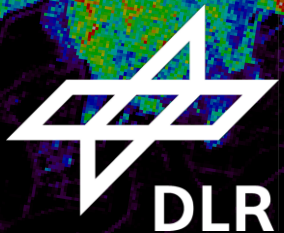


# Polarimetric SAR Interferometry

**Matteo Pardini, Konstantinos P. Papathanassiou**

German Aerospace Center (DLR)  
Microwaves and Radar Institute (DLR-HR)

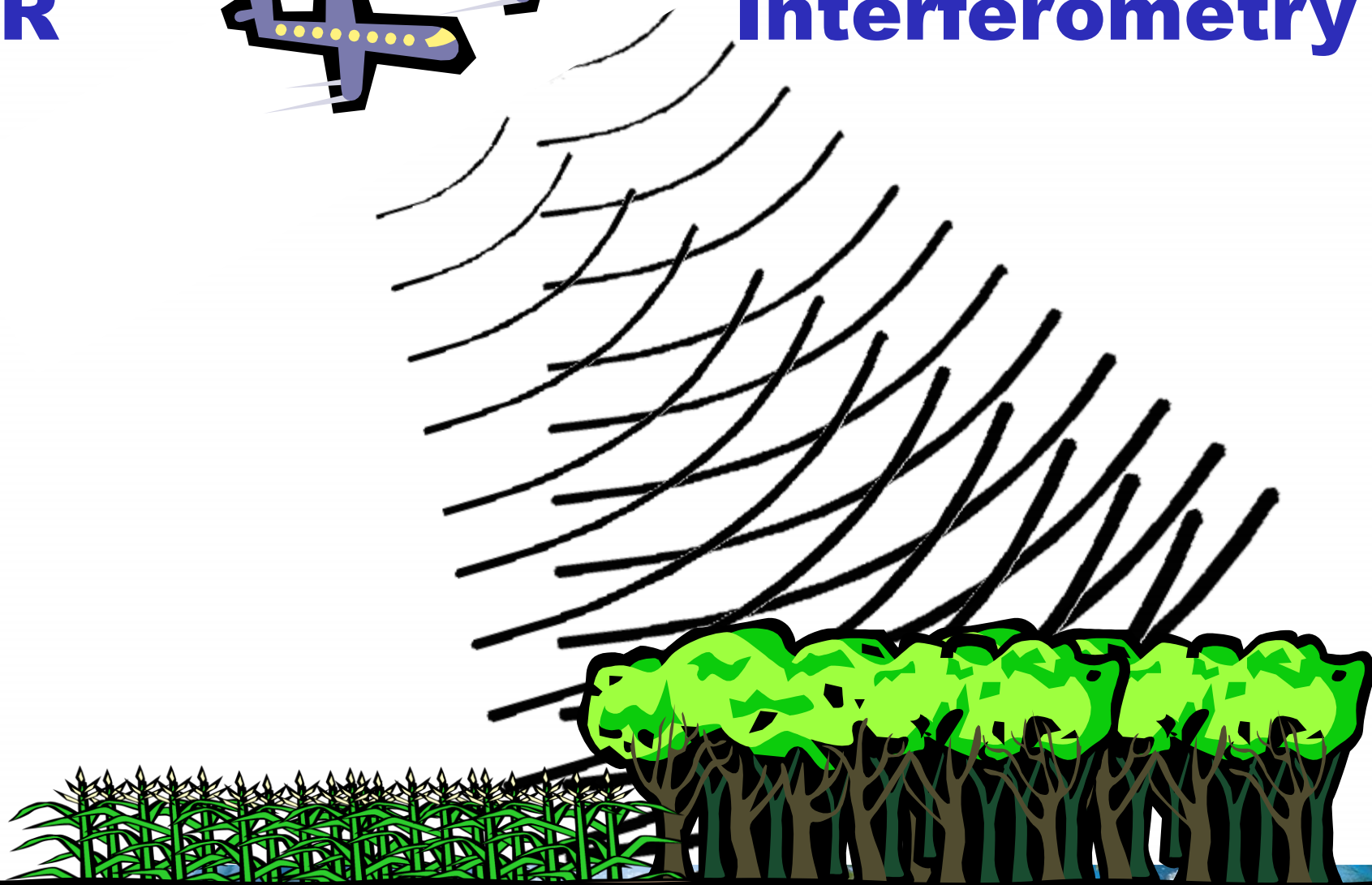
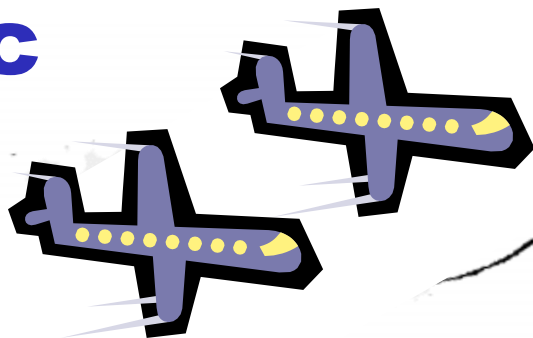
[kostas.papathanassiou@dlr.de](mailto:kostas.papathanassiou@dlr.de)  
[matteo.pardini@dlr.de](mailto:matteo.pardini@dlr.de)



**Polarimetric**

**SAR**

**Interferometry**





**VV Channel Image**

# SAR Polarimetry (PoISAR)

Allows the identification / decomposition of different scattering processes occurring inside the resolution cell



# SAR Interferometry (InSAR)

Allows the location of the effective scattering center inside the resolution cell

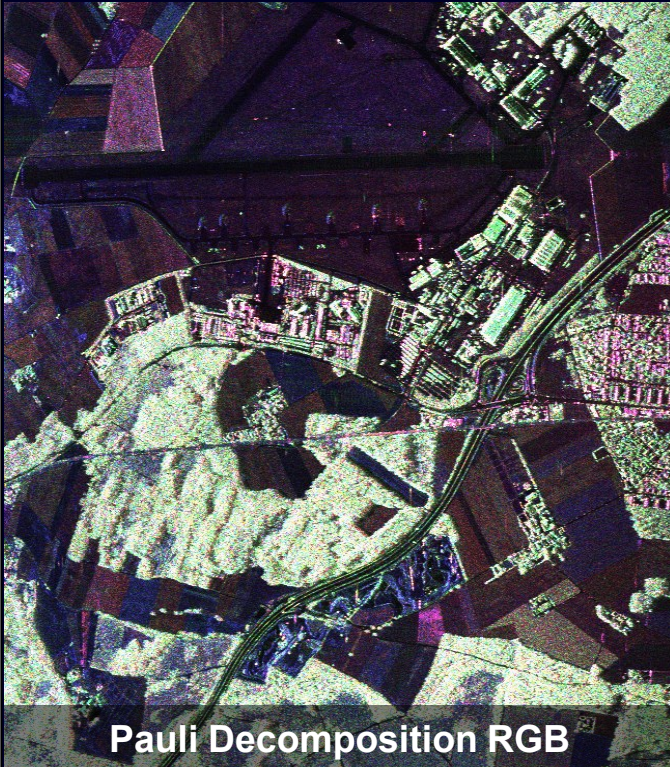


## SAR Polarimetry (PolSAR)

Allows the identification / decomposition of different scattering processes occurring inside the resolution cell

## SAR Interferometry (InSAR)

Allows the location of the effective scattering center inside the resolution cell



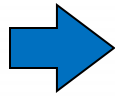
## Polarimetric SAR Interferometry (Pol-InSAR)

Potential to separate in height different scattering processes occurring inside the resolution cell.

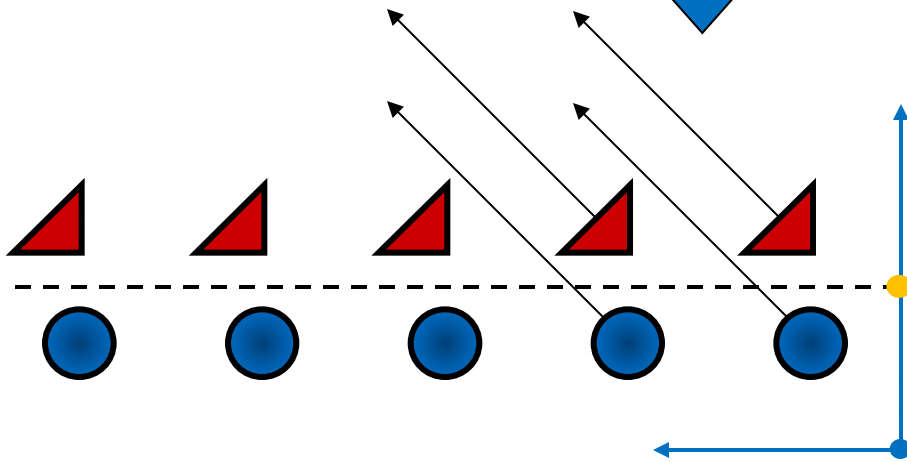
# Interferometry vs. Polarimetry

$$S_{HH}^1 = A_D^1 + A_S^1$$

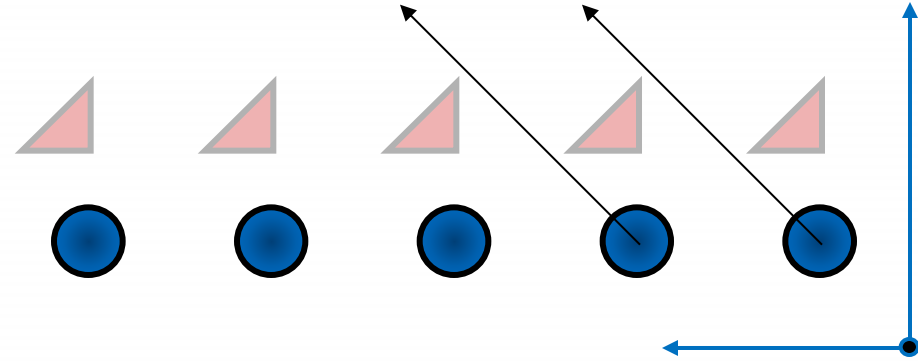
$$S_{HH}^2 = A_D^2 + A_S^2$$



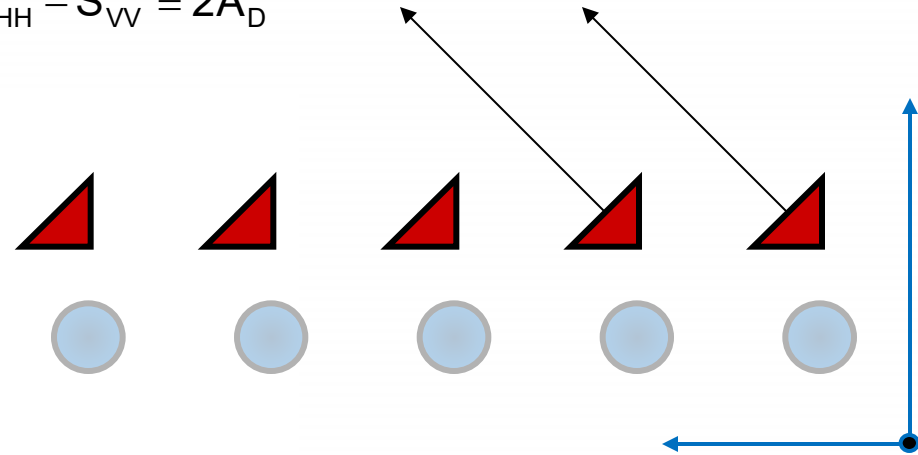
$$\varphi = \arg\{ S_{HH}^1 S_{HH}^{2*} \}$$



$$S_{HH} + S_{VV} = 2A_S$$



$$S_{HH} - S_{VV} = 2A_D$$



$$\begin{matrix} \triangle \\ \circ \end{matrix} [S_D] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} = A_D \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{matrix} \triangle \\ \circ \end{matrix} [S_S] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} = A_S \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

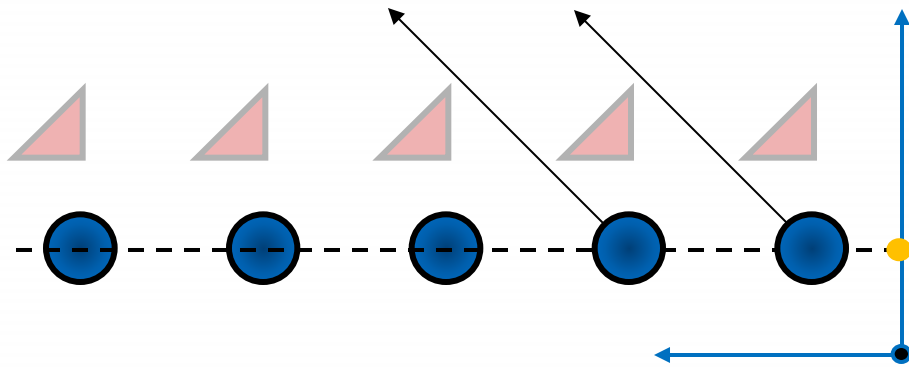


# Polarimetric Interferometry

$$i_{1S} = S_{HH}^1 + S_{VV}^1 = 2A_S^1$$

$$\phi_S = \arg\{ i_{1S} i_{2S}^* \}$$

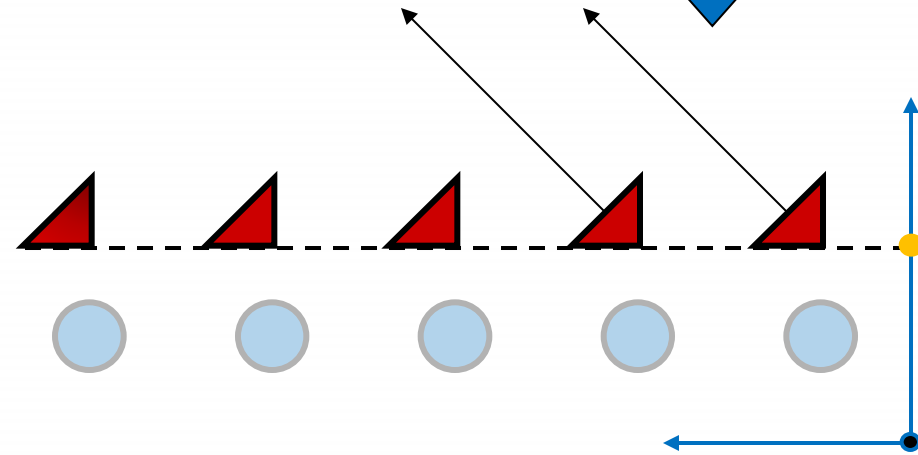
$$i_{2S} = S_{HH}^2 + S_{VV}^2 = 2A_S^2$$



$$i_{1D} = S_{HH}^1 - S_{VV}^1 = 2A_D^1$$

$$\phi_D = \arg\{ i_{1D} i_{2D}^* \}$$

$$i_{2D} = S_{HH}^2 - S_{VV}^2 = 2A_D^2$$



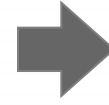
$$\begin{matrix} \triangle \\ [S_D] \end{matrix} = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} = A_D \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{matrix} \circ \\ [S_S] \end{matrix} = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} = A_S \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$





$$[S] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$$



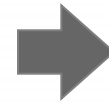
### Polarimetric Coherences

$$\tilde{\gamma}(S_{ij} S_{mn}) = \frac{\langle S_{ij} S_{mn}^* \rangle}{\sqrt{\langle S_{ij} S_{ij}^* \rangle \langle S_{mn} S_{mn}^* \rangle}}$$

### PoISAR



$$[S_1 \ S_2]$$



### Interferometric Coherences

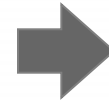
$$\tilde{\gamma}(S_1 S_2) = \frac{\langle S_1 S_2^* \rangle}{\sqrt{\langle S_1 S_1^* \rangle \langle S_2 S_2^* \rangle}}$$

### InSAR



$$[S_1] = \begin{bmatrix} S_{HH}^1 & S_{HV}^1 \\ S_{VH}^1 & S_{VV}^1 \end{bmatrix}$$

$$[S_2] = \begin{bmatrix} S_{HH}^2 & S_{HV}^2 \\ S_{VH}^2 & S_{VV}^2 \end{bmatrix}$$



### Polarimetric / Interferometric Coherences

$$\tilde{\gamma}(S_{ij}^1 S_{mn}^2) = \frac{\langle S_{ij}^1 S_{mn}^{2*} \rangle}{\sqrt{\langle S_{ij}^1 S_{ij}^{1*} \rangle \langle S_{mn}^2 S_{mn}^{2*} \rangle}}$$

### Pol-InSAR



# Complex Coherences on the Unit Circle (UC)

$$\tilde{Y} := \frac{\sum_{k=1}^N S_1(k)S_2^*(k)}{\sqrt{\sum_{k=1}^N S_1(k)S_1^*(k) \sum_{k=1}^N S_2(k)S_2^*(k)}} = \exp(i \text{Arg}(\tilde{Y})) \cdot |\tilde{Y}|$$

Correlation Coefficient  $0 \leq |\tilde{Y}| = \gamma \leq 1$

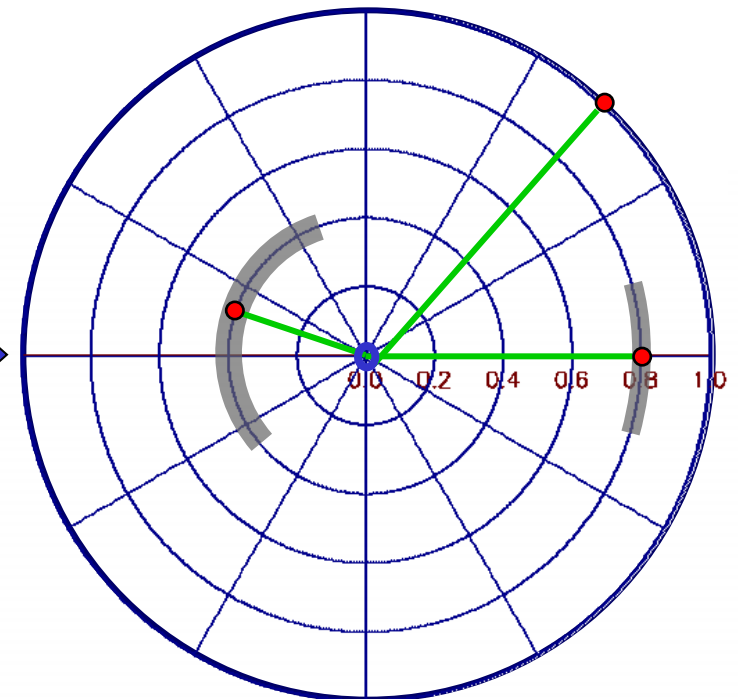
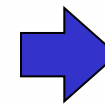
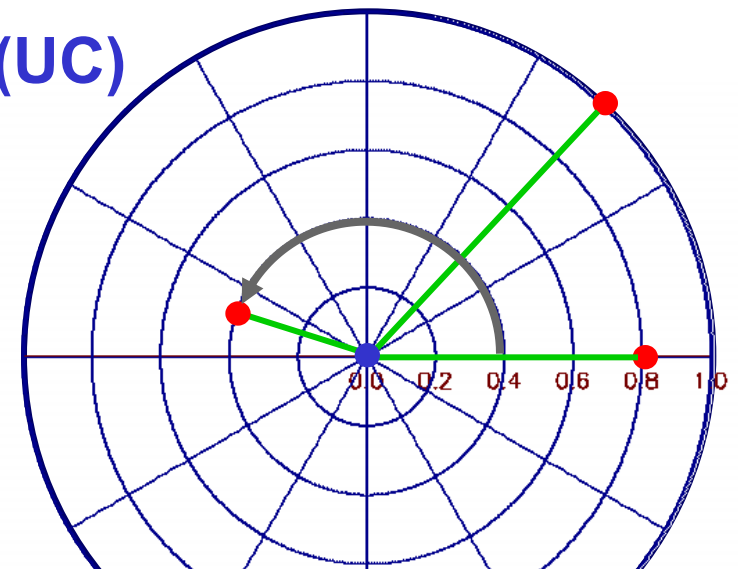
Interferometric Phase  $0 \leq \text{Arg}(\tilde{Y}) = \varphi \leq 2\pi$

Cramer Rao Bounds:

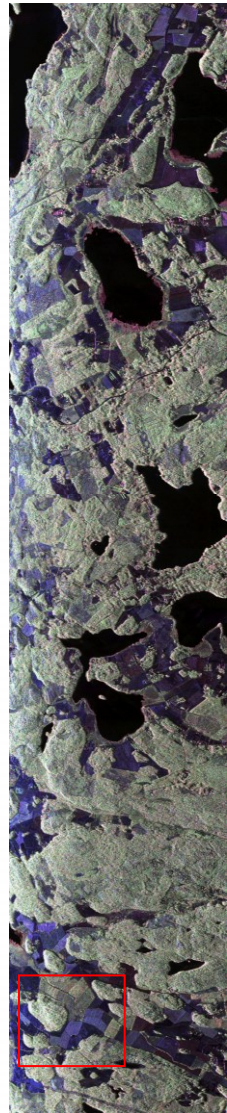
Correlation Coefficient  $\text{VAR}(|\tilde{Y}|)_{\text{CR}} = \frac{(1 - |\gamma|^2)^2}{2N}$

Interferometric Phase  $\text{VAR}(\varphi)_{\text{CR}} = \frac{1 - |\gamma|^2}{2N|\gamma|^2}$

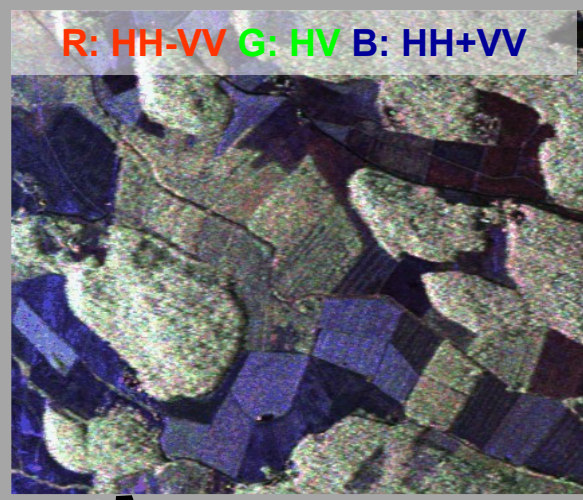
$\varphi = \text{arg}(\tilde{Y})$  and N is the number of Looks



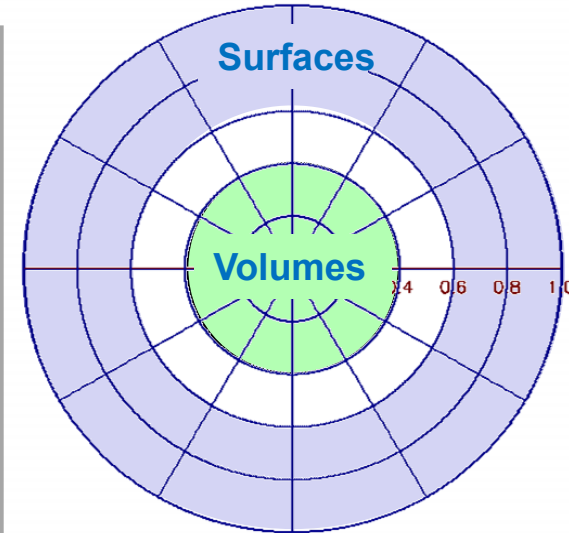
# Why is Interferometry important for Volume Scatterers?



