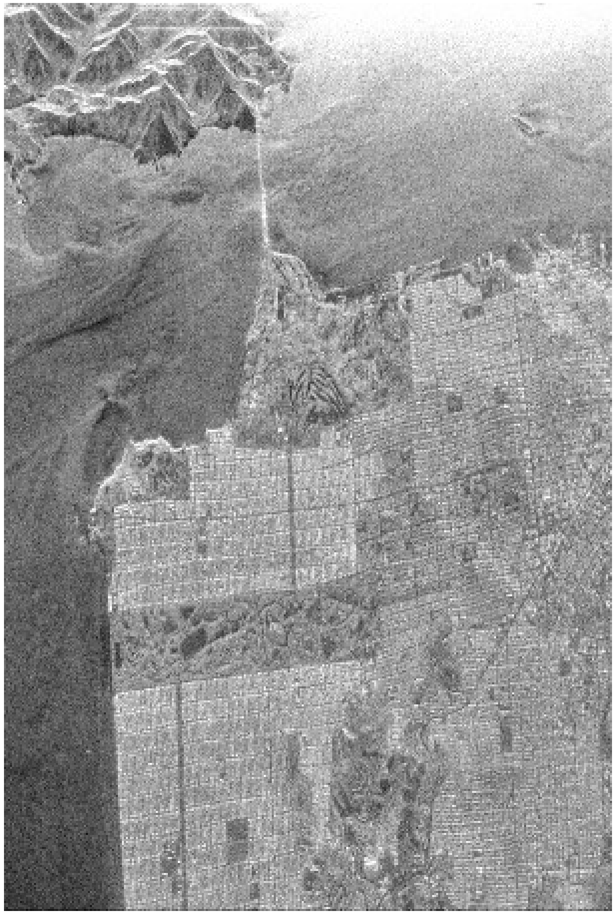


$$T_{11} = 2A_0 = |S_{XX} + S_{YY}|^2$$

$$T_{33} = B_0 - B = 2|S_{XY}|^2$$

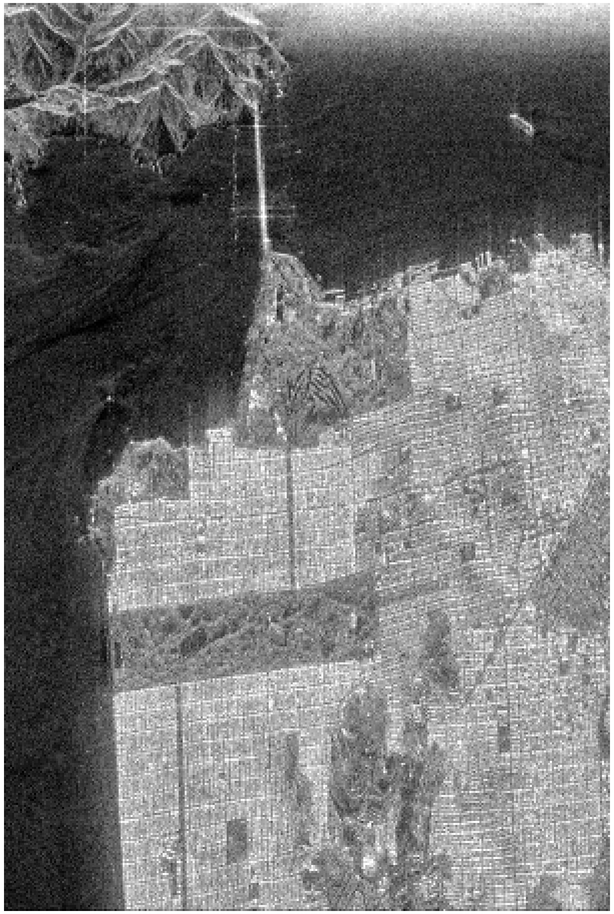
$$T_{22} = B_0 + B = |S_{XX} - S_{YY}|^2$$