E-SAR / Test Site: Helsinki, Finland



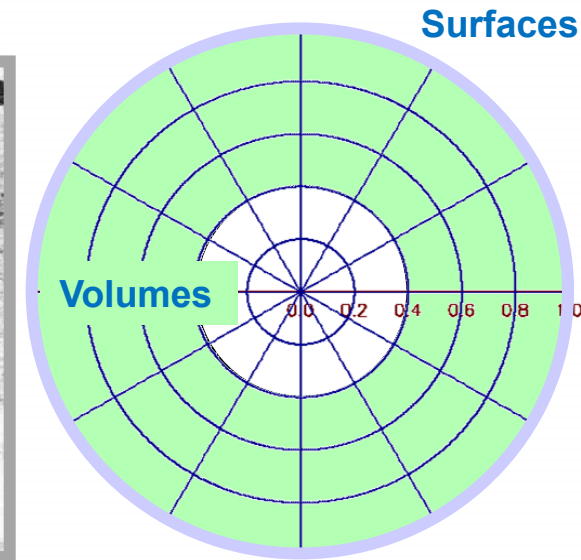
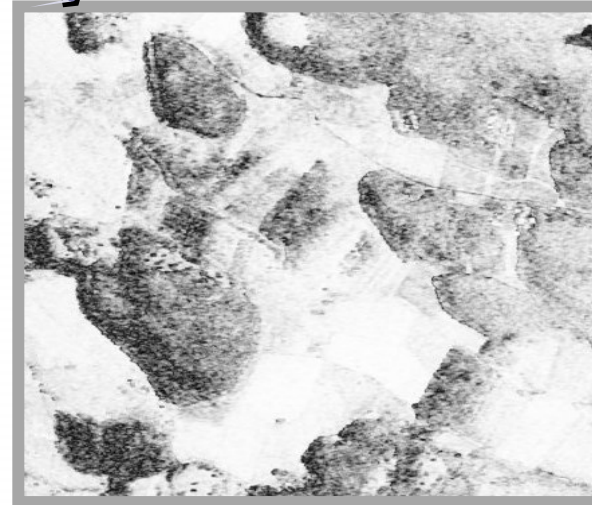
HH-VV Coherence

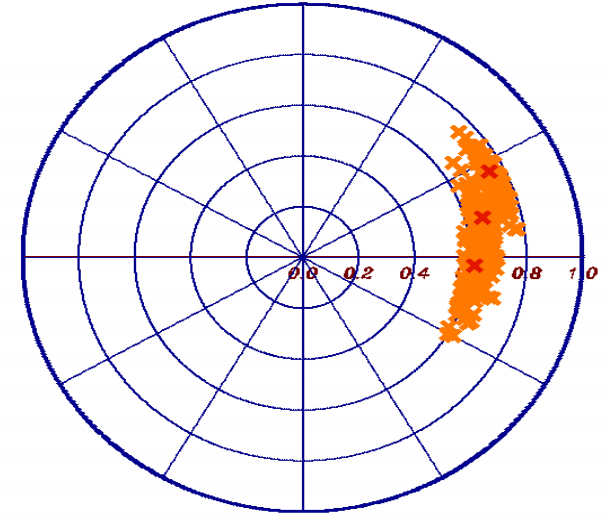
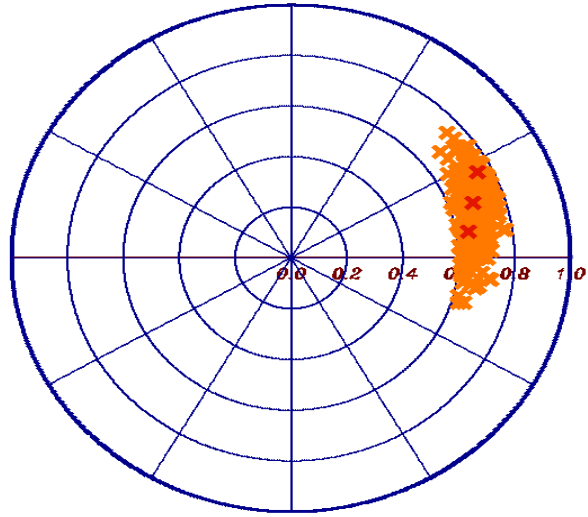
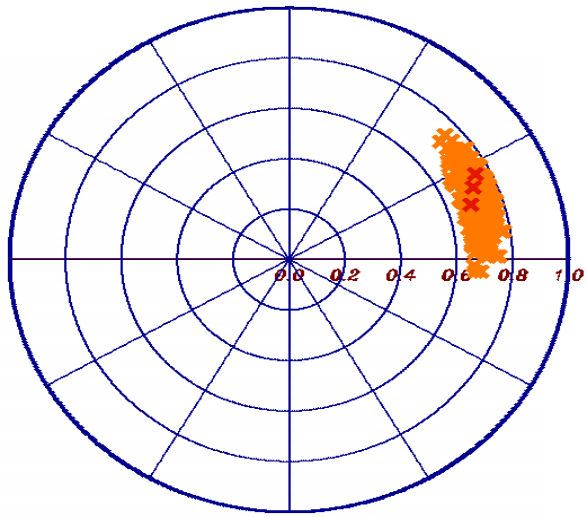


HH-HH Coherence

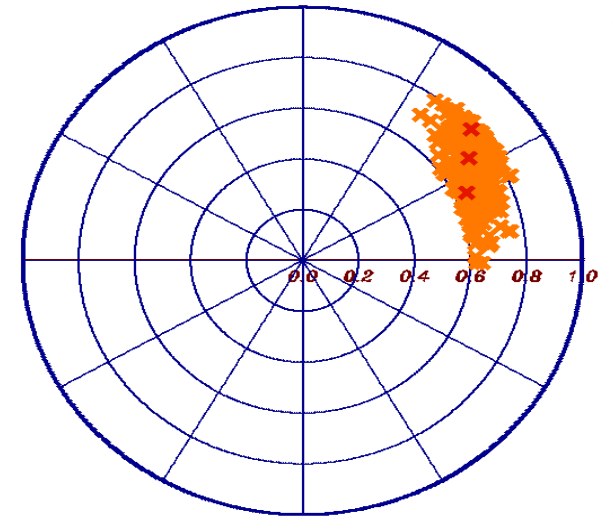
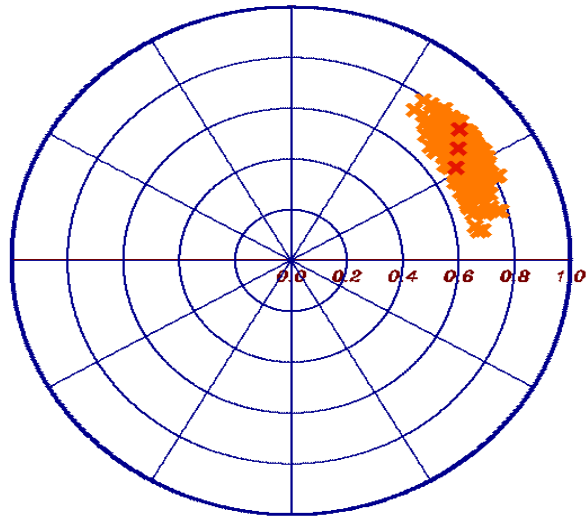
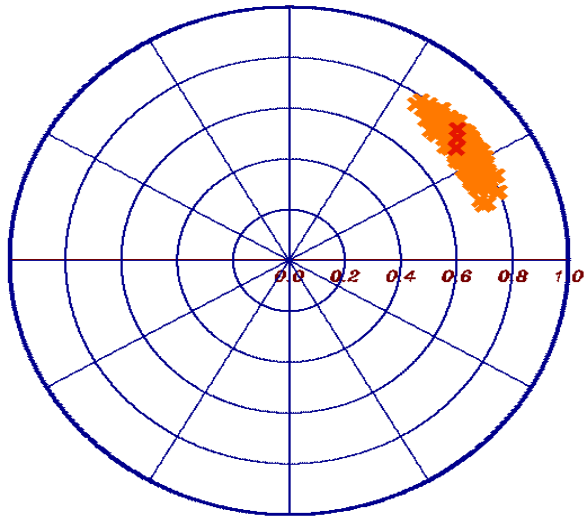


HH-HH Coherence





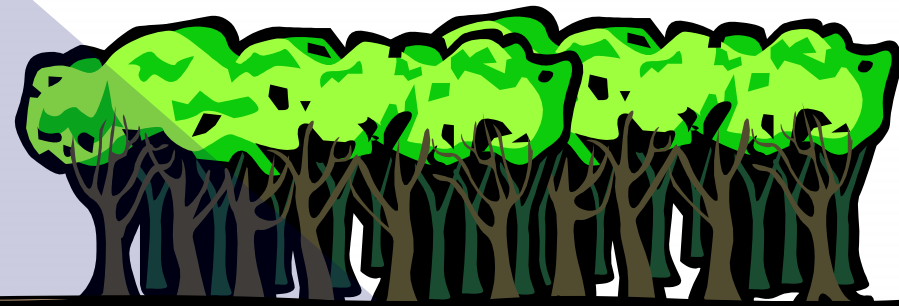
# Pol-InSAR: Basic Principles & Ideas





$S_1$

# SAR Interferometry for Volume Structure





S<sub>1</sub>

S<sub>2</sub>



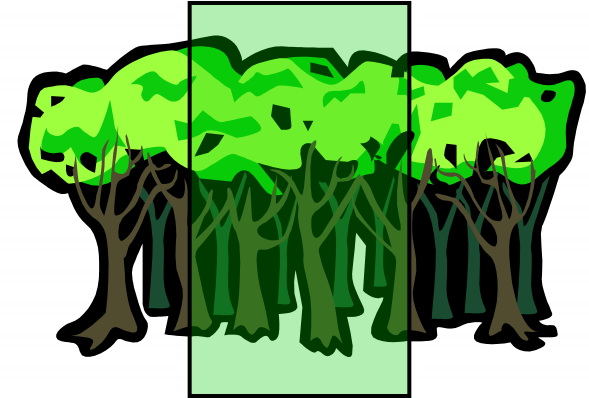
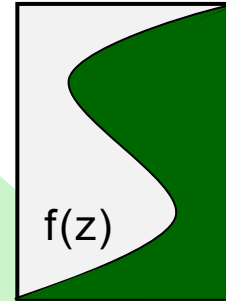
## Interferometric Coherence

$$\tilde{\gamma}(S_1, S_2) = \frac{\langle S_1 S_2^* \rangle}{\sqrt{\langle S_1 S_1^* \rangle \langle S_2 S_2^* \rangle}}$$

# SAR Interferometry for Volume Structure

## Volume Coherence

$$\tilde{\gamma}_{\text{Vol}}(f(z), k_z) = e^{ik_z z_0} \frac{\int_0^{h_v} f(z) e^{ik_z z} dz}{\int_0^{h_v} f(z) dz}$$



f(z) ... vertical reflectivity function

Vertical Wavenumber:  $k_z = \frac{\kappa \Delta \theta}{\sin(\theta_0)}$

$$\tilde{\gamma} = \tilde{\gamma}_{\text{Temporal}} \gamma_{\text{SNR}} \tilde{\gamma}_{\text{Vol}}$$

- $\tilde{\gamma}_{\text{Temporal}}$  ... temporal decorrelation
- $\gamma_{\text{SNR}}$  ... additive noise decorrelation
- $\tilde{\gamma}_{\text{Volume}}$  ... geometric decorrelation





S<sub>1</sub>

S<sub>2</sub>



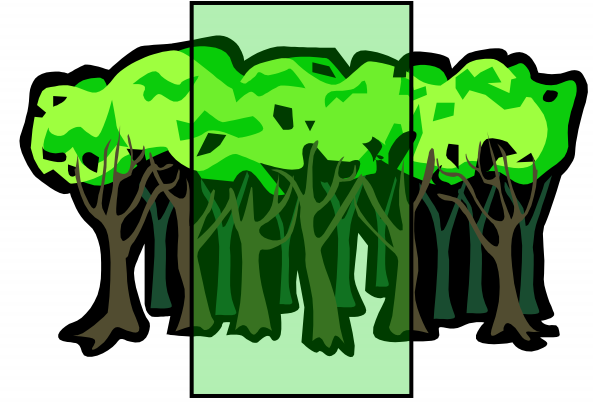
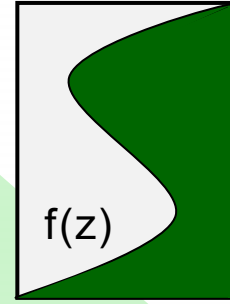
### Interferometric Coherence

$$\tilde{\gamma}(S_1, S_2) = \frac{\langle S_1 S_2^* \rangle}{\sqrt{\langle S_1 S_1^* \rangle \langle S_2 S_2^* \rangle}}$$

## SAR Interferometry for Volume Structure

### Volume Coherence

$$\tilde{\gamma}_{Vol}(f(z), k_z) = e^{ik_z z_0} \frac{\int_0^{h_v} f(z) e^{ik_z z} dz}{\int_0^{h_v} f(z) dz}$$



f(z) ... vertical reflectivity function

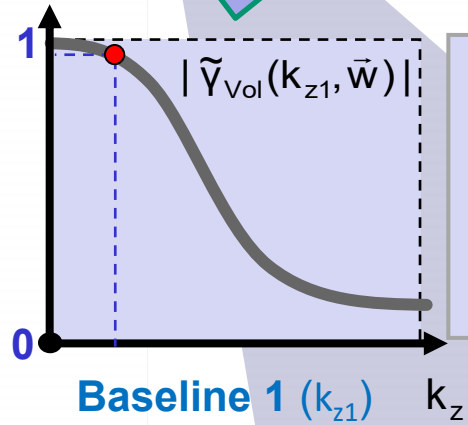
Vertical Wavenumber:  $k_z = \frac{\kappa \Delta \theta}{\sin(\theta_0)}$

$$\tilde{\gamma} = \tilde{\gamma}_{Temporal} \gamma_{SNR} \tilde{\gamma}_{Vol}$$

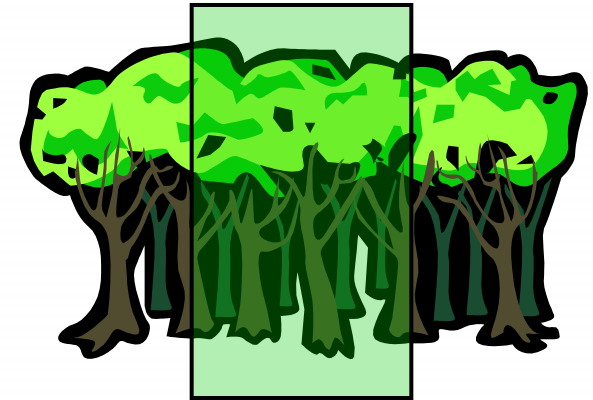
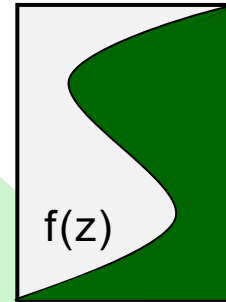
- $\tilde{\gamma}_{Temporal}$  ... temporal decorrelation
- $\gamma_{SNR}$  ... additive noise decorrelation
- $\tilde{\gamma}_{Volume}$  ... geometric decorrelation

SAR interferometry allows to reconstruct the vertical reflectivity function f(z) of a volume scatterer by means of interferometric (volume) coherence measurements at different vertical wavenumbers  $k_z$ , i.e. at different spatial baselines.

Normalised Fourier Transform of the vertical reflectivity function  $f(z)$



$$\tilde{Y}_{\text{Vol}}(k_{z1}) = e^{ik_{z1}z_0} \frac{\int_0^{h_y} f(z) e^{ik_{z1}z} dz}{\int_0^{h_y} f(z) dz}$$



$f(z)$  ... vertical reflectivity function

Vertical Wavenumber:  $k_z = \frac{\kappa \Delta \theta}{\sin(\theta_0)}$



# Multibaseline SAR Interferometry



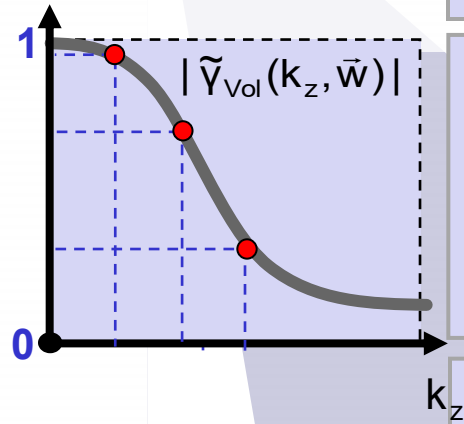
S<sub>1</sub>

Baseline 3 (k<sub>z3</sub>)

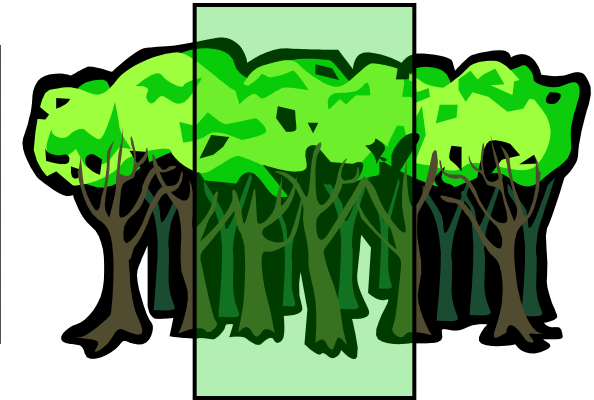
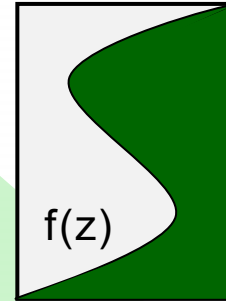
$$\tilde{Y}_{Vol}(k_{z3}) = e^{ik_{z3}z_0} \frac{\int_0^{h_y} f(z) e^{ik_{z3}z} dz}{\int_0^{h_y} f(z) dz}$$

$$\tilde{Y}_{Vol}(k_{z1}) = e^{ik_{z1}z_0} \frac{\int_0^{h_y} f(z) e^{ik_{z1}z} dz}{\int_0^{h_y} f(z) dz}$$

$$\tilde{Y}_{Vol}(k_{z2}) = e^{ik_{z2}z_0} \frac{\int_0^{h_y} f(z) e^{ik_{z2}z} dz}{\int_0^{h_y} f(z) dz}$$



Baseline 2 (k<sub>z2</sub>)



f(z) ... vertical reflectivity function

Vertical Wavenumber:  $k_z = \frac{\kappa \Delta \theta}{\sin(\theta_0)}$



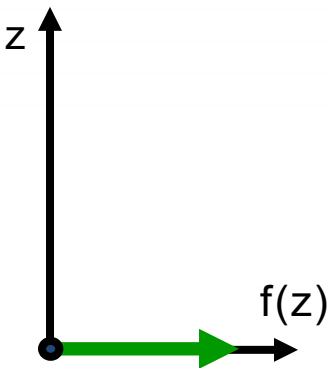
# Multibaseline SAR Interferometry

Multi-baseline measurements allow to sample the spectrum of the vertical reflectivity  $FT\{f(z)\}$  @ different (spatial) frequencies ( $k_z$ ).

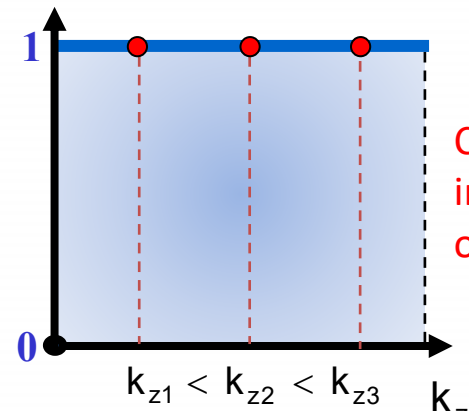
# Vertical Reflectivity Function $f(z)$

# InSAR Volume Coherence $|\tilde{\gamma}_{Vol}(k_z)|$

Surface Scatterer

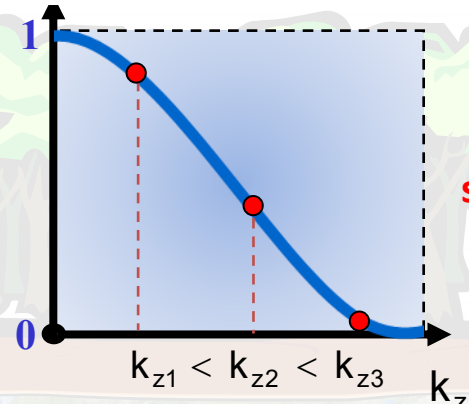
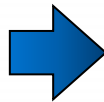
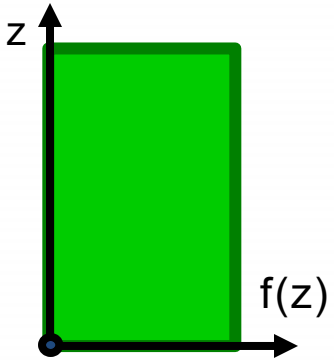


$$|\tilde{\gamma}_{Vol}(k_z)| = \frac{\left| \int_0^{h_v} f(z) e^{ik_z z} dz \right|}{\int_0^{h_v} f(z) dz}$$



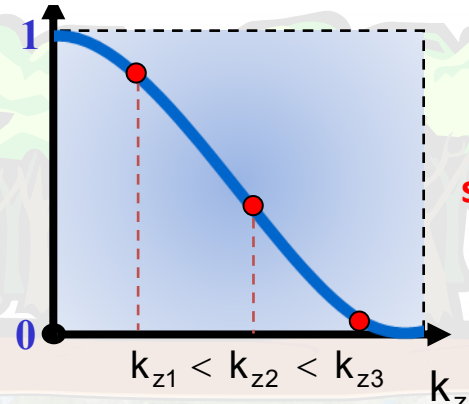
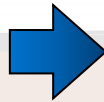
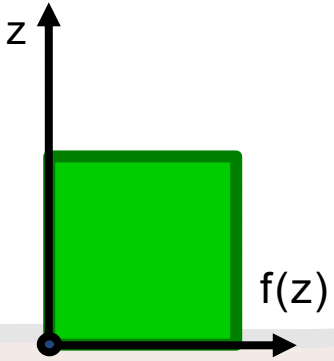
Coherence is independent of baseline