TARGET GENERATORS

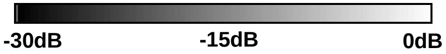


$|HH+VV|_{dB}$

$|HV|_{dB}$



$|HH-VV|_{dB}$



TARGET GENERATORS

© Google Earth

(H,V) POLARISATION BASIS

|HH+VV| **|HV |** **|HH-VV|**

ELLIPTICAL BASIS TRANSFORMATION

SPECIAL UNITARY SU(2) GROUP

$$[U_2]=\begin{bmatrix}\cos(\varphi) & -\sin(\varphi)\\ \sin(\varphi) & \cos(\varphi)\end{bmatrix}\begin{bmatrix}\cos(\tau) & j\sin(\tau)\\ j\sin(\tau) & \cos(\tau)\end{bmatrix}\begin{bmatrix}e^{-j\alpha} & 0\\ 0 & e^{j\alpha}\end{bmatrix}$$

$$[U_2(\varphi)] \qquad [U_2(\tau)] \qquad [U_2(\alpha)]$$

SPECIAL UNITARY SU(3) GROUP

$$\begin{bmatrix}1 & 0 & 0\\ 0 & \cos(2\varphi) & \sin(2\varphi)\\ 0 & -\sin(2\varphi) & \cos(2\varphi)\end{bmatrix}\begin{bmatrix}\cos(2\tau) & 0 & j\sin(2\tau)\\ 0 & 1 & 0\\ j\sin(2\tau) & 0 & \cos(2\tau)\end{bmatrix}\begin{bmatrix}\cos(2\alpha) & -j\sin(2\alpha) & 0\\ -j\sin(2\alpha) & \cos(2\alpha) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$[U_3(2\phi)] \qquad [U_3(2\tau)] \qquad [U_3(2\alpha)]$$

ELLIPTICAL BASIS TRANSFORMATION

SINCLAIR MATRIX

$$E^{s}_{(A,A)} = \left[S_{(A,A)}\right] E^{i}_{(A,A)} \qquad E^{s}_{(B,B)} = \left[S_{(B,B)}\right] E^{i}_{(B,B)}$$

$$\left[S_{(B,B)}\right] = \left[U_{(A,A)\mapsto(B,B)}\right]^{T} \left[S_{(A,A)}\right] \left[U_{(A,A)\mapsto(B,B)}\right]$$

CON-SIMILARITY TRANSFORMATION

COHERENCY MATRIX

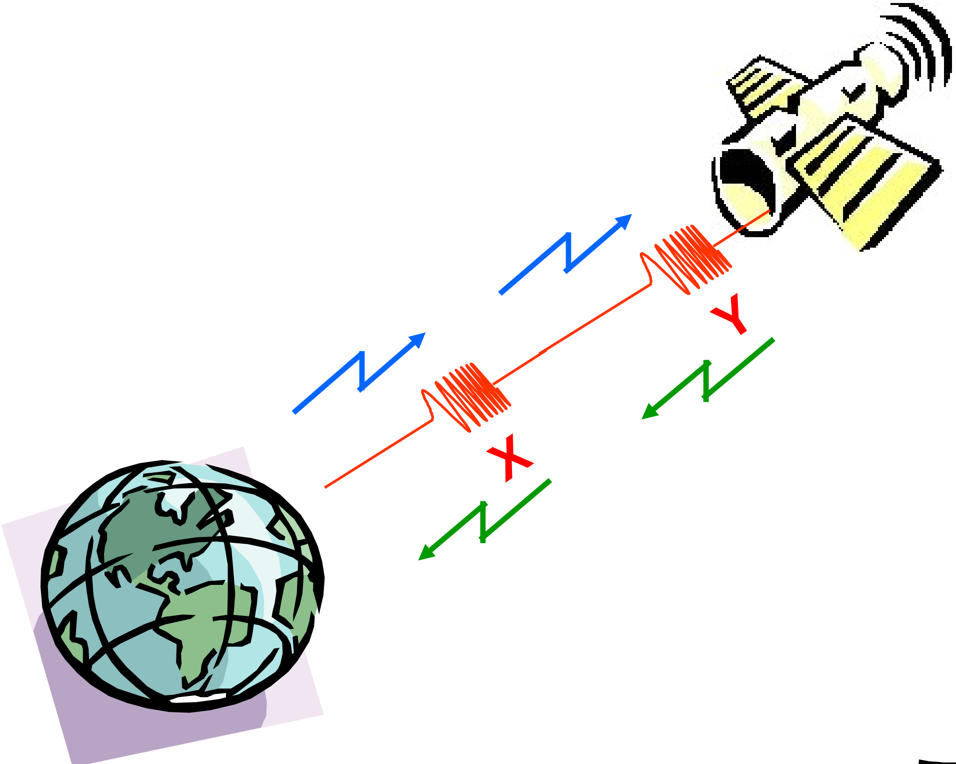
$$\left[T_{(B,B)}\right] = \left[U_{3(A,A)\mapsto(B,B)}\right] \left[T_{(A,A)}\right] \left[U_{3(A,A)\mapsto(B,B)}\right]^{-1}$$

SIMILARITY TRANSFORMATION

$$\left[U_{3(A,A)\mapsto(B,B)}\right]$$

U(3) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
k, Ω	Target Vectors
[K]	KENNAUGH Matrix
[T]	Coherency Matrix
[C]	**Covariance Matrix**

STATISTICAL DESCRIPTION

PARTIAL SCATTERING POLARIMETRY

COVARIANCE MATRIX

MONOSTATIC CASE

LEXICOGRAPHIC SCATTERING VECTOR $\underline{\Omega}$

$$\underline{\Omega} = \begin{bmatrix} S_{XX} & \sqrt{2}\,S_{XY} & S_{YY} \end{bmatrix}^T$$

$$\Downarrow$$

COVARIANCE MATRIX $[C]$

$$[C] = \underline{\Omega} \cdot \underline{\Omega}^T = \begin{bmatrix} S_{XX}S_{XX} & \sqrt{2}\,S_{XX}S_{XY} & S_{XX}S_{YY} \\ \sqrt{2}\,S_{XY}S_{XX} & 2\,S_{XY}S_{XY} & \sqrt{2}\,S_{XY}S_{YY} \\ S_{YY}S_{XX} & \sqrt{2}\,S_{YY}S_{XY} & S_{YY}S_{YY} \end{bmatrix}$$

HERMITIAN POSITIVE SEMI DEFINITE MATRIX - RANK 1

COVARIANCE-COHERENCY MATRICES

COHERENCY MATRIX

$$[T] = \underline{k} \cdot \underline{k}^{T}$$

$$\underline{k} = [D_{3\,or\,4}]\,\underline{\Omega}$$

COVARIANCE MATRIX

$$[C] = \underline{\Omega} \cdot \underline{\Omega}^{T}$$

UNITARY TRANSFORMATION

$$[T] = [D_{3\,or\,4}]\,[C]\,[D_{3\,or\,4}]^{T*¿}\,¿$$

$[T]$ and $[C]$ HAVE THE SAME EIGENVALUES

Both contain the same information about Polarimetric Scattering Amplitudes, Phase Angles and Correlations

$[T]$ is closer related to Physical and Geometrical Properties of the Scattering Process, and thus allows a better and direct physical interpretation

$[C]$ is directly related to the system measurables

$[T]$ is directly related to the Kennaugh matrix and the Huynen parameters

POLARIMETRIC DESCRIPTORS

SINCLAIR MATRIX

$$[S]=\begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

EQUIVALENCE ?

SCATTERING VECTOR $\underline{k}$

$$\underline{k}=\frac{1}{\sqrt{2}}\left[S_{XX}+S_{YY} \quad S_{XX}-S_{YY} \quad 2S_{XY}\right]^T$$