Tall Vegetation

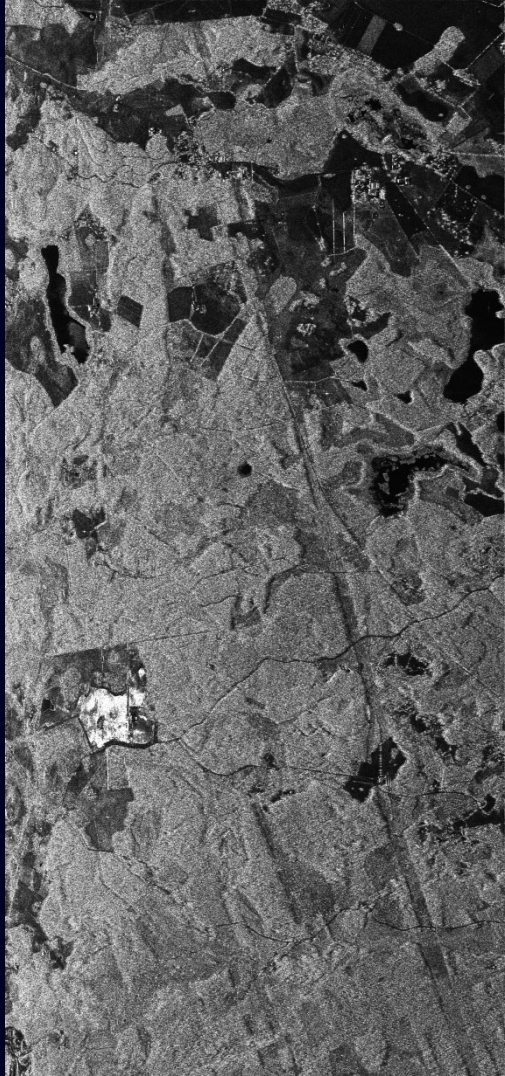


Coherence decreases slower with increasing baseline

Short Vegetation



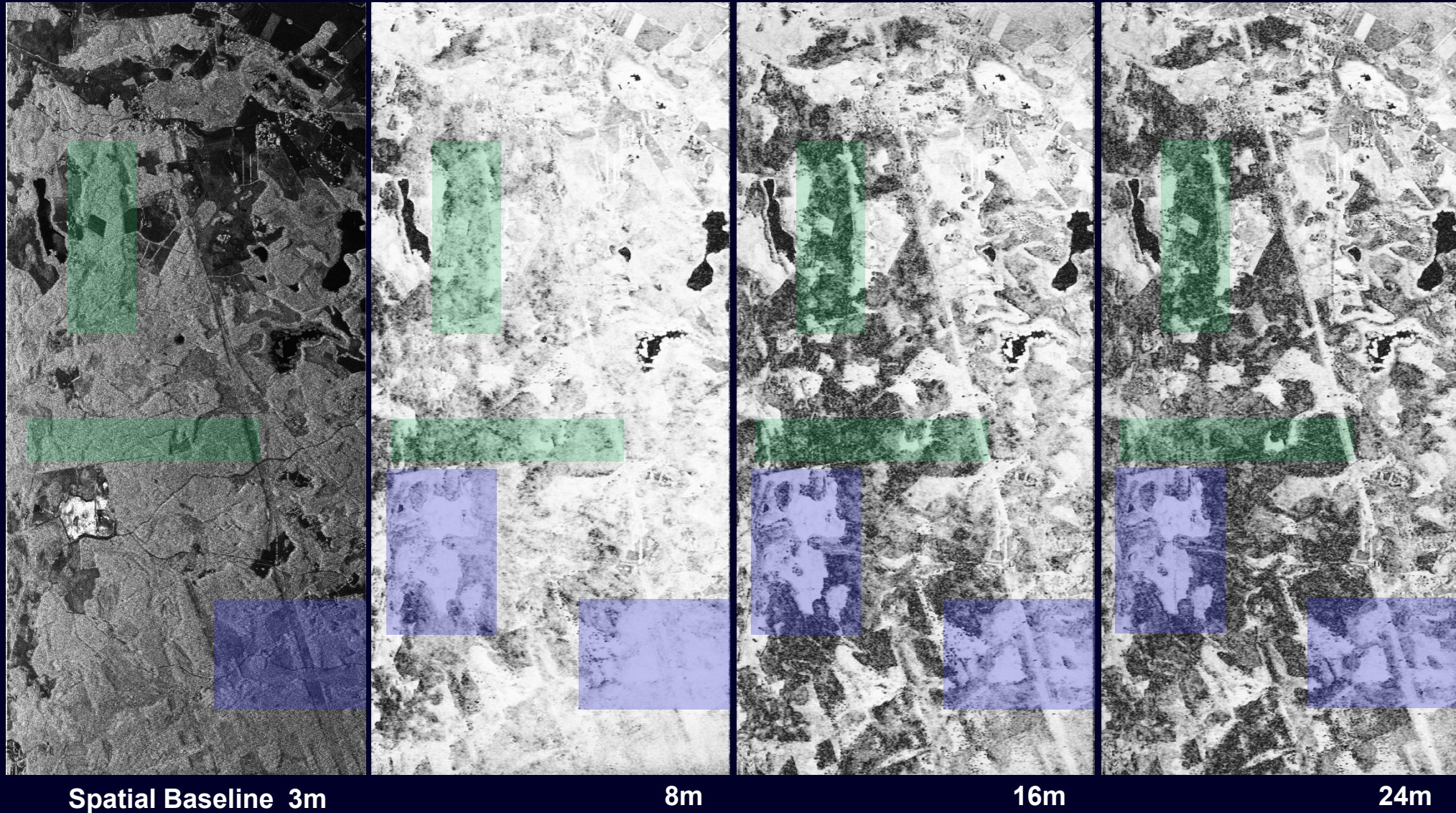
# Amplitude Image



Amplitude Image HH



# Interferometric Coherence: Volume Decorrelation





0

1



By DLR-HR-STL

500x500 m<sup>2</sup> resolution



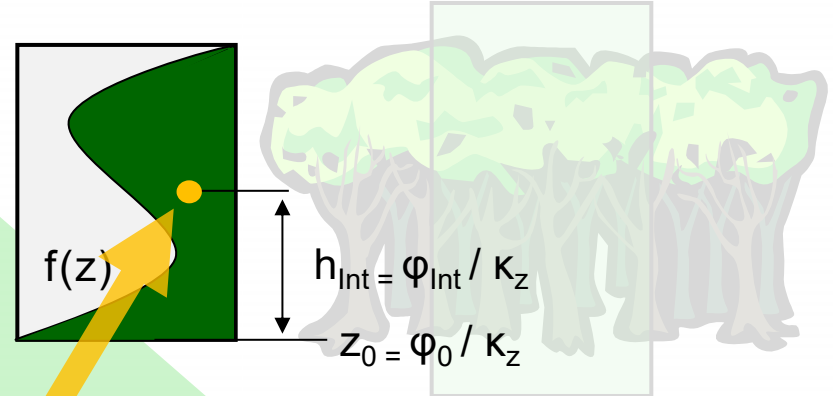
## Interferometric Coherence

$$\tilde{\gamma}(S_1, S_2) = \frac{\langle S_1 S_2^* \rangle}{\sqrt{\langle S_1 S_1^* \rangle \langle S_2 S_2^* \rangle}}$$

# SAR Interferometry for Volume Structure: The Phase Center

## Volume Coherence

$$\tilde{\gamma}_{Vol}(f(z), k_z) = \frac{\int_0^{h_v} f(z) e^{ik_z z} dz}{\int_0^{h_v} f(z) dz} e^{ik_z z_0}$$



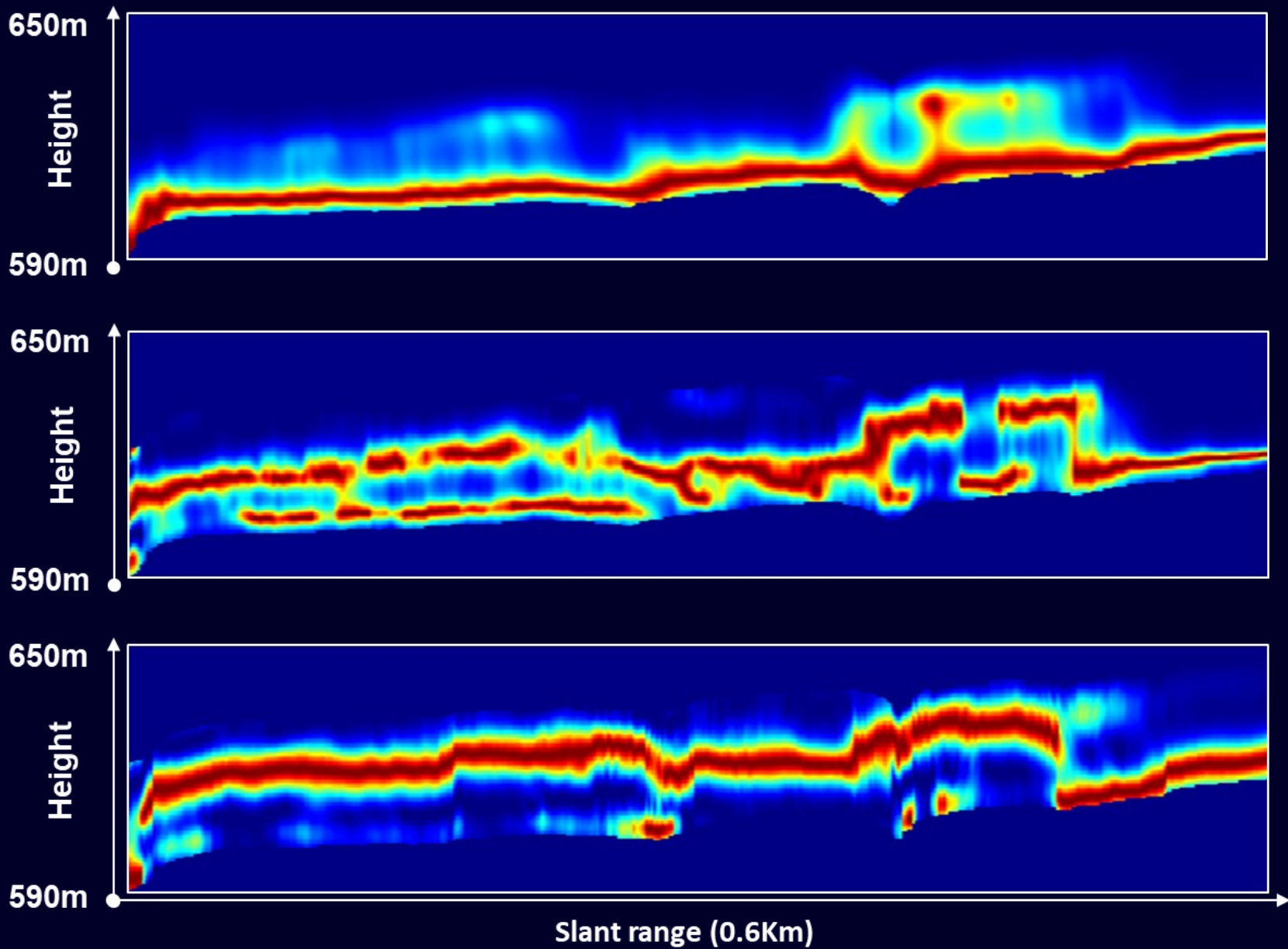
$f(z)$  ... vertical reflectivity function

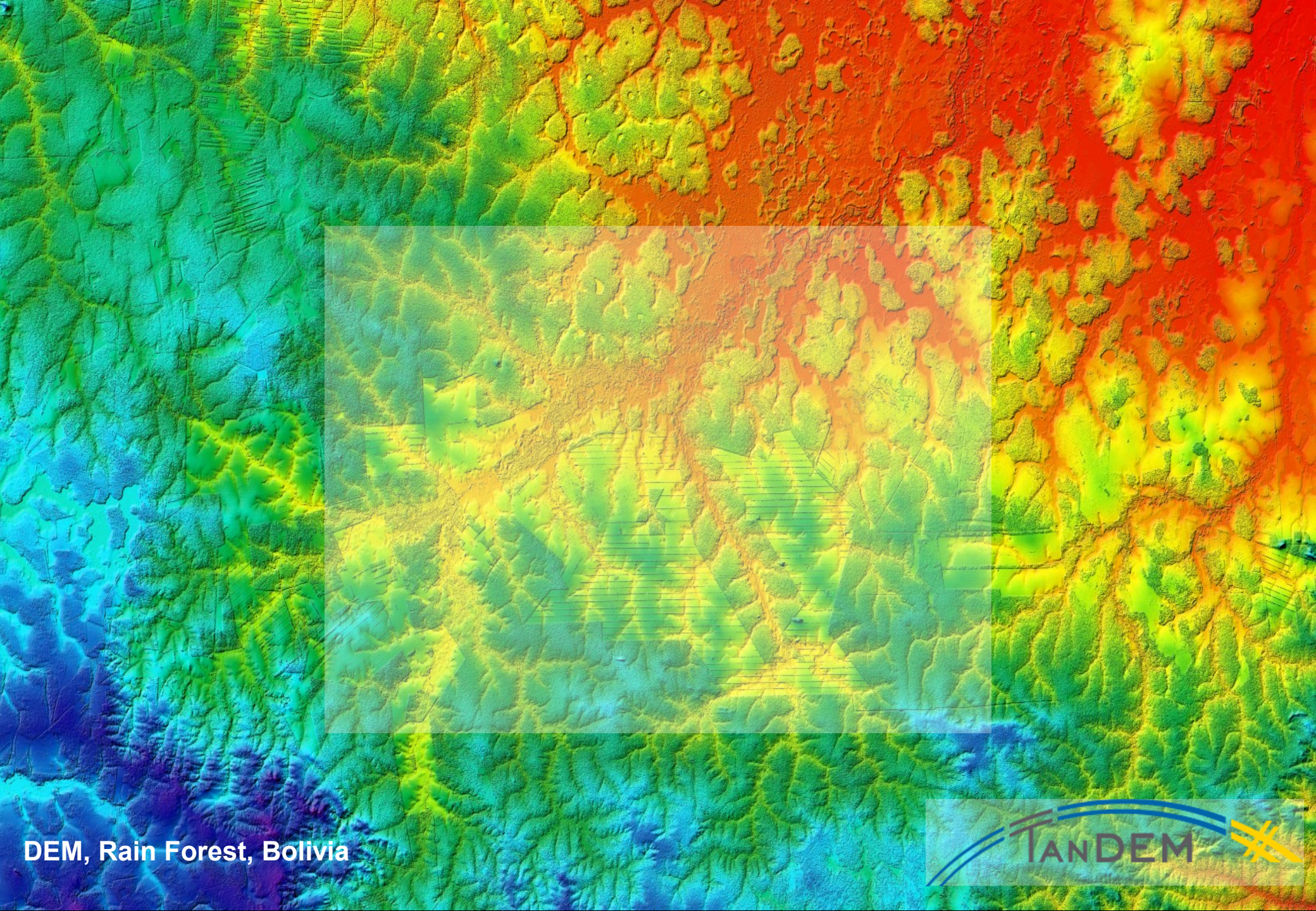
Vertical Wavenumber:  $k_z = \frac{\kappa \Delta \theta}{\sin(\theta_0)}$

$$\tilde{\gamma} = \tilde{\gamma}_{Temporal} \gamma_{SNR} \tilde{\gamma}_{Vol}$$

- $\tilde{\gamma}_{Temporal}$  ... temporal decorrelation
- $\gamma_{SNR}$  ... additive noise decorrelation
- $\tilde{\gamma}_{Volume}$  ... geometric decorrelation

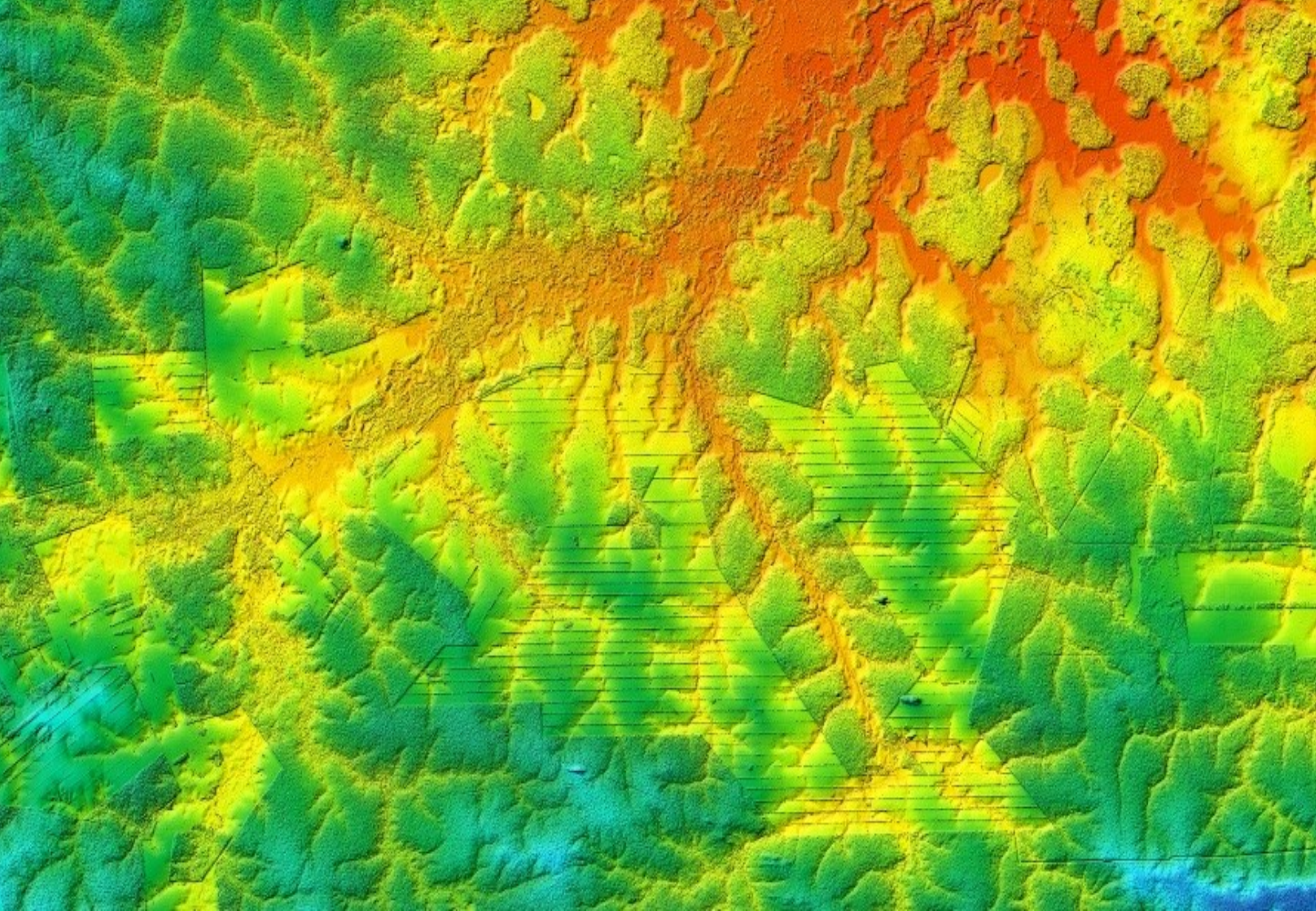
The **phase (center)** of  $\tilde{\gamma}_{Vol}$  is associated to the **center of mass of  $f(z)$** : The phase center height  $h_{Int} = \phi_{Int} / k_z$  corresponds to the height of the center of mass of  $f(z)$  with respect to  $z_0$  !!!





DEM, Rain Forest, Bolivia





# Polarimetric SAR Interferometry

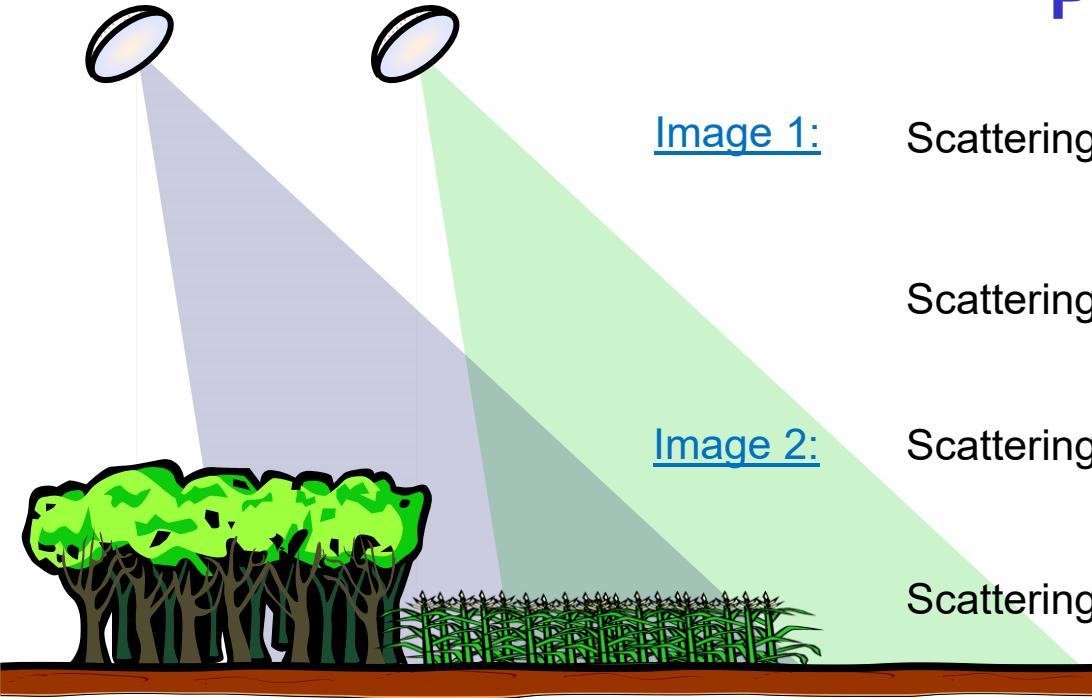


Image 1:

Scattering Matrix:  $[S_1] = \begin{bmatrix} S_{HH}^1 & S_{HV}^1 \\ S_{VH}^1 & S_{VV}^1 \end{bmatrix}$

Scattering Vector:  $\vec{k}_1 = \frac{1}{\sqrt{2}} [S_{HH}^1 + S_{VV}^1 \quad S_{HH}^1 - S_{VV}^1 \quad 2S_{HV}^1]^T$

Image 2:

Scattering Matrix:  $[S_2] = \begin{bmatrix} S_{HH}^2 & S_{HV}^2 \\ S_{VH}^2 & S_{VV}^2 \end{bmatrix}$

Scattering Vector:  $\vec{k}_2 = \frac{1}{\sqrt{2}} [S_{HH}^2 + S_{VV}^2 \quad S_{HH}^2 - S_{VV}^2 \quad 2S_{HV}^2]^T$

Image formation:

$i_1 = \vec{w}_1^+ \cdot \vec{k}_1$  and  $i_2 = \vec{w}_2^+ \cdot \vec{k}_2$  ... projection of the scattering vector on a (complex) unitary vector  $\vec{w}_i$

$\vec{w}_i$  used to select a given polarisation out of all possible polarisations provided by [S]

Example:  $S_{HH} + S_{VV}$  image:  $\vec{w} = [1 \ 0 \ 0]^T \rightarrow i = \vec{w}^+ \cdot \vec{k}_j = \frac{1}{\sqrt{2}} (S_{HH}^j + S_{VV}^j)$

$S_{HH}$  image:  $\vec{w} = [1/\sqrt{2} \ 1/\sqrt{2} \ 0]^T \rightarrow i_j = \vec{w}^+ \cdot \vec{k}_j = S_{HH}^j$