⬇

COHERENCY MATRIX $[T]$

$$[T]=\underline{k}\cdot\underline{k}^T$$

SCATTERING VECTOR $\underline{\Omega}$

$$\underline{\Omega}=\left[S_{XX} \quad \sqrt{2}S_{XY} \quad S_{YY}\right]^T$$

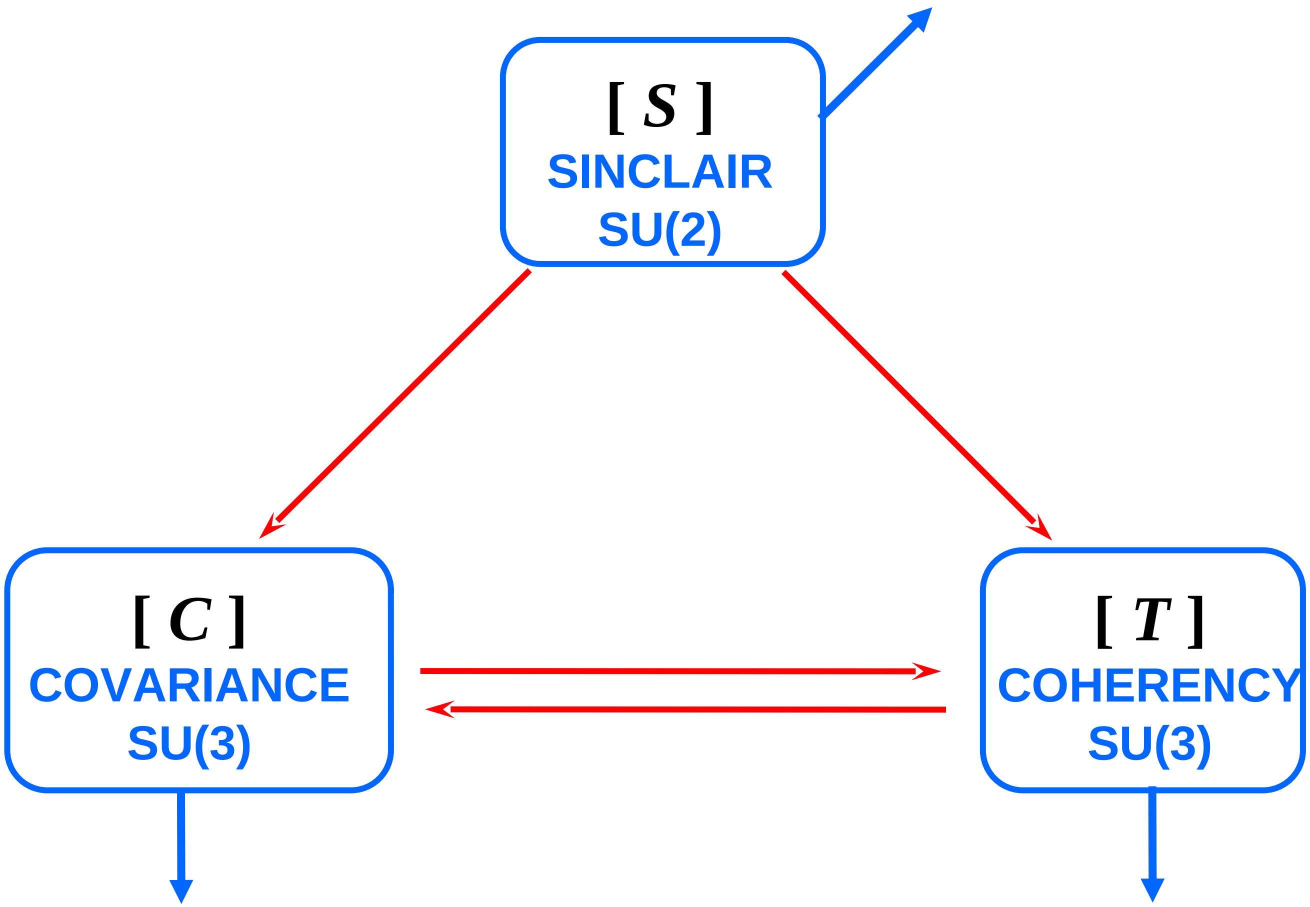
⬇

COVARIANCE MATRIX $[C]$

$$[C]=\underline{\Omega}\,\underline{\Omega}^{T*¿}¿$$

POLARIMETRIC DESCRIPTORS

$$[\,S'\,]=[\,U_2\,]^T[\,S\,]\,[\,U_2\,]$$



$$[\,C'\,]=[\,U_3\,]\,[\,C\,]\,[\,U_3\,]^{-1}$$

$$[\,T'\,]=[\,U_3\,]\,[\,T\,]\,[\,U_3\,]^{-1}$$

ELLIPTICAL BASIS TRANSFORMATION

SPECIAL UNITARY SU(2) GROUP

$$\begin{bmatrix} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

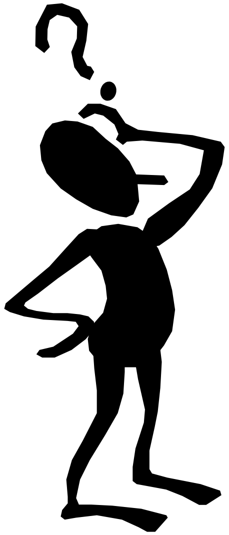
$$[U_2(\varphi)] \qquad [U_2(\tau)] \qquad [U_2(\alpha)]$$

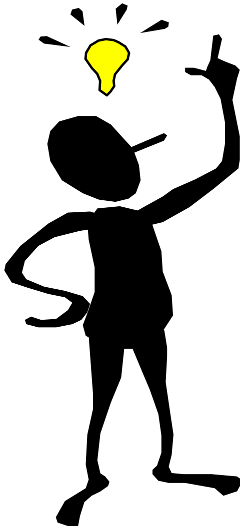
SPECIAL UNITARY SU(3) GROUP (T Matrix)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\varphi) & \sin(2\varphi) \\ 0 & -\sin(2\varphi) & \cos(2\varphi) \end{bmatrix} \begin{bmatrix} \cos(2\tau) & 0 & j\sin(2\tau) \\ 0 & 1 & 0 \\ j\sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix} \begin{bmatrix} \cos(2\alpha) & -j\sin(2\alpha) & 0 \\ -j\sin(2\alpha) & \cos(2\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[U_3(2\phi)] \qquad [U_3(2\tau)] \qquad [U_3(2\alpha)]$$

TARGET EQUATIONS





POLARIMETRIC GOLDEN NUMBER

POLARIMETRIC TARGET DIMENSION

TARGET EQUATIONS

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

5 DEGREES OF FREEDOM

$$|S_{XX}|, |S_{XY}|, |S_{YY}|$$

$$\varphi_{XY-XX}, \varphi_{YY-XX}$$

TARGET MONOSTATIC POLARIMETRIC « *DIMENSION* »

=

5

KENNAUGH MATRIX $[K]$

COHERENCY MATRIX $[T]$

9 HUYNEN REAL PARAMETERS

(*A0, B0, B, C, D, E, F, G, H*)

COVARIANCE MATRIX $[C]$

9 REAL PARAMETERS

$|XX|$, $|XY|$, $|YY|$,

Re$(XXXY^*)$, Im$(XXXY^*)$

Re$(XXYY^*)$, Im$(XXYY^*)$

Re$(XYYY^*)$, Im$(XYYY^*)$

9 - 5 = 4 TARGET EQUATIONS

TARGET EQUATIONS

PURE TARGET – MONOSTATIC CASE

$$[T] = \underline{k} \cdot \underline{k}^T = \begin{bmatrix} 2A_0 & C - jD & H + jG \\ C + jD & B_0 + B & E + jF \\ H - jG & E - jF & B_0 - B \end{bmatrix}$$

3x3 HERMITIAN MATRIX - RANK 1

↓

9 PRINCIPAL MINORS = 0

$$2A_0(B_0 + B) - C^2 - D^2 = 0 \qquad 2A_0(B_0 - B) - G^2 - H^2 = 0$$

$$-2A_0 E + CH - DG = 0 \qquad B_0^2 - B^2 - E^2 - F^2 = 0$$

$$C(B_0 - B) - EH - GF = 0 \qquad -D(B_0 - B) + FH - GE = 0$$

$$2A_0 F - CG - DH = 0 \qquad -G(B_0 + B) + FC - ED = 0$$

$$H(B_0 + B) - CE - DF = 0$$

TARGET EQUATIONS

$$[S]=\begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

5 DEGREES OF FREEDOM

$$|S_{XX}|,|S_{XY}|,|S_{YY}|$$

$$\varphi_{XY-XX},\varphi_{YY-XX}$$

COHERENCY MATRIX $[T]$

9 HUYNEN REAL PARAMETERS

(*A0, B0, B, C, D, E, F, G, H*)

TARGET MONOSTATIC POLARIMETRIC « *DIMENSION* »