# Polarimetric SAR Interferometry

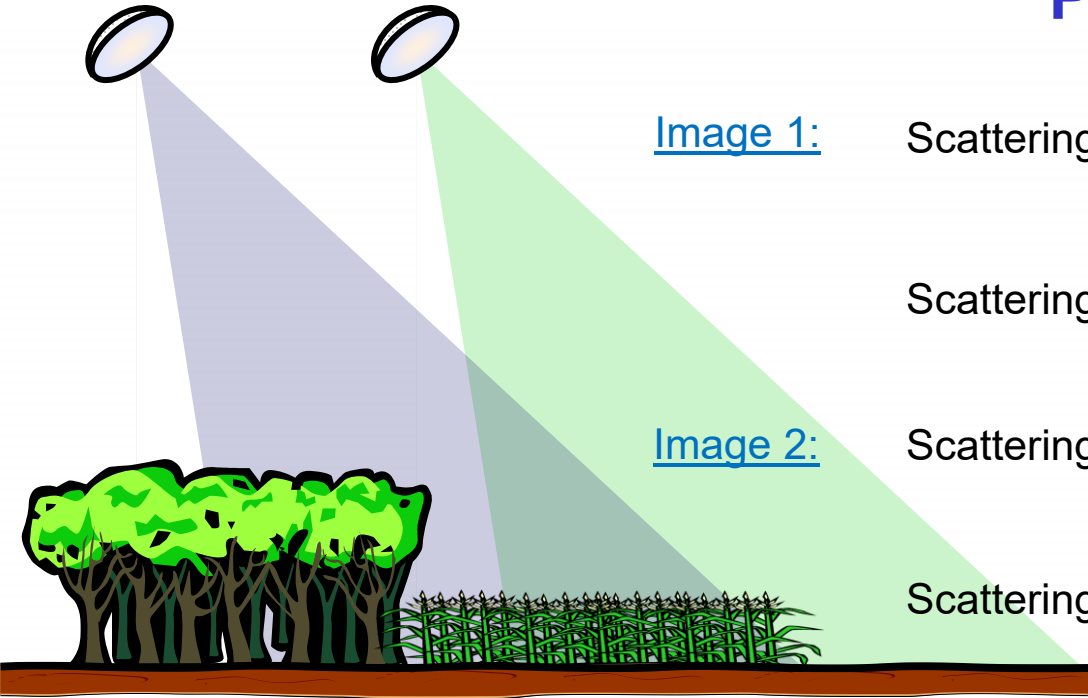


Image 1:

Scattering Matrix:  $[S_1] = \begin{bmatrix} S_{HH}^1 & S_{HV}^1 \\ S_{VH}^1 & S_{VV}^1 \end{bmatrix}$

Scattering Vector:  $\vec{k}_1 = \frac{1}{\sqrt{2}} [S_{HH}^1 + S_{VV}^1 \quad S_{HH}^1 - S_{VV}^1 \quad 2S_{HV}^1]^T$

Image 2:

Scattering Matrix:  $[S_2] = \begin{bmatrix} S_{HH}^2 & S_{HV}^2 \\ S_{VH}^2 & S_{VV}^2 \end{bmatrix}$

Scattering Vector:  $\vec{k}_2 = \frac{1}{\sqrt{2}} [S_{HH}^2 + S_{VV}^2 \quad S_{HH}^2 - S_{VV}^2 \quad 2S_{HV}^2]^T$

Image formation:  $i_1 = \vec{w}_1^+ \cdot \vec{k}_1$  and  $i_2 = \vec{w}_2^+ \cdot \vec{k}_2$  where  $\vec{w}_i$  are complex unitary vectors\*

Interferogram formation:  $i_1 i_2^* = (\vec{w}_1^+ \cdot \vec{k}_1)(\vec{w}_2^+ \cdot \vec{k}_2)^+ = \vec{w}_1^+(\vec{k}_1 \cdot \vec{k}_2^+)\vec{w}_2 = \vec{w}_1^+[\Omega]\vec{w}_2$

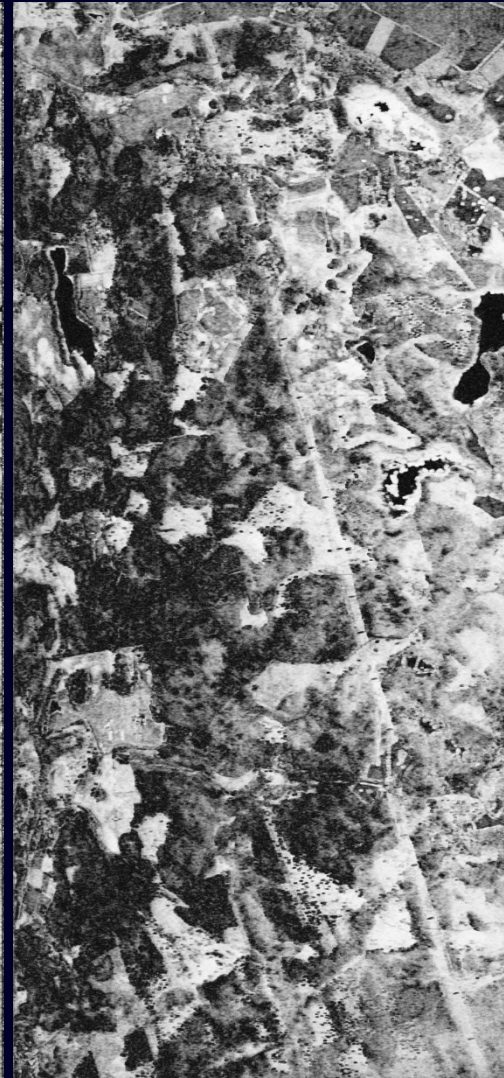
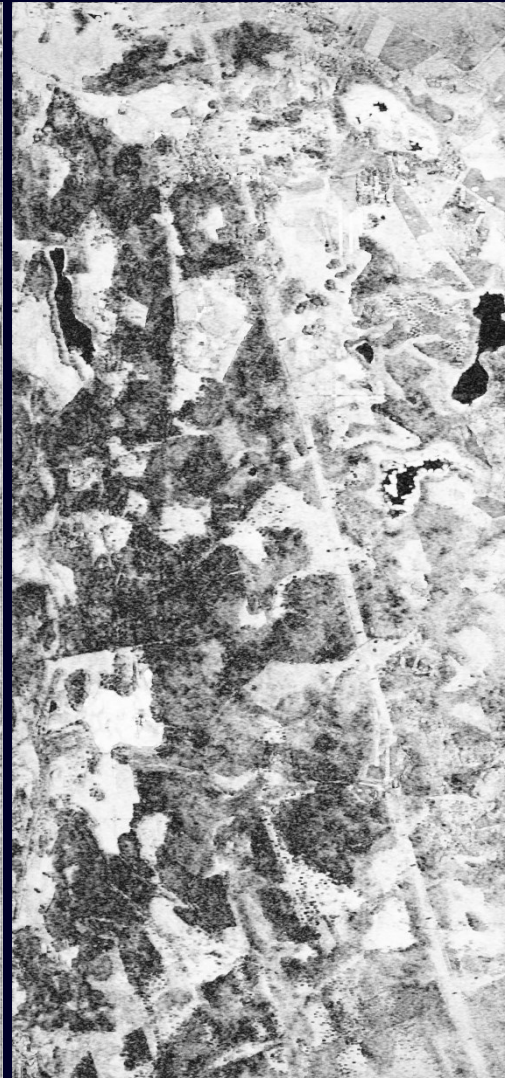
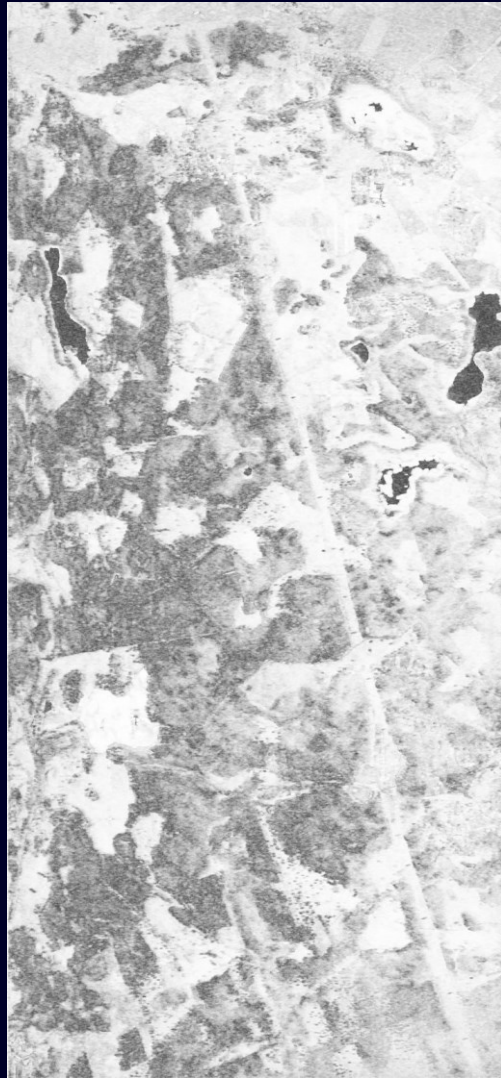
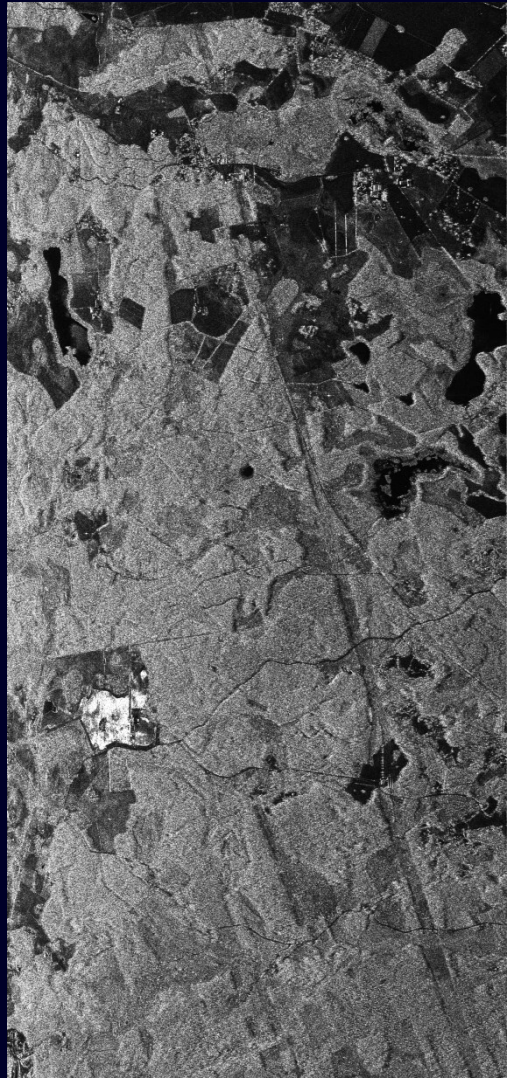
Interferometric Coherence:

$$\tilde{\gamma}(\vec{w}_1, \vec{w}_2) = \frac{\langle i_1 i_2^* \rangle}{\sqrt{\langle i_1 i_1^* \rangle \langle i_2 i_2^* \rangle}} = \frac{\langle \vec{w}_1^+[\Omega]\vec{w}_2 \rangle}{\sqrt{\langle \vec{w}_1^+[T_{11}]\vec{w}_1 \rangle \langle \vec{w}_2^+[T_{22}]\vec{w}_2 \rangle}}$$

where  $[T_{11}] = \langle \vec{k}_1 \cdot \vec{k}_1^+ \rangle$   $[T_{22}] = \langle \vec{k}_2 \cdot \vec{k}_2^+ \rangle$  and  $[\Omega] = \langle \vec{k}_1 \cdot \vec{k}_2^+ \rangle$

$\vec{w}_i$  used to select a polarisation state out of all possible polarisations provided by the scattering matrix [S]

# Interferometric Coherence: Volume Decorrelation



Amplitude Image HH

Sp. Baseline 16m Opt 1

HH

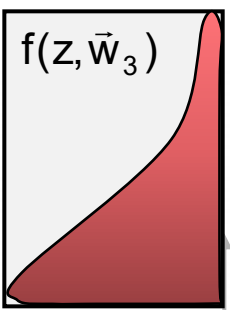
Opt 3





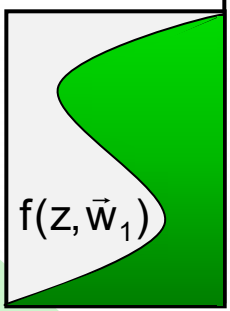
**Polarisation 3 ( $\underline{w}_3$ ):**

$$\tilde{\Upsilon}_{\text{Vol}}(f(z, \vec{w}_3)) = e^{ik_z z_0} \frac{\int_0^{h_v} f(z, \vec{w}_3) e^{ik_z z} dz}{\int_0^{h_v} f(z, \vec{w}_3) dz}$$



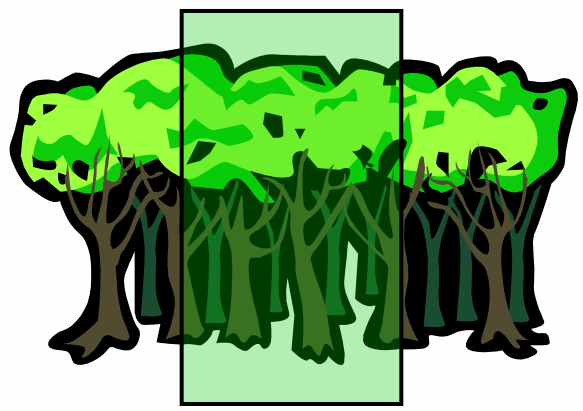
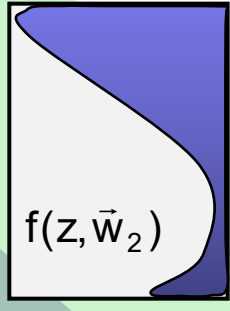
**Polarisation 1 ( $\underline{w}_1$ ):**

$$\tilde{\Upsilon}_{\text{Vol}}(f(z, \vec{w}_1)) = e^{ik_z z_0} \frac{\int_0^{h_v} f(z, \vec{w}_1) e^{ik_z z} dz}{\int_0^{h_v} f(z, \vec{w}_1) dz}$$



**Polarisation 2 ( $\underline{w}_2$ ):**

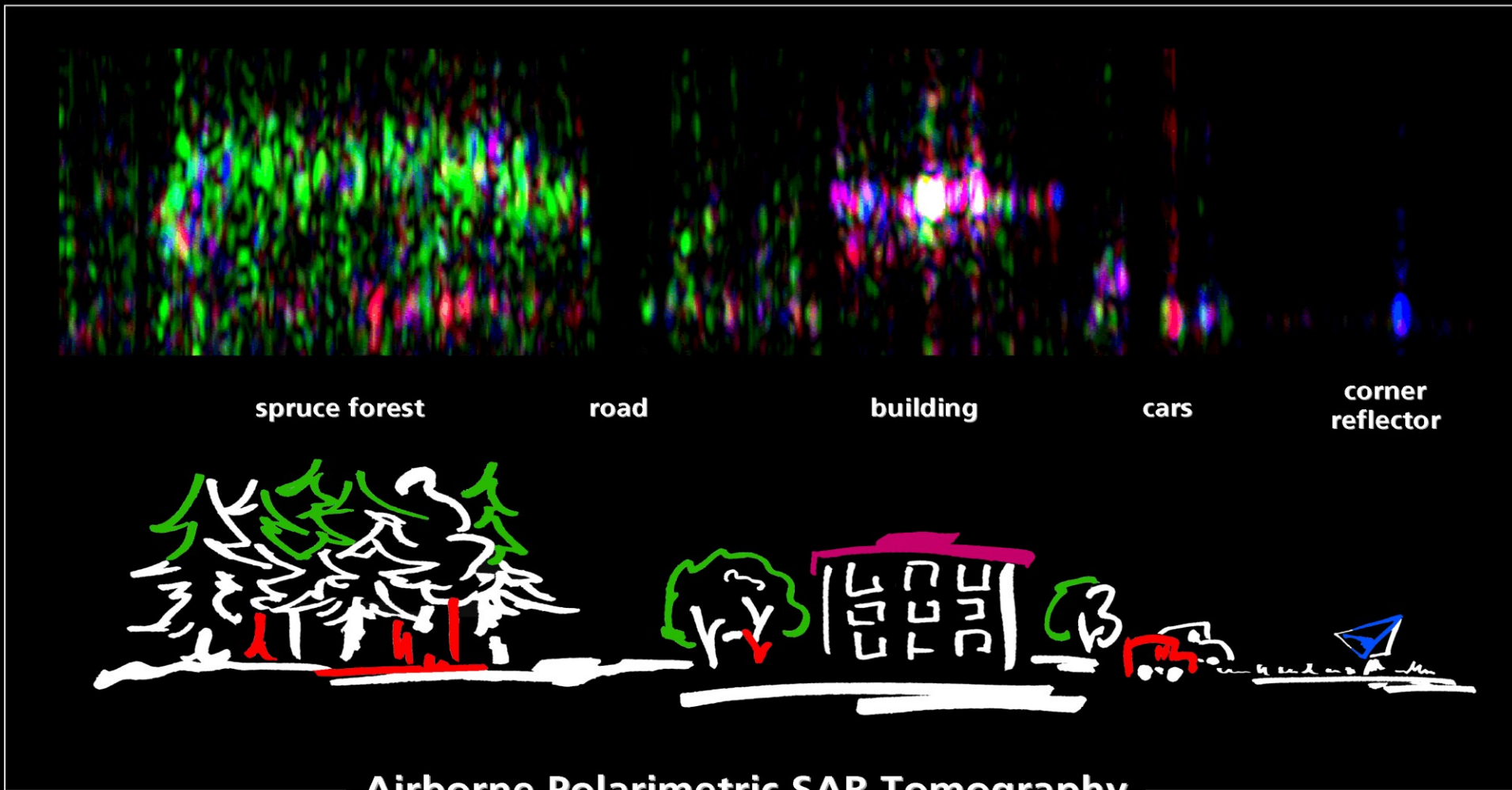
$$\tilde{\Upsilon}_{\text{Vol}}(f(z, \vec{w}_2)) = e^{ik_z z_0} \frac{\int_0^{h_v} f(z, \vec{w}_2) e^{ik_z z} dz}{\int_0^{h_v} f(z, \vec{w}_2) dz}$$