=

5

9 - 5 = 4 TARGET EQUATIONS

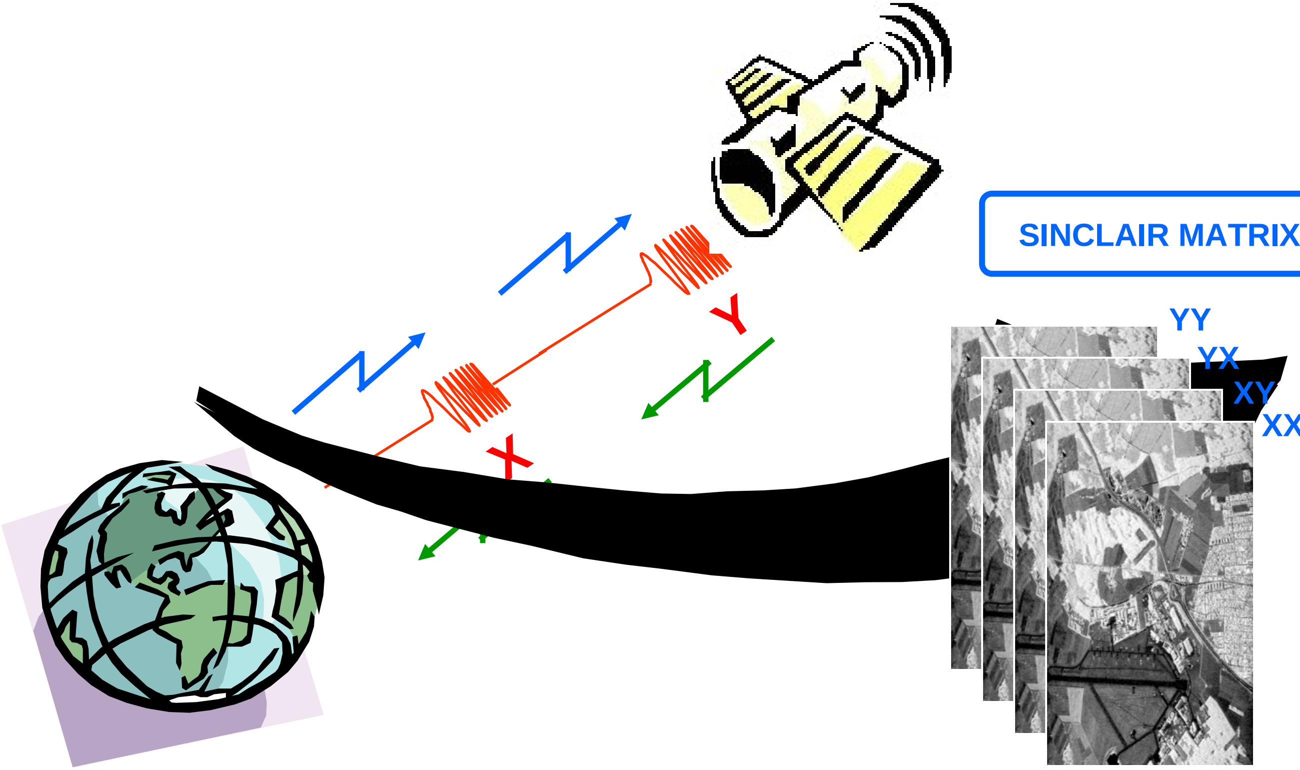
$$2A_0(B_0+B) \qquad C^2+D^2$$

$$2A_0(B_0-B) \qquad G^2+H^2$$

$$2A_0E \qquad CH-DG$$

$$2A_0F \qquad CG+DH$$

SCATTERING POLARIMETRY



SINCLAIR MATRIX $[S]$

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

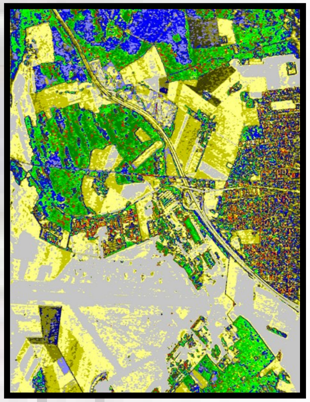
[S]	SINCLAIR Matrix
k, <u>Ω</u>	Target Vectors
[K]	KENNAUGH Matrix
[T]	Coherency Matrix
[C]	Covariance Matrix

TRANSMITTER: X & Y
RECEIVERS: X & Y

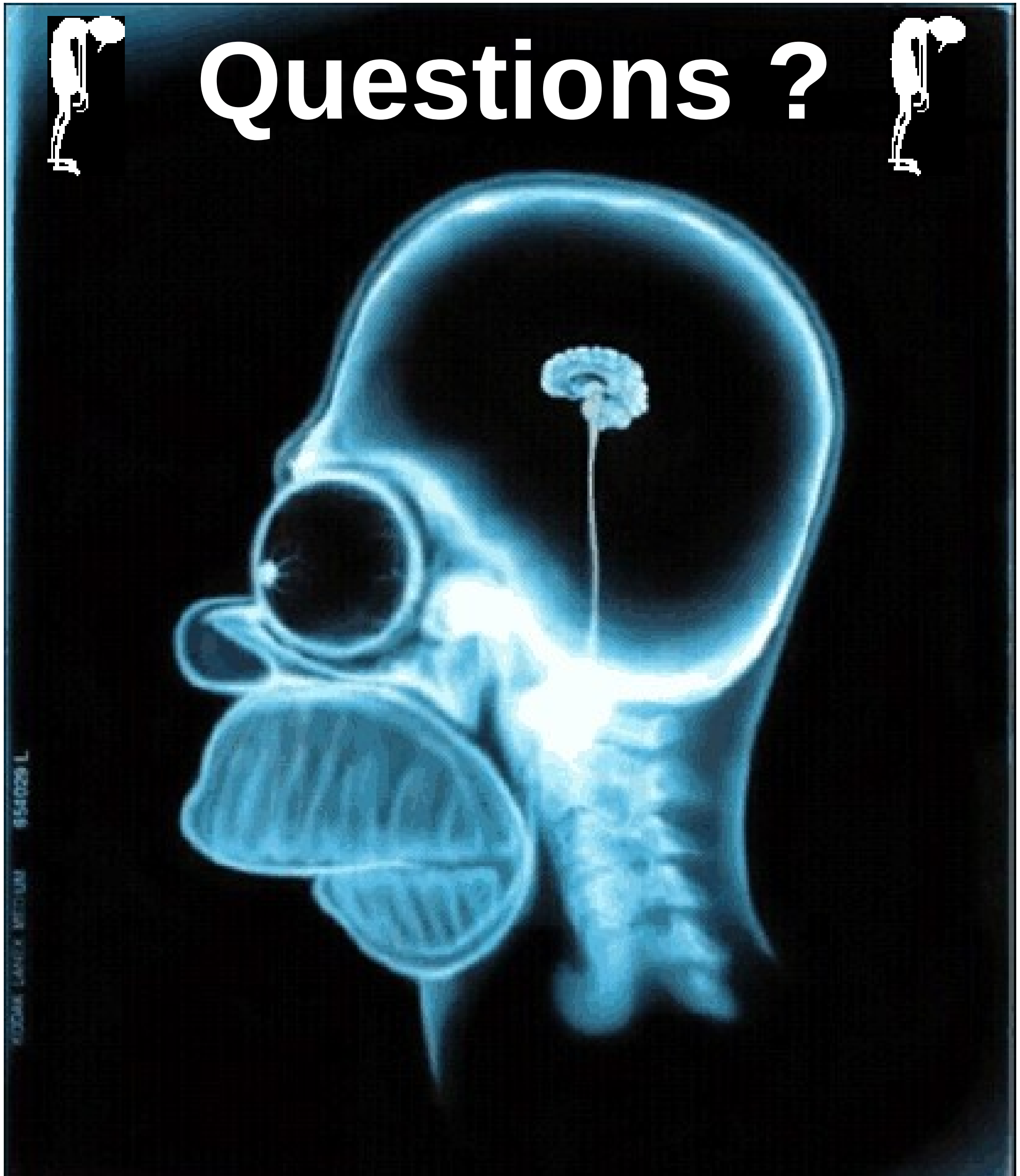
POLARIMETRIC REMOTE SENSING

POLARIMETRIC REMOTE SENSING

POL-SAR PROCESSING
PHENOMENOLOGIC
QUALITATIVE ANALYSIS



POLARIMETRIC SPECKLE FILTERING → **POLARIMETRIC TARGET DECOMPOSITION** → **POLARIMETRIC CLASSIFICATION**
MONO/DUAL CHANNELS


Questions ?