$f(z, \vec{w})$  ...vertical reflectivity function

# Polarimetric SAR Interferometry



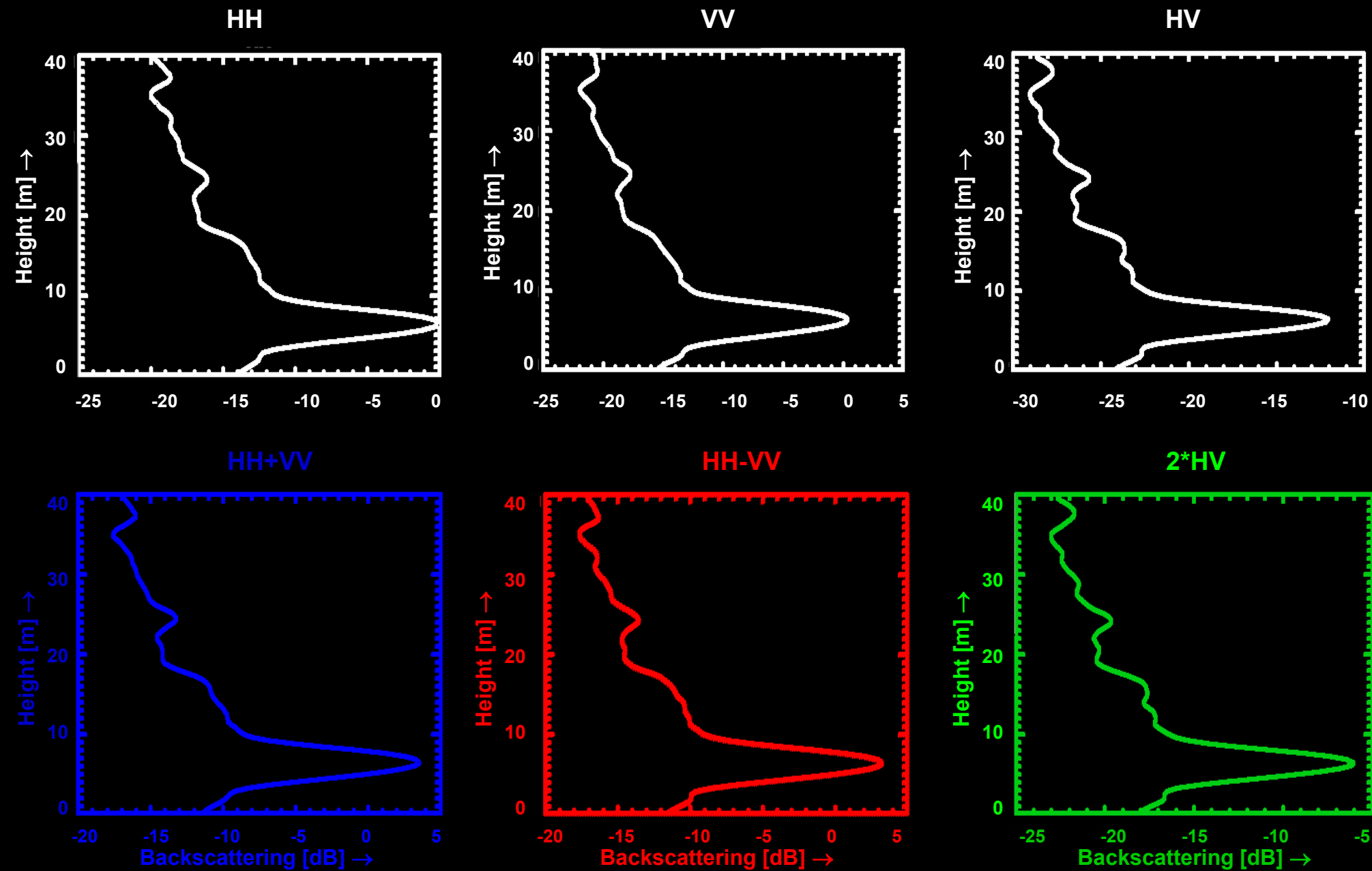


### Airborne Polarimetric SAR Tomography

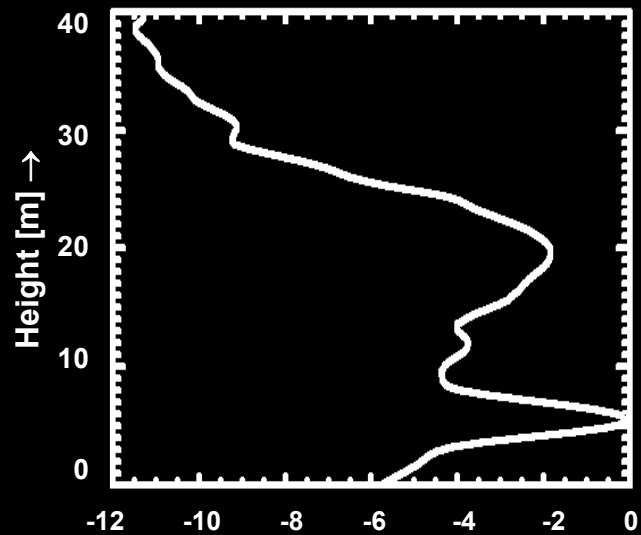
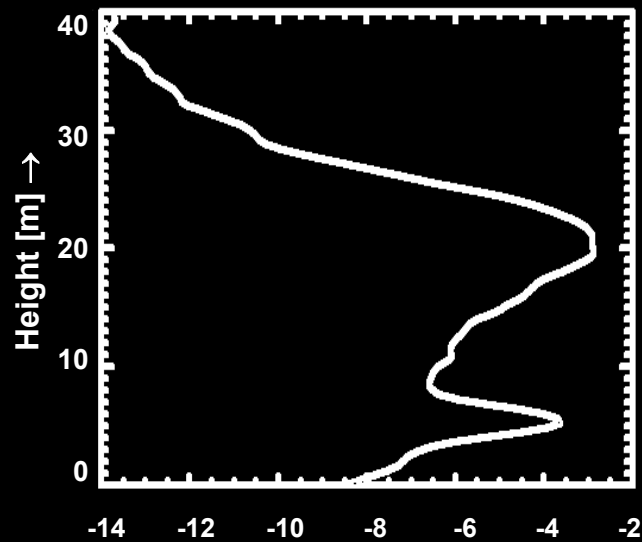
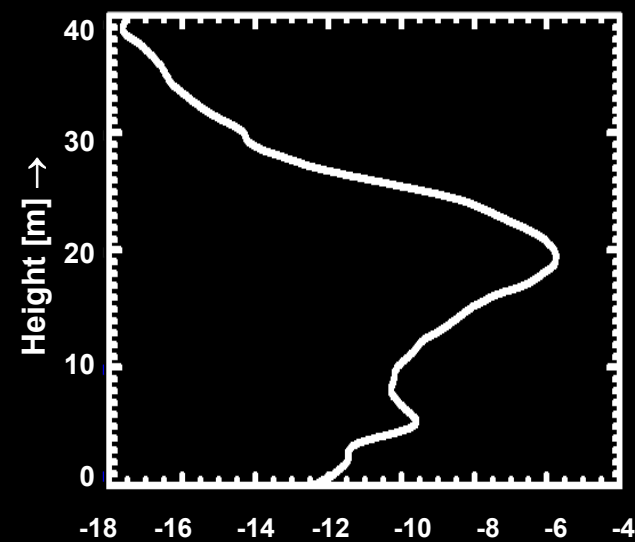
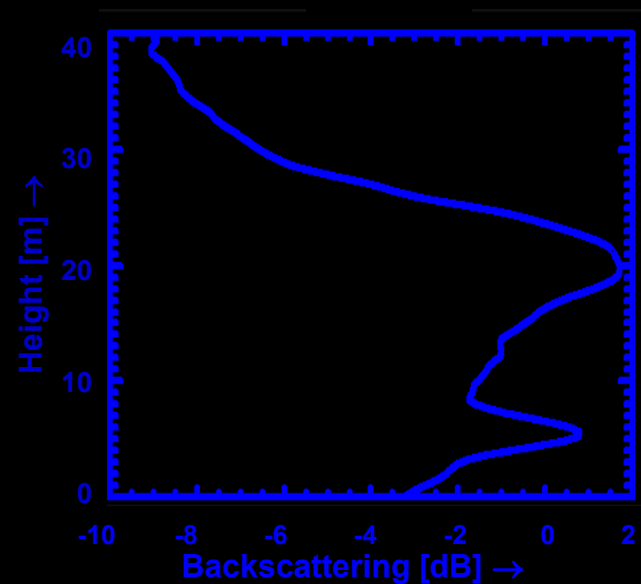
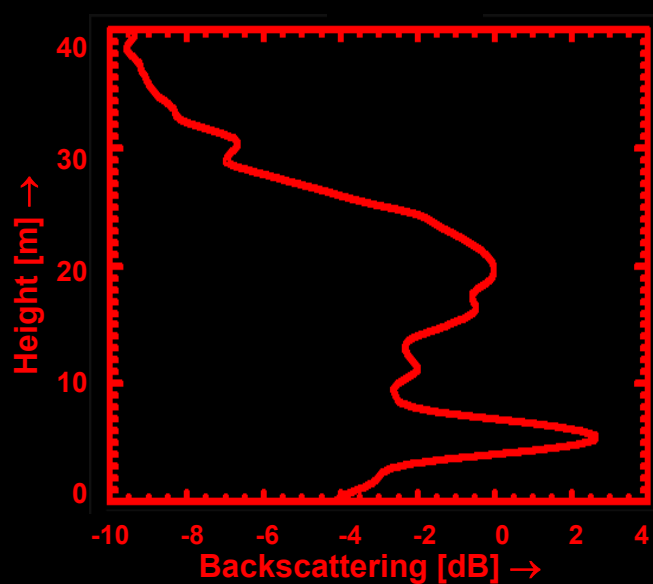
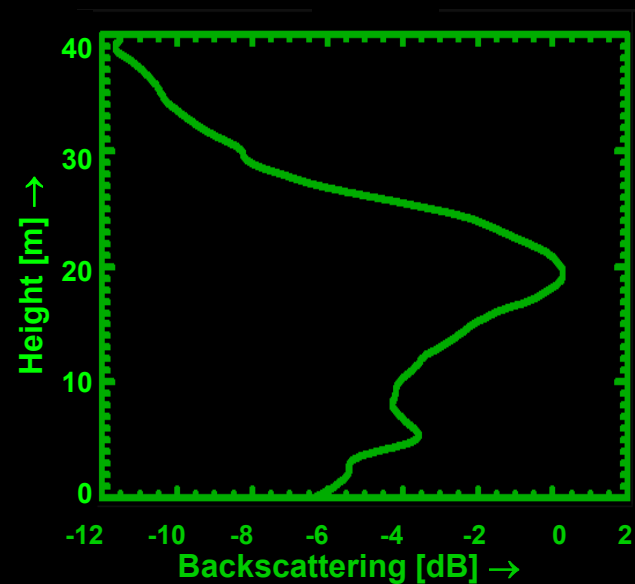
Upper image: Polarimetric color composite (L-band) of a tomographic slice in the height/azimuth-direction

■ HH+VV, ■ HH-VV, ■ 2\*HV

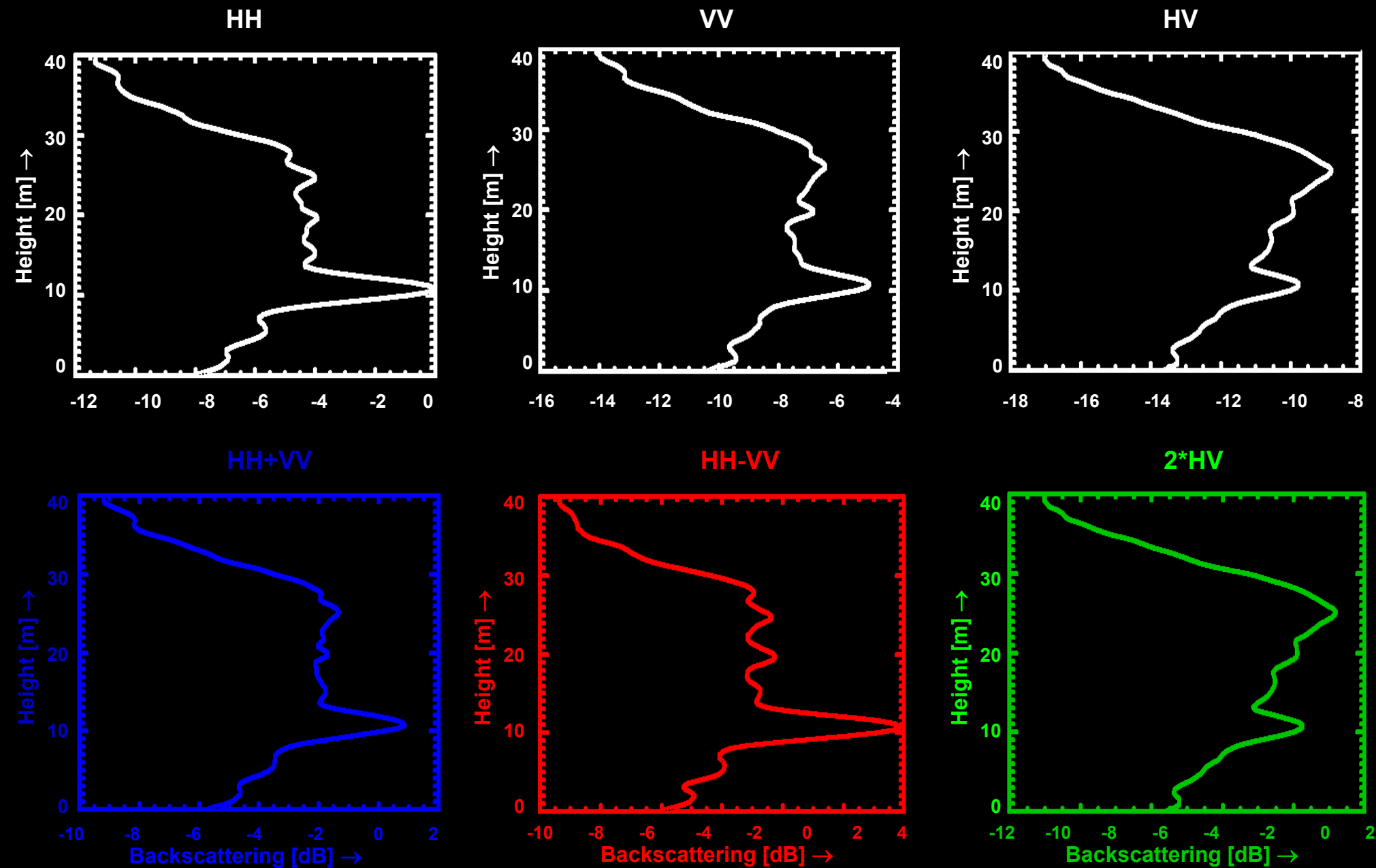
Lower image: Schematic view of the imaged area



**Bare Surface Backscattering Profiles (12-20 m height)**

**HH****VV****HV****HH+VV****HH-VV****2\*HV**

**Spruce Forest Backscattering Profiles (15-20 m height)**



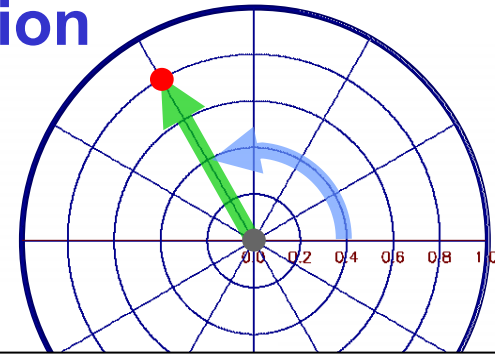
**Mixed Forest Backscattering Profiles (12-20 m height)**

# Geometrical Representation: The Coherence Region

Interferometric Coherence:  $\tilde{\gamma}(\vec{w}_i, \vec{w}_i) = |\tilde{\gamma}(\vec{w}_i, \vec{w}_i)| \cdot \exp(i \text{Arg}\{ \tilde{\gamma}(\vec{w}_i, \vec{w}_i) \})$

Radius

Angle



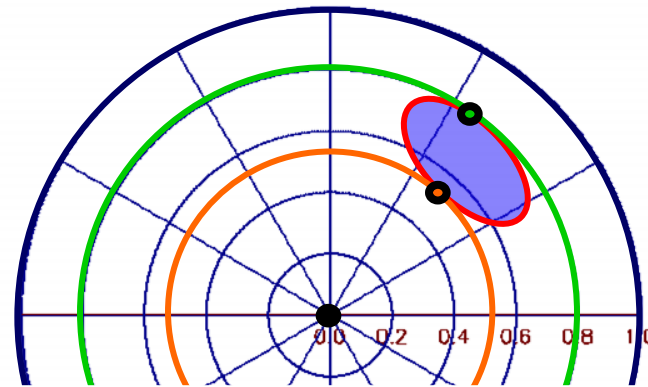
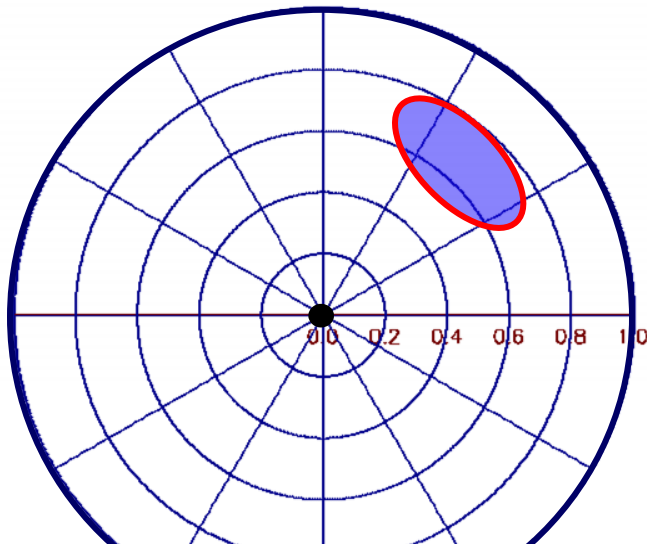
► can be represented by a single point on the unit circle (UC)

**Coherence Region:**

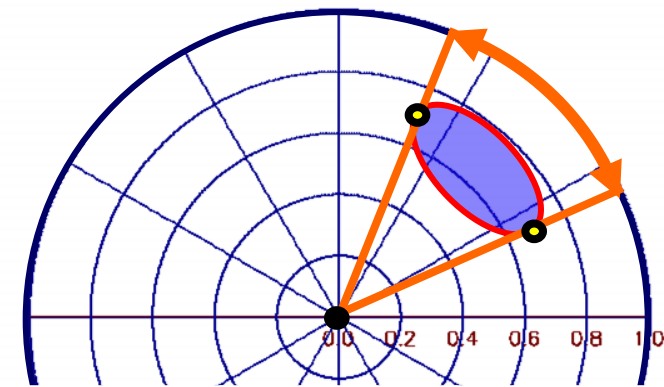
$$\tilde{\gamma}(\vec{w}_i, \vec{w}_i) \quad \forall \quad \vec{w}_i = \begin{bmatrix} \cos \alpha \exp(i\phi_1) \\ \sin \alpha \cos \beta \exp(i\phi_2) \\ \sin \alpha \sin \beta \exp(i\phi_3) \end{bmatrix} \quad \text{with} \quad 0 \leq \alpha \leq \frac{\pi}{2} \quad \text{and} \quad -\pi \leq \beta \leq \pi$$

The set of interferometric coherences obtained for all the possible polarizations  $\vec{w}_i$  plotted on the unit circle (UC) defines the so-called **coherence region**.

Its shape & size depend on the structure of the underlying scatterer and on acquisition parameters.

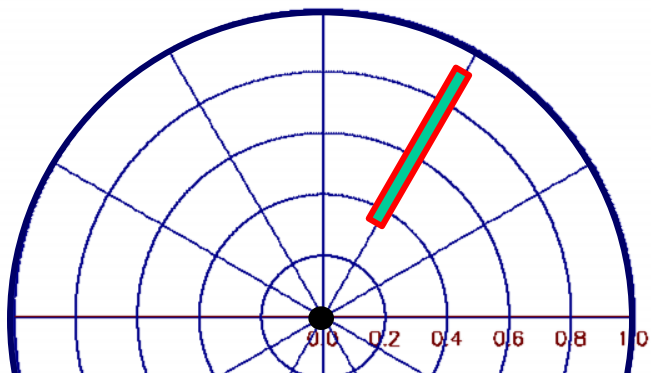


Max./ Min. Interferometric Coherence  
as function of  $\vec{w}_i$



Max. Phase Difference between  
interferograms formed with  $\vec{w}_i$  and  $\vec{w}_j$

# Coherence Region Interpretation

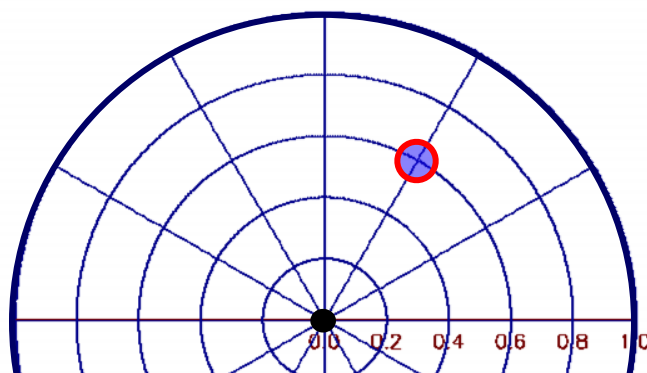
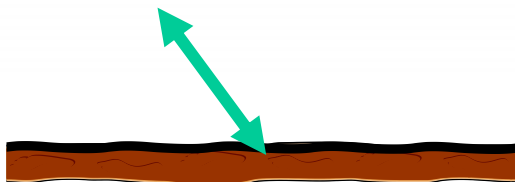


## Radial Shaped CR

i.e. InSAR coherence amplitude changes with polarisation but not the location of the phase center.

## Surface Scattering

$$\tilde{Y}(\vec{W}) = Y_{\text{SNR}}(\vec{W}) \tilde{Y}_{\text{Vol}}^{\tilde{V}_{\text{Vol}}:=1} = Y_{\text{SNR}}(\vec{W})$$

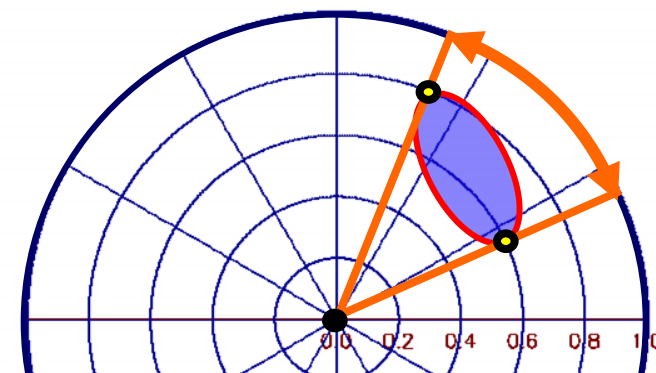
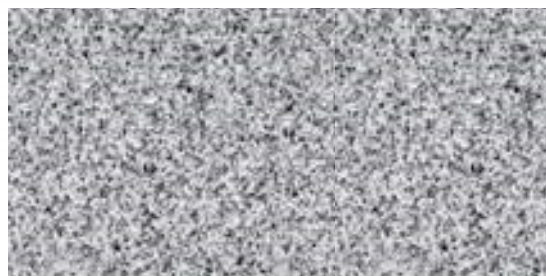


## Point Like Coherence Region

i.e. InSAR Coherence and Phase are independent of polarisation.

Pol-InSAR does not provide any additional information compared to InSAR !!!

## (Random) Volume scattering

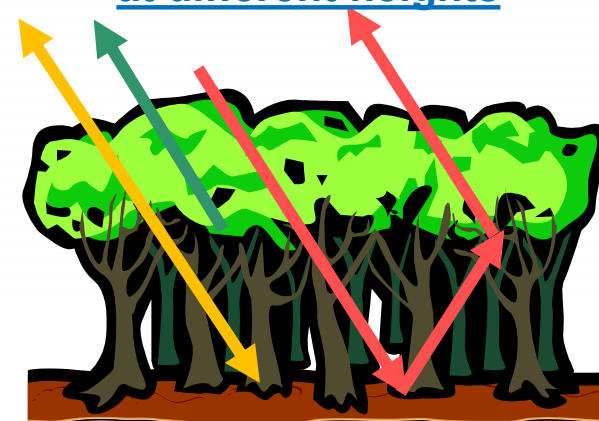


## Elliptical Shaped CR

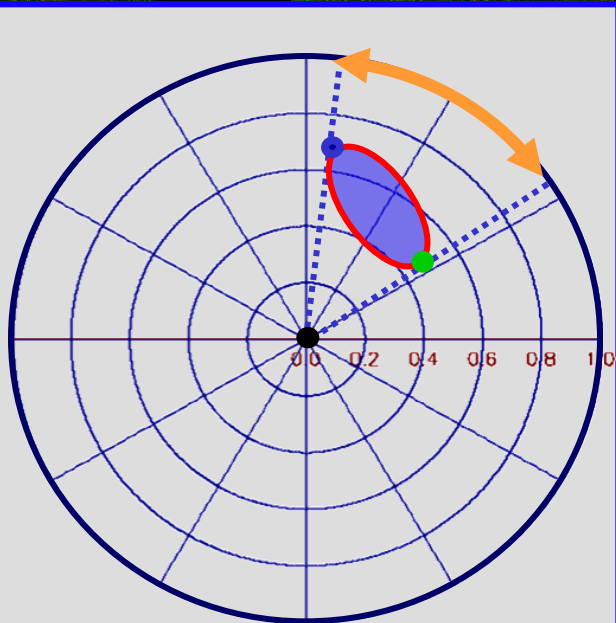
i.e. InSAR coherence magnitude and phase center location changes with polarisation.

## (Depolarising) Scaterrers

### at different heights



# First Bistatic Dual Pol-InSAR Data Takes



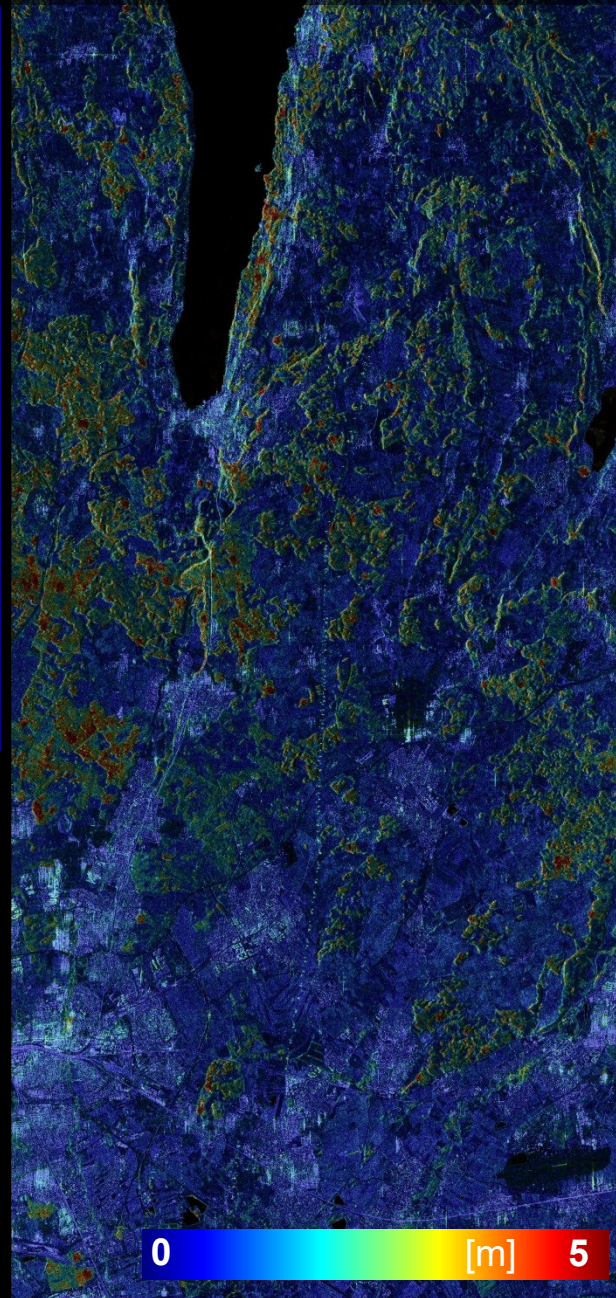
Max. Phase Difference  
between Polarizations

Dual-Pol HH-VV Stripmap

Test Site Location: OP

InSAR Mode: Bistatic

Vertical Wavenumber: 0.1m



0 [m] 5



# Structure Parameters & Applications

## Forest

- Forest Height
- Forest (Vertical) Structure
- Forest Biomass
- Underlying Topography



- Forest Ecology
- Forest Management
- Ecosystem Modeling
- Climate Change

## Agriculture

- Underlying Soil Moisture
- Moisture of Vegetation Layer
- Height of Vegetation Layer
- Soil Roughness



- Farming Management
- Ecosystem Modeling
- Water Cycle / CC
- Desertification

## Snow & Ice

- Ice Layer Structure
- Penetration Depth (Ice)
- Snow Layer Thickness
- Snow Water Equivalent



- Ecosystem Change
- Water Cycle
- Water Management



# Pol-InSAR In Orbit

**ALOS-2 (4)**



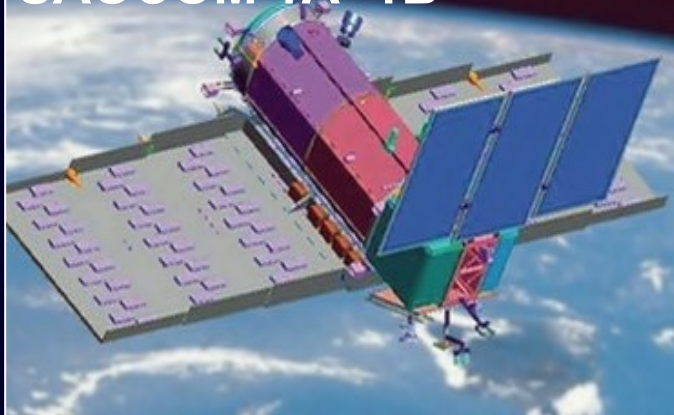
**RadarSAT 2**



**Sentinel 1a+1b (1c + 1d)**



**SAOCOM 1A+1B**



**TanDEM-X**

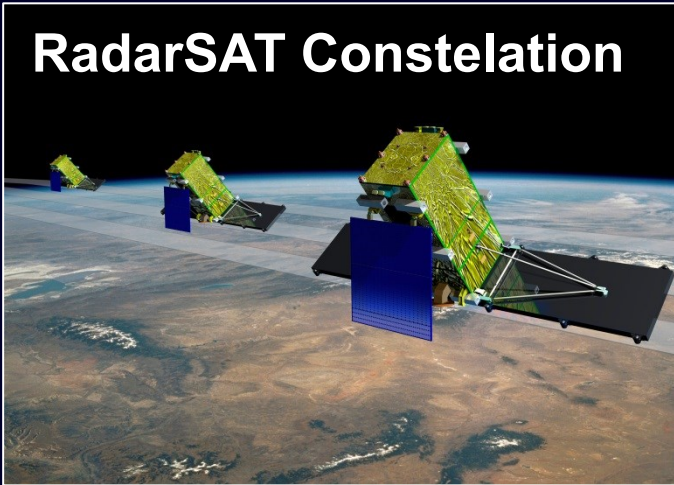


**RISAT-1**

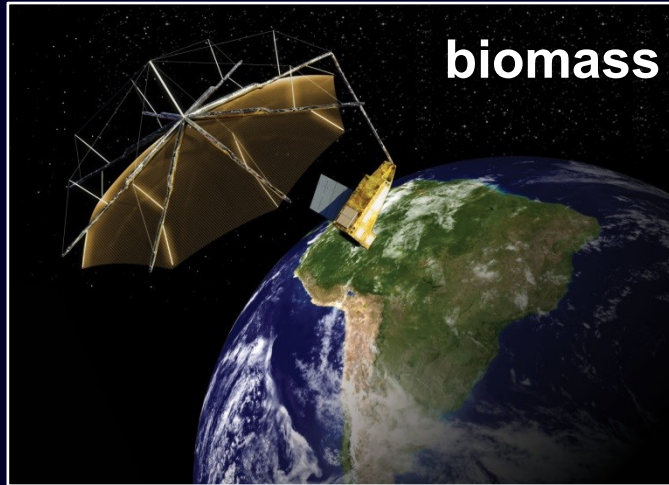


# Pol-InSAR In Orbit

## RadarSAT Constellation



## biomass



## NISAR



## Missions with (pasive or

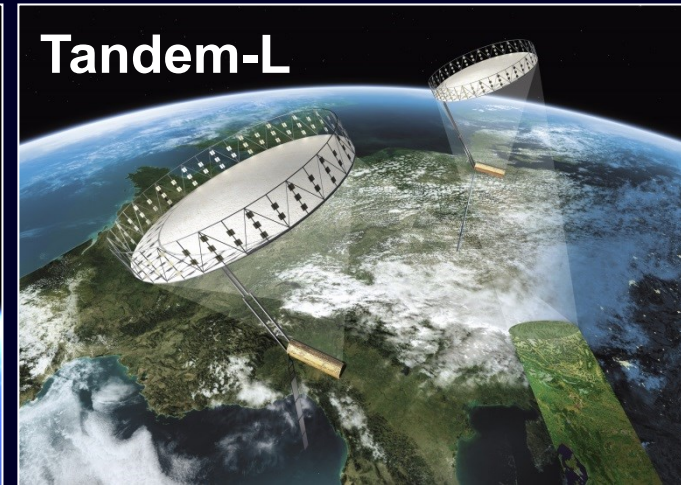


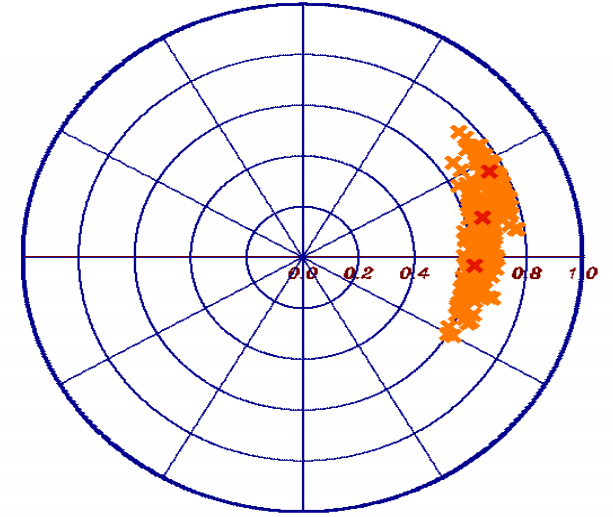
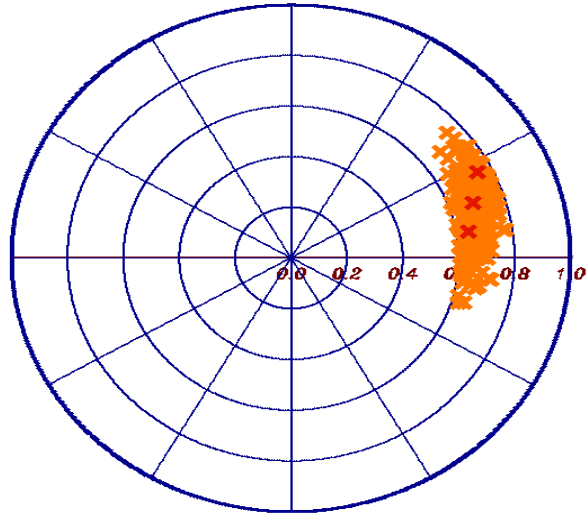
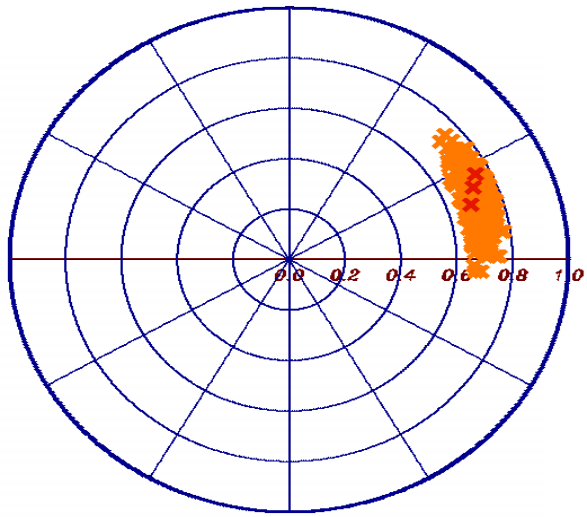
## active) Pol. Companions: Harmony

## Rose-L (+)

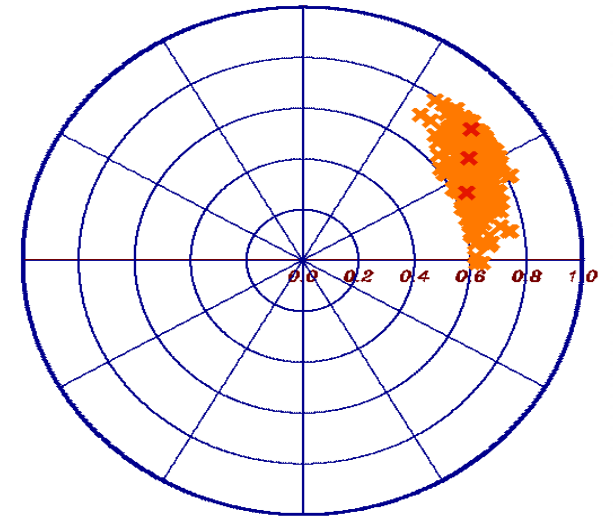
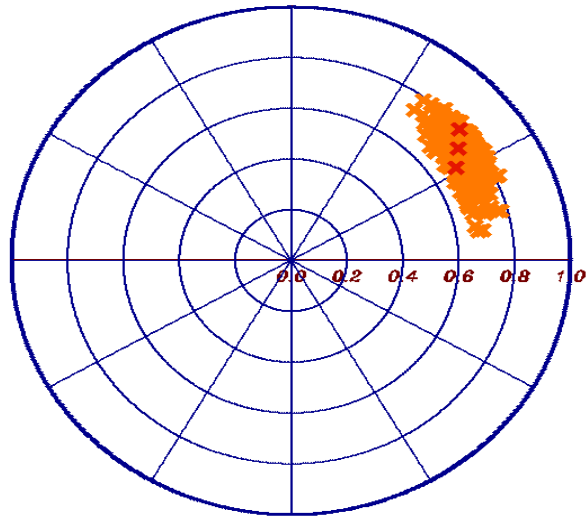
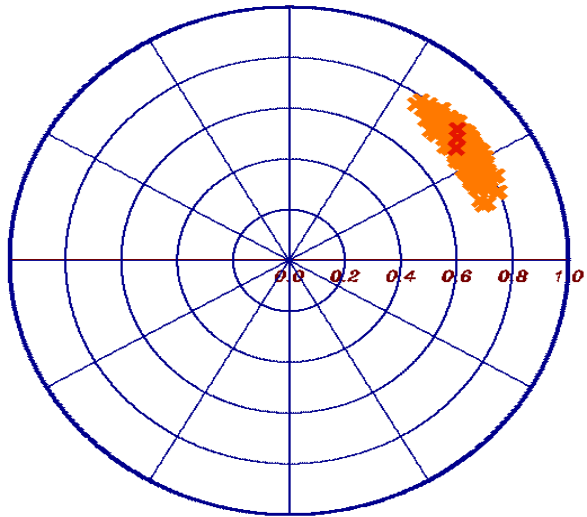


## Tandem-L





## Po-InSAR: Modelling and Inversion



# Forest Height inversion from InSAR Data



$$\tilde{Y}(s_1, s_2) = \frac{\langle s_1 s_2^* \rangle}{\sqrt{\langle s_1 s_1^* \rangle \langle s_2 s_2^* \rangle}}$$

Interferometric Coherence (complex)

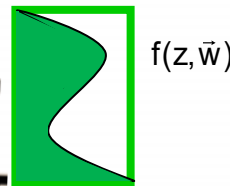
$$\tilde{Y}(s_1, s_2) = Y_{\text{Sys}} Y_{\text{Tmp}} \tilde{Y}_{\text{Vol}}$$

$\tilde{Y}_{\text{Vol}}$  ... volume decorrelation     $\tilde{Y}_{\text{Tmp}}$  ... temporal decorrelation

$$\tilde{Y}_{\text{Vol}}(\bar{w}, \kappa_z) = e^{i\kappa_z z_0} \frac{\int_0^{h_y} f(z, \bar{w}) e^{i\kappa_z z} dz}{\int_0^{h_y} f(z, \bar{w}) dz}$$

$f(z, \bar{w})$  ... vertical reflectivity function     $\kappa_z$  ... vertical wavenumber

Interferometric (volume) decorrelation is sensitive to the „visible“ (forest) **height** and to the **vertical reflectivity function**  $f(z, \bar{w})$  within the interferometric resolution cell.



## Forest Height inversion challenges:

- The parameterisation / description of  $f(z, \bar{w})$
- The presence of  $\tilde{Y}_{\text{Tmp}}$

# 2 Layer Inversion Model: (Random) Volume over Ground (RVoG)



Volume Layer Ground Layer

$$f(z, \vec{w}) = m_V f_V(z) + m_G(\vec{w}) \delta(z - z_0)$$

Volume Layer Coherence

$$\tilde{Y}_V = \frac{I}{I_0} \left\{ \begin{array}{l} I = \int_0^{h_V} \exp(ik_z z') f_V(z') dz' \\ I_0 = \int_0^{h_V} f_V(z') dz' \end{array} \right.$$

$$\tilde{Y}_{Vol}(\vec{w}, \kappa_z) = \exp(i\phi_0) \frac{\tilde{Y}_V(\kappa_z) + m(\vec{w})}{1 + m(\vec{w})}$$

$$m(\vec{w}) = \frac{m_G(\vec{w})}{m_V(\vec{w}) I_0} \quad \kappa_z = \frac{\kappa \Delta \theta}{\sin(\theta_0)}$$

$f_V(z)$  ... volume reflectivity function  
 $\phi_0 = k_z z_0$  ... underlying topography

## Single Baseline Observations

single- / dual- / quad-pol

$$\tilde{Y}_{Vol}(\vec{w}_1, \kappa_z) \quad \tilde{Y}_{Vol}(\vec{w}_2, \kappa_z) \quad \tilde{Y}_{Vol}(\vec{w}_3, \kappa_z)$$

1, 2, or 3 complex coherences

Total Coherence

$$\tilde{Y}(\vec{w}, \kappa_z) = Y_{Temp}(\kappa_z) \tilde{Y}_{Vol}(\vec{w}, \kappa_z)$$

## For a Single Baseline

3+N unknown parameters

Volume Height  $h_V$

Topography  $\phi_0$

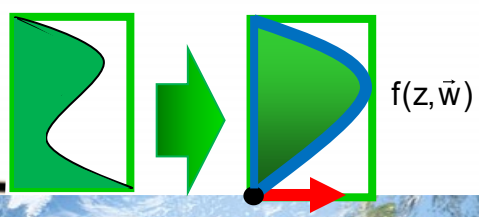
G/V Ratio  $m(\vec{w}) = f(\text{pol})$

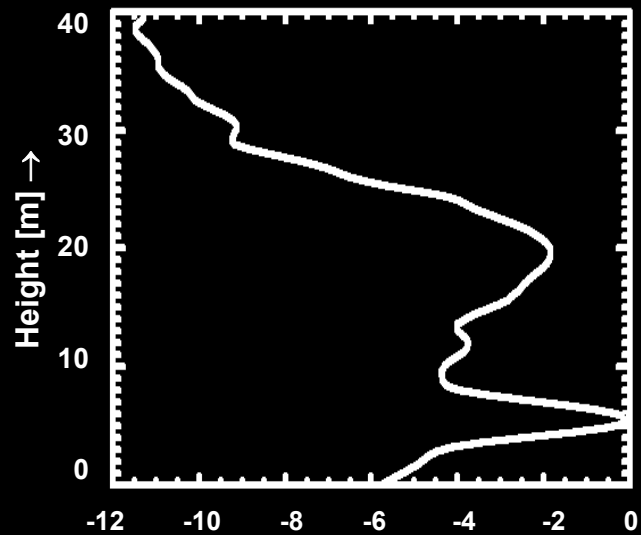
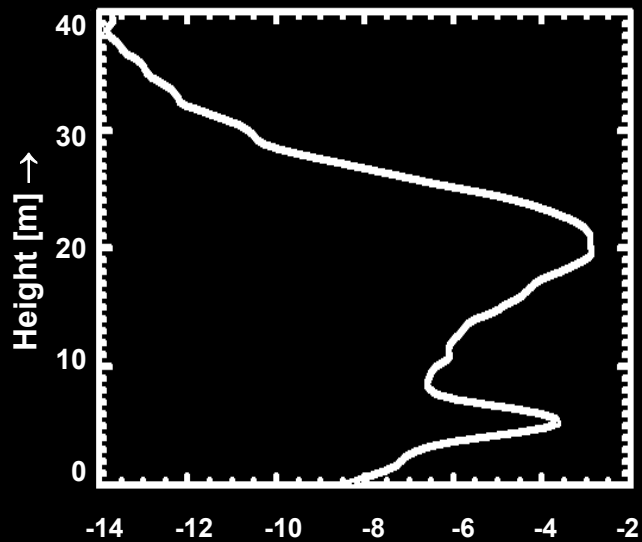
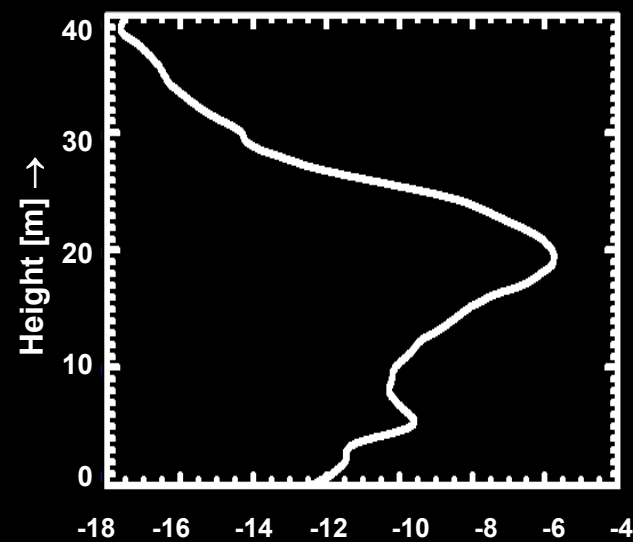
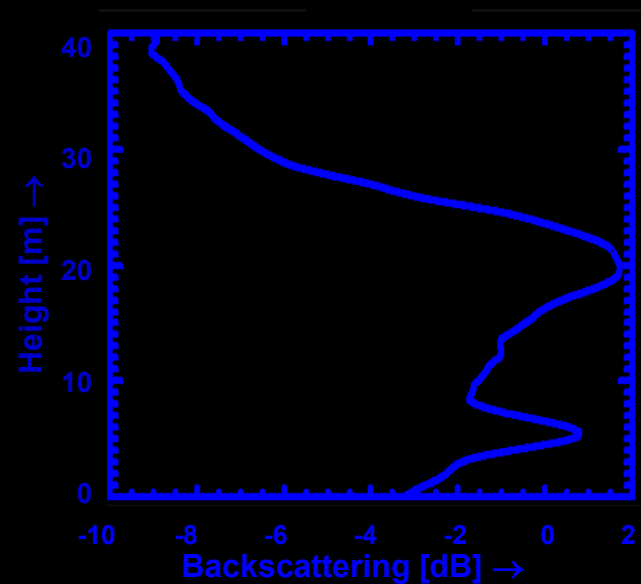
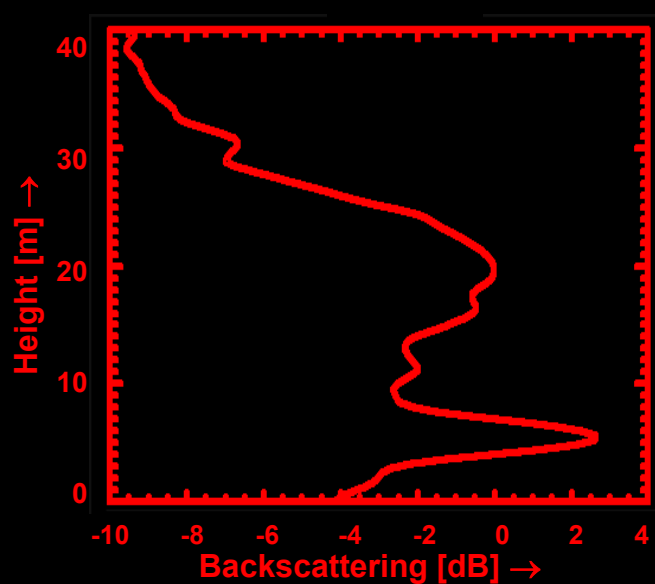
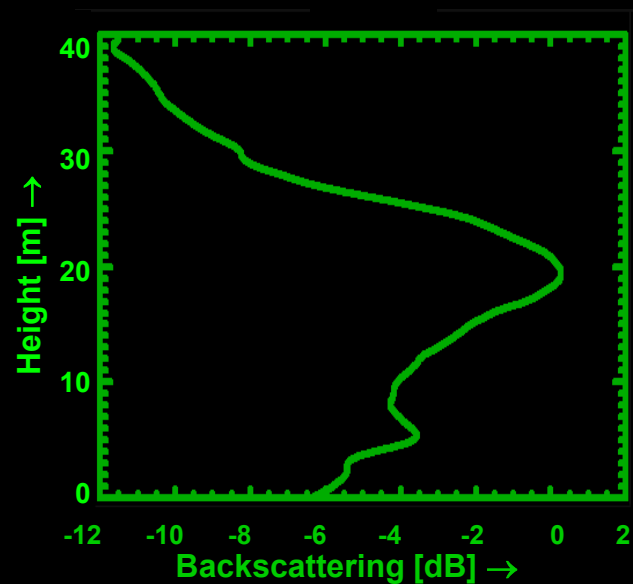
$f_V(z)$  ... N parameters

plus one more

Temporal Deco  $Y_{Temp}$

for repeat-pass implementations



**HH****VV****HV****HH+VV****HH-VV****2\*HV****Spruce Forest Backscattering Profiles (15-20 m height)**

# 2 Layer Inversion Model: (Random) Volume over Ground (RVoG)



Volume Layer    Ground Layer

$$f(z, \vec{w}) = m_V f_V(z) + m_G(\vec{w}) \delta(z - z_0)$$

$\tilde{Y}_{Vol}(\vec{w}, \kappa_z) = \exp(i\phi_0) \frac{\tilde{Y}_V(\kappa_z) + m(\vec{w})}{1 + m(\vec{w})}$

Volume Layer Coherence

$$\tilde{Y}_V = \frac{I}{I_0} \left\{ \begin{array}{l} I = \int_0^{h_V} \exp(i\kappa_z z') f_V(z') dz' \\ I_0 = \int_0^{h_V} f_V(z') dz' \end{array} \right.$$

$$m(\vec{w}) = \frac{m_G(\vec{w})}{m_V(\vec{w}) I_0} \quad \kappa_z = \frac{\kappa \Delta \theta}{\sin(\theta_0)}$$

$f_V(z)$  ... volume reflectivity function  
 $\phi_0 = \kappa_z z_0$  ... underlying topography

**Single Baseline Observations**  
 single- / dual- / quad-pol  
 $\tilde{Y}_{Vol}(\vec{w}_1, \kappa_z) \quad \tilde{Y}_{Vol}(\vec{w}_2, \kappa_z) \quad \tilde{Y}_{Vol}(\vec{w}_3, \kappa_z)$   
 1, 2, or 3 complex coherences

**Total Coherence**

$$\tilde{Y}(\vec{w}, \kappa_z) = Y_{Temp}(\kappa_z) \tilde{Y}_{Vol}(\vec{w}, \kappa_z)$$

**For a Single Baseline**  
 3+N unknown parameters

Volume Height	$h_V$
Topography	$\phi_0$
G/V Ratio	$m(\vec{w}) = f(\text{pol})$
$f_V(z)$	... N parameters

plus one more

Temporal Deco     $Y_{Temp}$   
 for repeat-pass implementations



# 2 Layer Inversion Model with exponential volume reflectivity



Volume Layer    Ground Layer

$$f(z, \bar{w}) = m_V f_V(z) + m_G(\bar{w}) \delta(z - z_0)$$

$\tilde{Y}_{Vol}(\bar{w}, \kappa_z) = \exp(i\phi_0) \frac{\tilde{Y}_V(\kappa_z) + m(\bar{w})}{1 + m(\bar{w})}$

Volume Layer Coherence  $\tilde{Y}_V = \frac{I}{I_0}$

$$I = \int_0^{h_V} \exp(ik_z z') e^{\left(\frac{2\sigma z'}{\cos\theta_0}\right)} dz' \quad m(\bar{w}) = \frac{m_G(\bar{w})}{m_V(\bar{w}) I_0} \quad \kappa_z = \frac{\kappa \Delta \theta}{\sin(\theta_0)}$$

$$I_0 = \int_0^{h_V} e^{\left(\frac{2\sigma z'}{\cos\theta_0}\right)} dz' \quad f_V(z) = \exp\left(\frac{2\sigma z}{\cos\theta_0}\right) \quad \text{exponential volume reflectivity}$$

## Single Baseline Observations

single- / dual- / quad-pol

$$\tilde{Y}_{Vol}(\bar{w}_1, \kappa_z) \quad \tilde{Y}_{Vol}(\bar{w}_2, \kappa_z) \quad \tilde{Y}_{Vol}(\bar{w}_3, \kappa_z)$$

1, 2, or 3 complex coherences

Total Coherence

$$\tilde{Y}(\bar{w}, \kappa_z) = Y_{Temp}(\kappa_z) \tilde{Y}_{Vol}(\bar{w}, \kappa_z)$$

## For a Single Baseline

4 unknown parameters

Volume Height  $h_V$

Topography  $\phi_0$

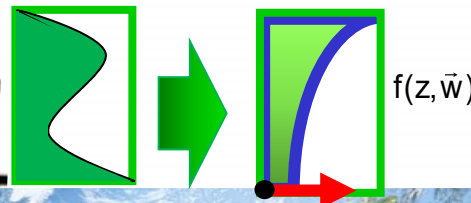
G/V Ratio  $m(\bar{w}) = f(\text{pol})$

$f_V(z)$  ... 1 parameter ( $\sigma$ )

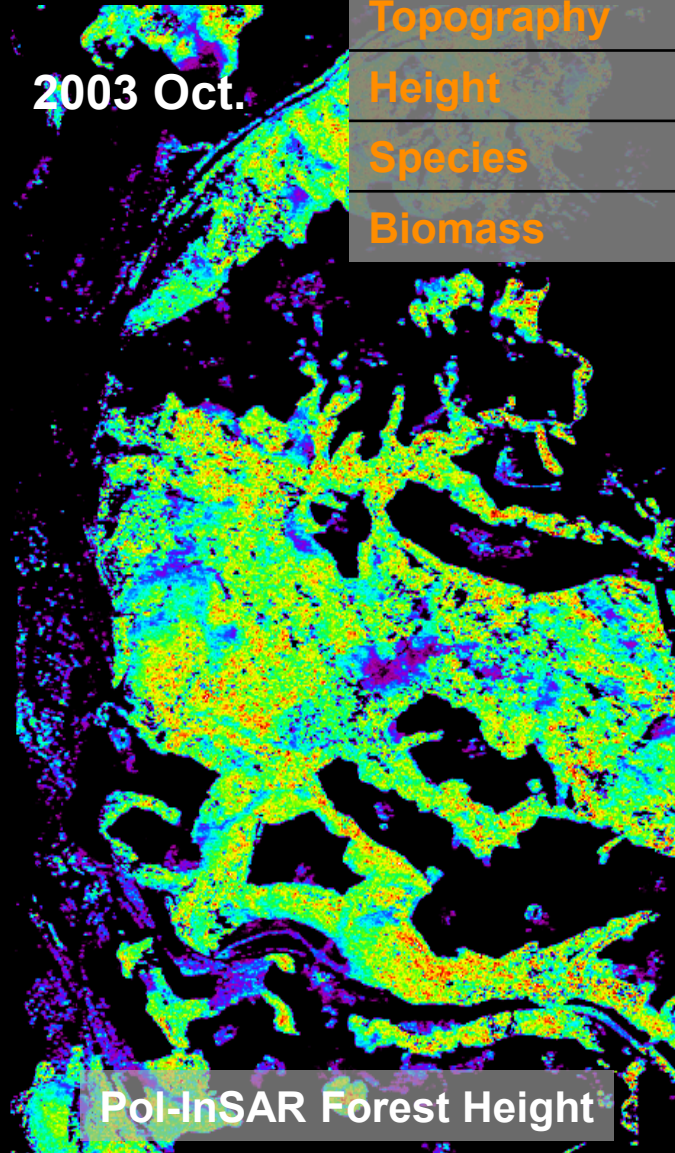
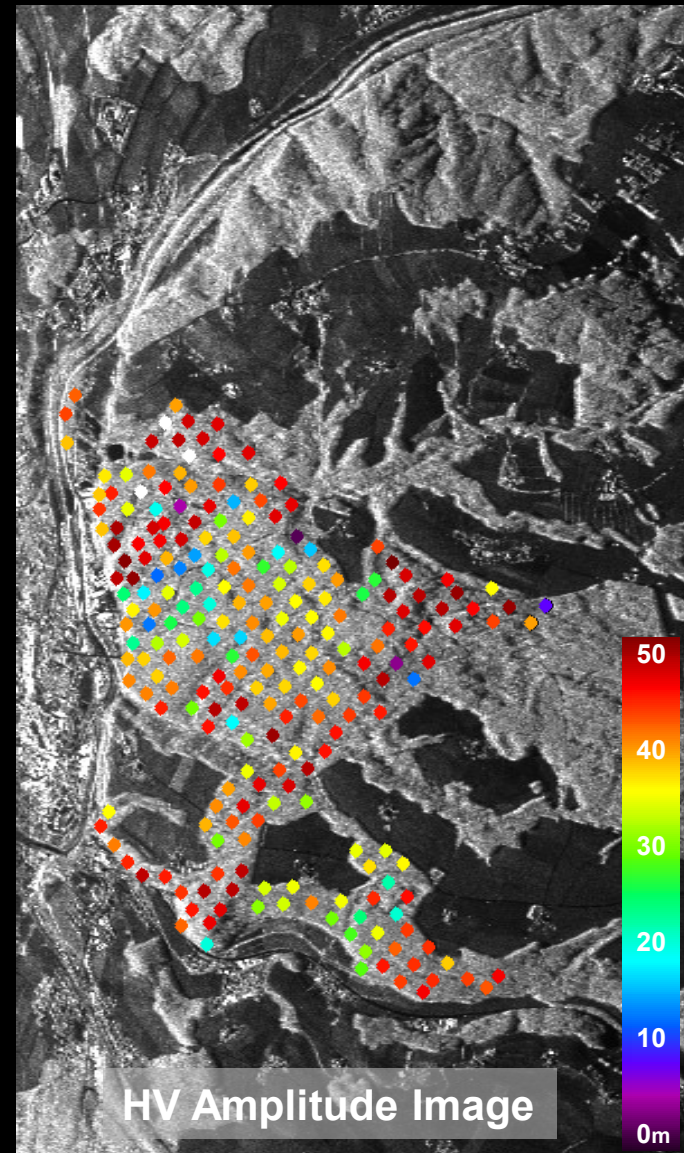
plus one more

Temporal Deco  $Y_{Temp}$

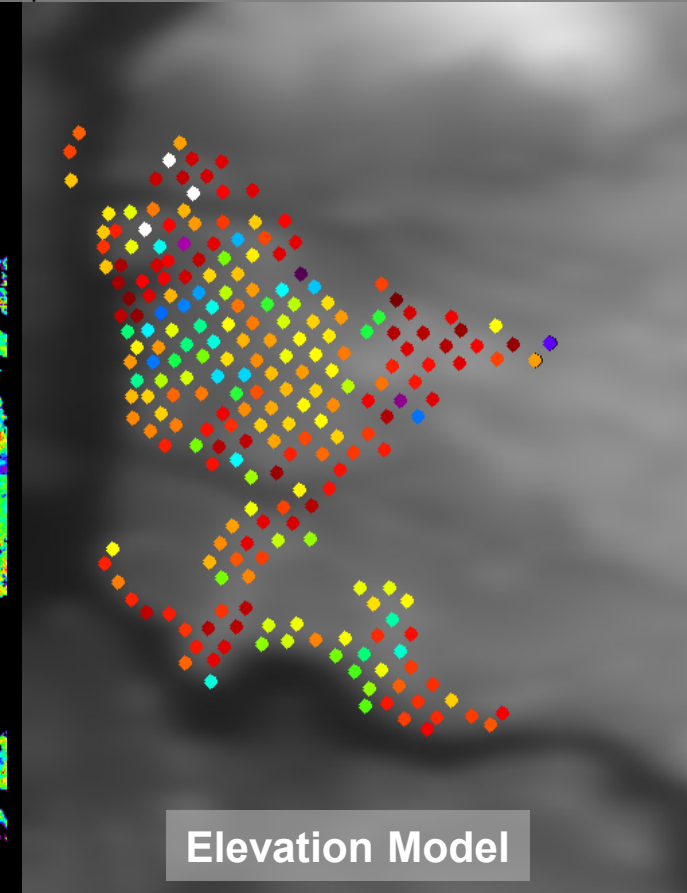
for repeat-pass implementations



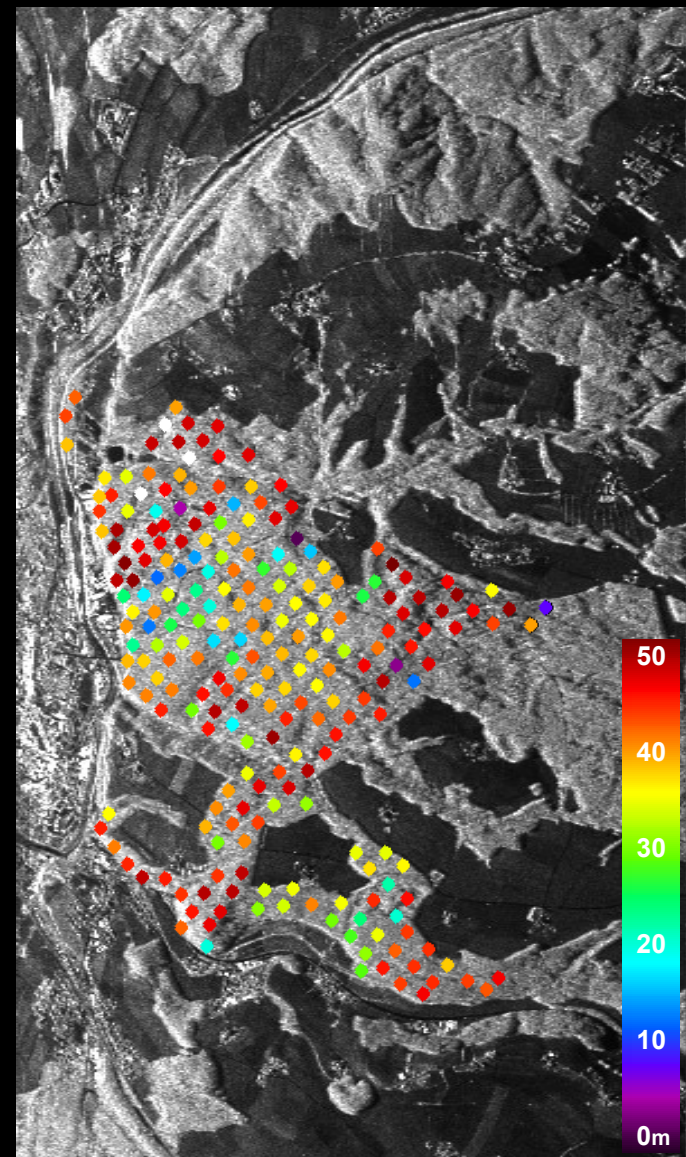
# Traunstein Test Site



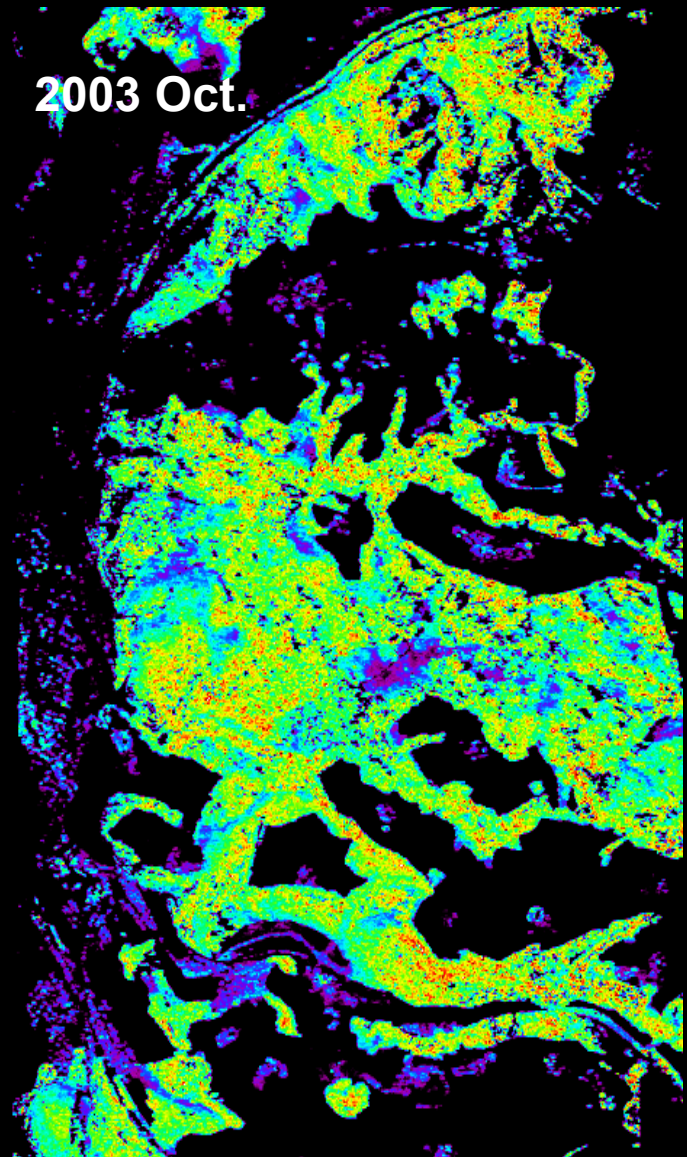
Forest type	Temperate
Topography	Moderate slopes
Height	25 ~ 35m
Species	N. Spruce, E. Beech, White Fir
Biomass	40 ~ 450 t/ha



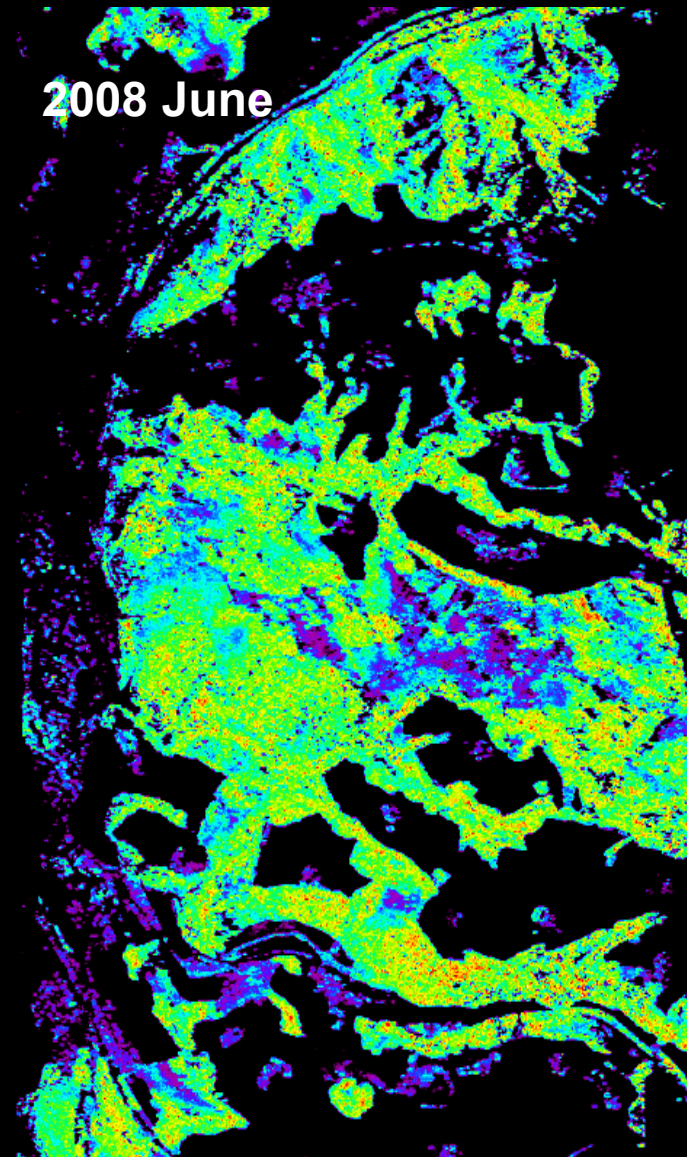
# Traunstein Test Site

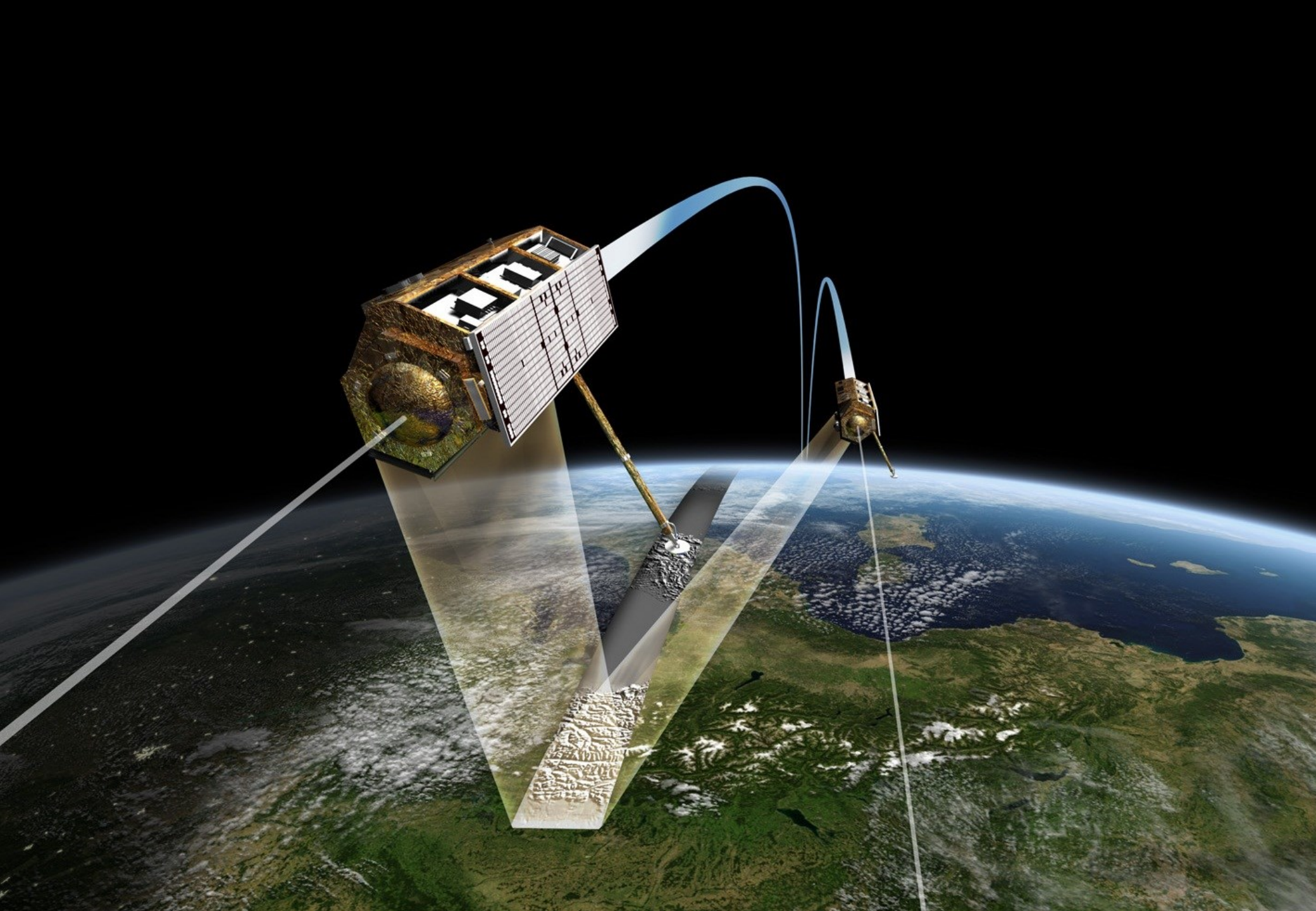


2003 Oct.

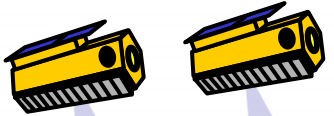


2008 June





# TanDEM-X: 2 Layer model with exp. volume reflectivity



Volume Layer    Ground Layer

$$f(z, \vec{w}) = m_V f_V(z) + m_G(\vec{w}) \delta(z - z_0)$$

➔

$$\tilde{Y}_{Vol}(\vec{w}, \kappa_z) = \exp(i\phi_0) \frac{\tilde{Y}_V(\kappa_z) + m(\vec{w})}{1 + m(\vec{w})}$$

Volume Layer  
Coherence

$$\tilde{Y}_V = \frac{I}{I_0}$$

{

$$I = \int_0^{h_V} \exp(ik_z z') e^{\left(\frac{2 \sigma z'}{\cos \theta_0}\right)} dz'$$

$$I_0 = \int_0^{h_V} e^{\left(\frac{2 \sigma z'}{\cos \theta_0}\right)} dz'$$

$$m(\vec{w}) = \frac{m_G(\vec{w})}{m_V(\vec{w}) I_0}$$

$$f_V(z) = \exp\left(\frac{2 \sigma z}{\cos \theta_0}\right)$$

exponential  
volume  
reflectivity

$$\kappa_z = \frac{\kappa \Delta \theta}{\sin(\theta_0)}$$

Single Baseline Observation(s)

single-pol

$$\tilde{Y}_{Vol}(\vec{w}_1, \kappa_z)$$

1 complex coherence

Total  
Coherence

$$\tilde{Y}(\vec{w}, \kappa_z) = \tilde{Y}_{Temp}(\vec{w}, \kappa_z) \tilde{Y}_{Vol}(\vec{w}, \kappa_z)$$

For a Single Baseline

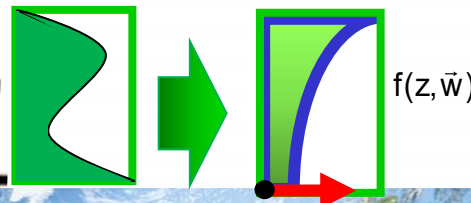
4 unknown parameters

Volume Height             $h_V$

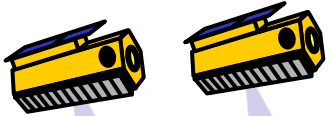
Topography                 $\phi_0$

G/V Ratio     $m(\vec{w}) = f(\text{pol})$

$f_V(z)$  ... 1 parameter ( $\sigma$ )



# TanDEM-X: 2 Layer model with exp. volume reflectivity + no ground



Volume Layer Ground Layer

$$f(z, \vec{w}) = m_V f_V(z) + m_G \delta(z - z_0)$$

$\tilde{Y}_{Vol}(\vec{w}, \kappa_z) = \exp(i\phi_0) \frac{\tilde{Y}_V(\kappa_z) + m(\vec{w})}{1 + m(\vec{w})}$

Volume Layer Coherence

$$\tilde{Y}_V = \frac{I}{I_0} \left\{ \begin{array}{l} I = \int_0^{h_V} \exp(ik_z z') e^{\left(\frac{2\sigma z'}{\cos\theta_0}\right)} dz' \\ I_0 = \int_0^{h_V} e^{\left(\frac{2\sigma z'}{\cos\theta_0}\right)} dz' \end{array} \right.$$

$f_V(z) = \exp\left(\frac{2\sigma z}{\cos\theta_0}\right)$  exponential volume reflectivity

Invertible with single-pol data only if the ground topography  $\phi_0$  is known !!!

Single Baseline Observation(s)  
 single-pol  
 $\tilde{Y}_{Vol}(\vec{w}_1, \kappa_z)$   
 1 complex coherence

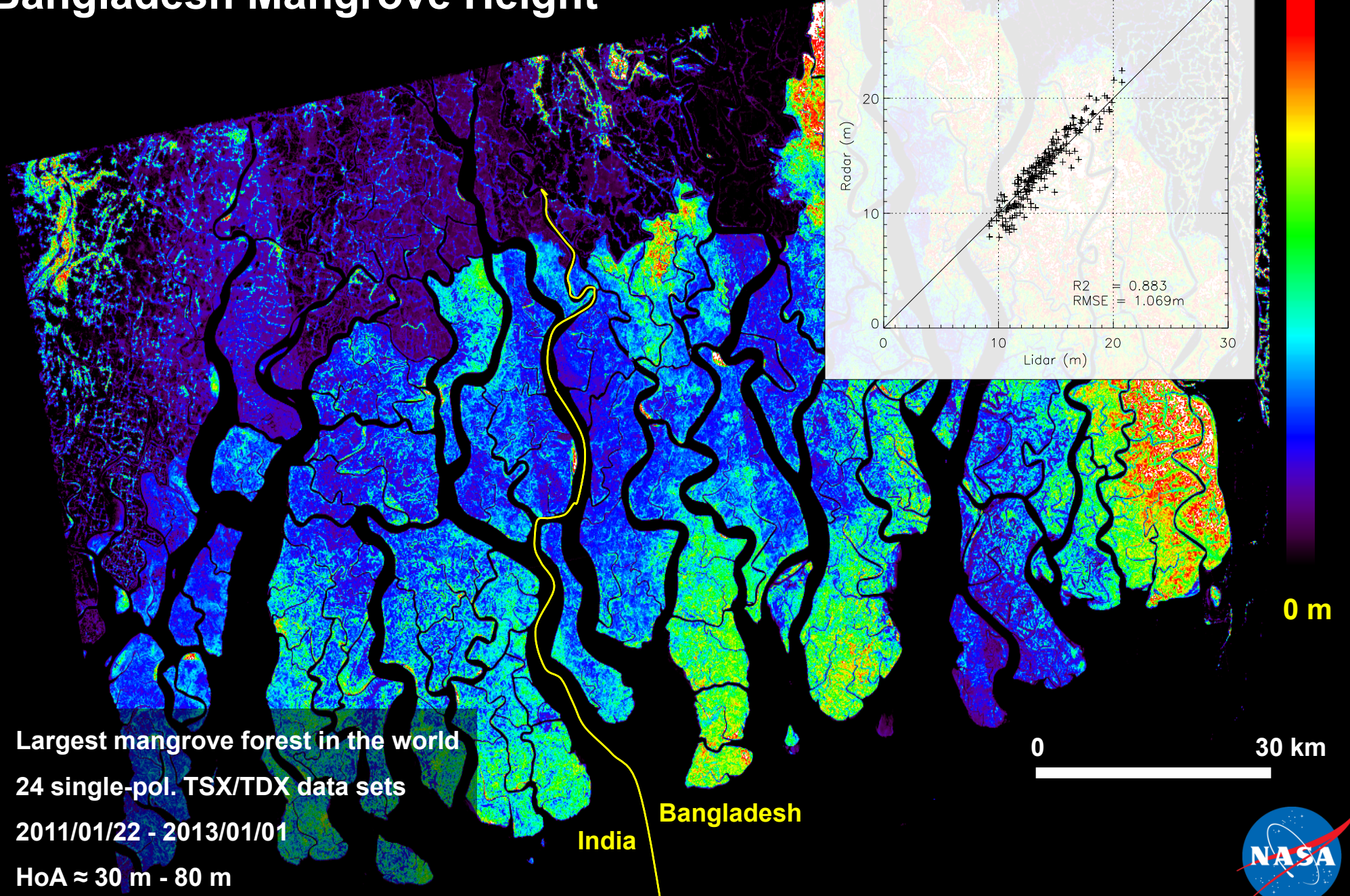
Total Coherence

$$\tilde{Y}(\vec{w}, \kappa_z) = \tilde{Y}_{Temp}(\vec{w}, \kappa_z) \tilde{Y}_{Vol}(\vec{w}, \kappa_z)$$

For a Single Baseline  
 3 unknown parameters  
 Volume Height  $h_V$   
 Topography  $\phi_0$   
~~SNR Ratio  $m(\vec{w})$  (pph)~~  
 $f_V(z)$  ... 1 parameter ( $\sigma$ )



# Bangladesh Mangrove Height



- Largest mangrove forest in the world
- 24 single-pol. TSX/TDX data sets
- 2011/01/22 - 2013/01/01
- HoA  $\approx$  30 m - 80 m

India

Bangladesh

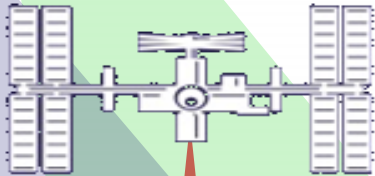
NASA



**TDX Interferometric Coherence**  
(after calibration for system effects)

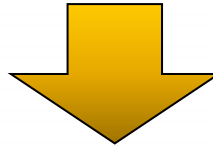
$$\tilde{Y}_{Vol}(k_z) = e^{ik_z z_0} \frac{\int_0^{h_y} f_{TDX}(z) e^{ik_z z} dz}{\int_0^{h_y} f_{TDX}(z) dz}$$

The TanDEM-X forest height inversion problem is underdetermined (**1 complex measurement** for at least **3 real unknowns**) and thus not solvable !



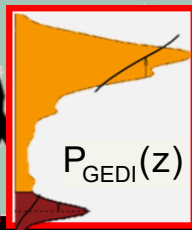
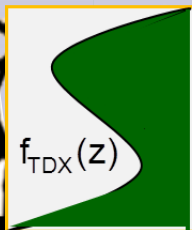
**GEDI waveforms** can be used to approximate the X-band (vertical) reflectivity profiles

$$f_{TDX}(z) \approx P_{GEDI}(z)$$

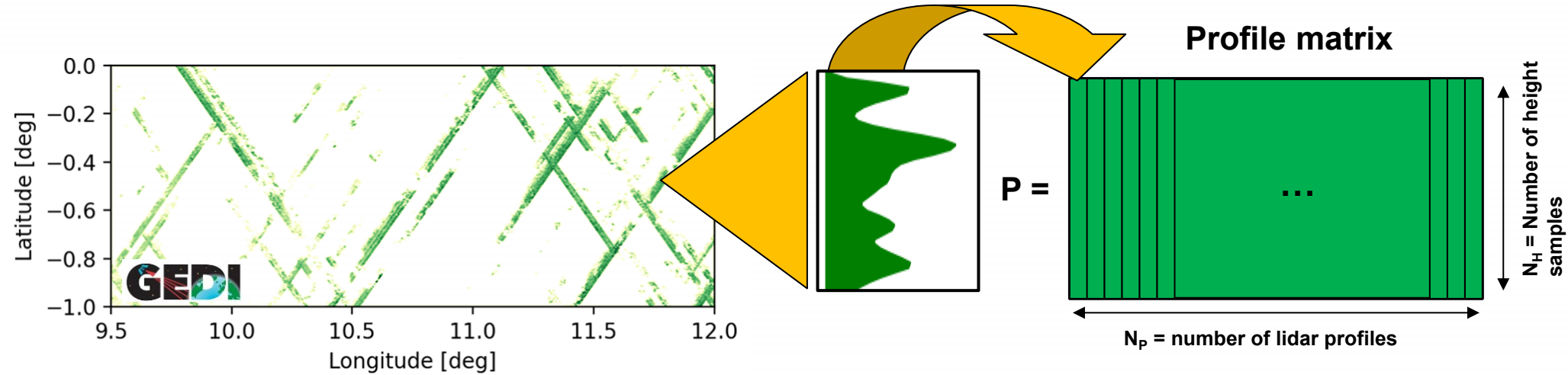


$$\tilde{Y}_{Vol}(k_z) = e^{ik_z z_0} \frac{\int_0^{h_y} P_{GEDI}(z) e^{ik_z z} dz}{\int_0^{h_y} f_{GEDI}(z) dz}$$

The TanDEM-X/GEDI forest height inversion problem is balanced (with **1 complex measurement** for **2 real unknowns**) and becomes solvable !



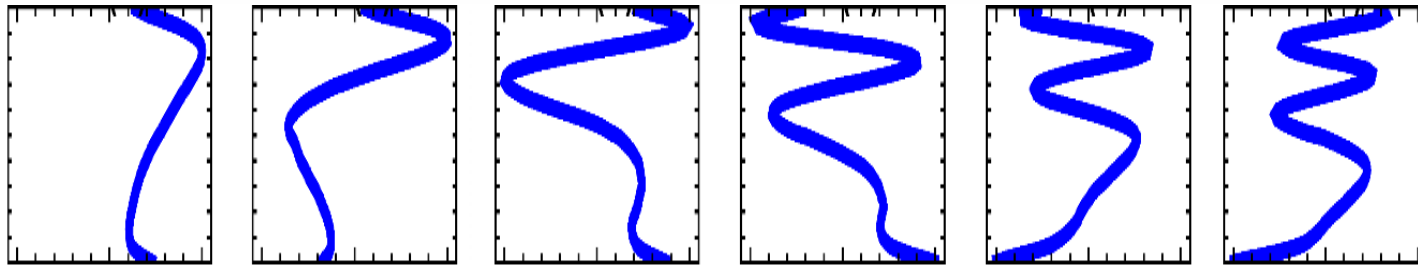
# TanDEM-X GEDI Fusion: Common Reflectivity Profile Estimation



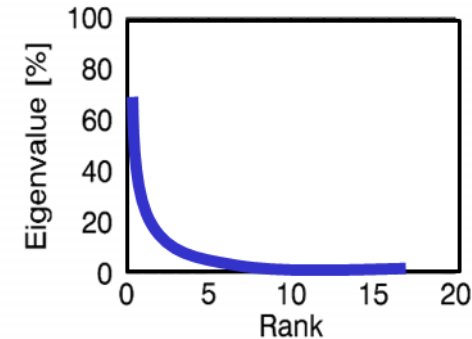
Profile covariance matrix:  $R = P P^T = U \Lambda U^T$  Eigen-decomposition of  $R$



**Eigen-Vectors (e.g. Eigen-Functions)**

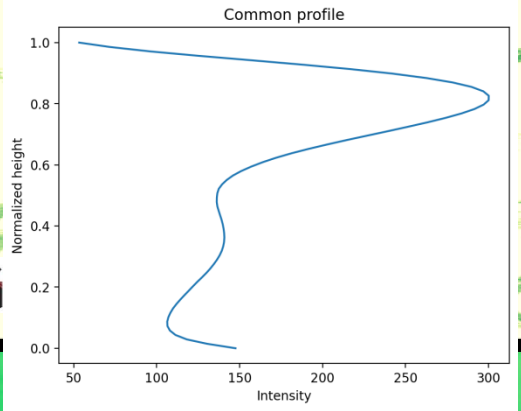
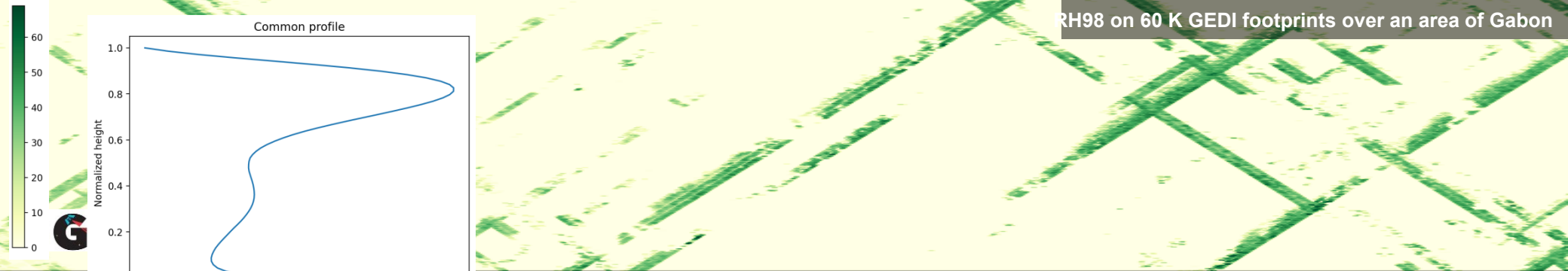


**Eigen-Values**

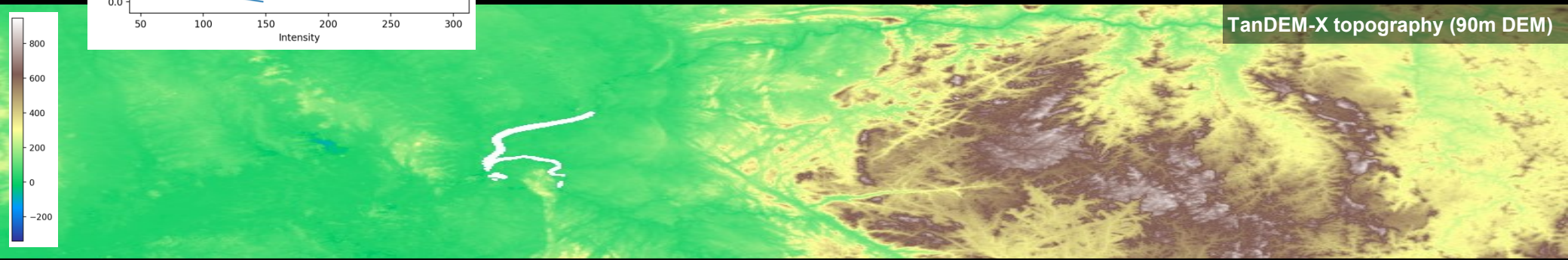


The Eigen-Vectors can be used derive a “mean” vertical profile

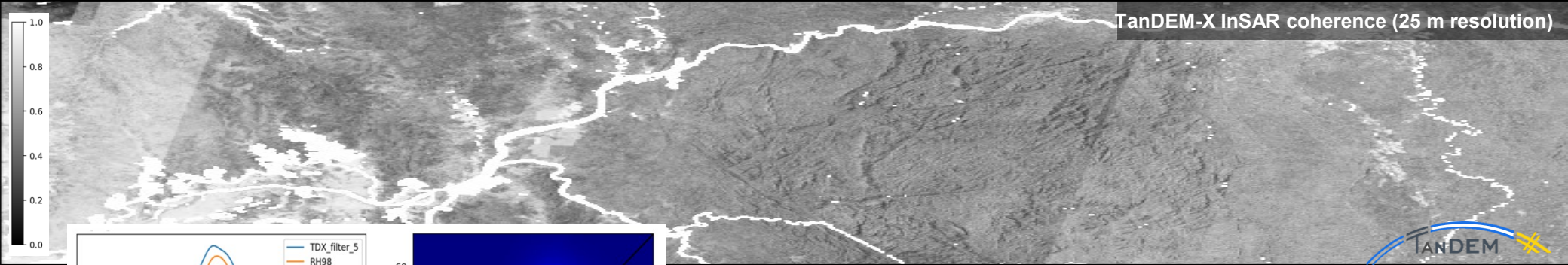
RH98 on 60 K GEDI footprints over an area of Gabon



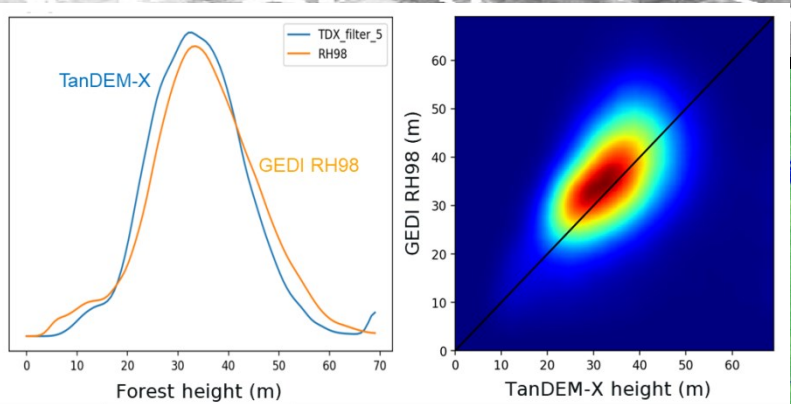
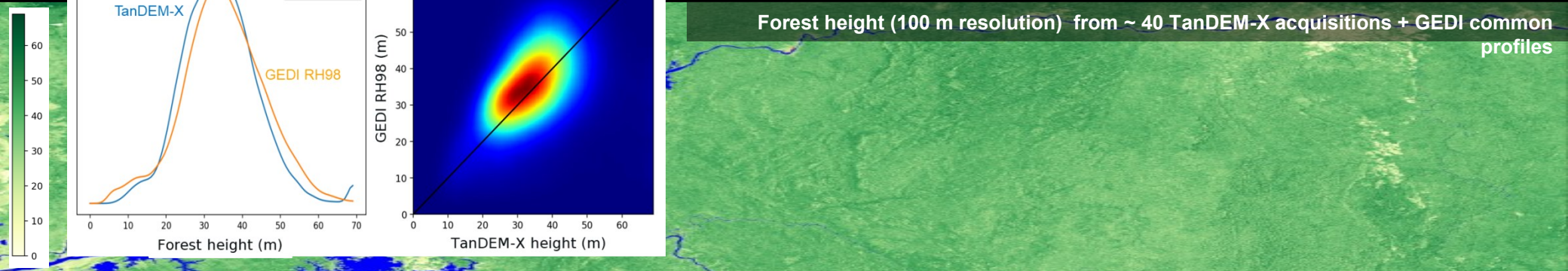
TanDEM-X topography (90m DEM)

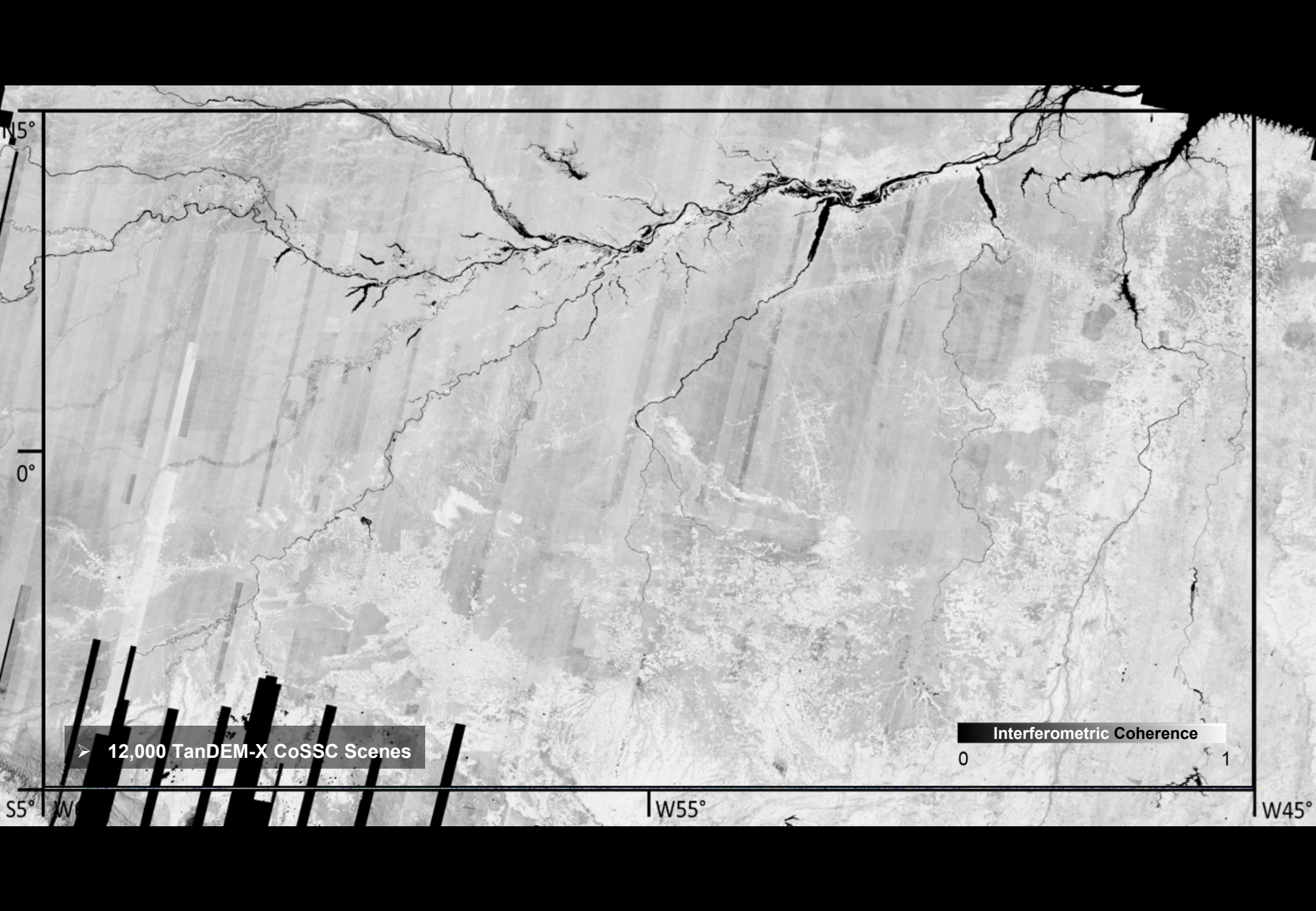


TanDEM-X InSAR coherence (25 m resolution)



Forest height (100 m resolution) from ~ 40 TanDEM-X acquisitions + GEDI common profiles





N5°

0°

S5°

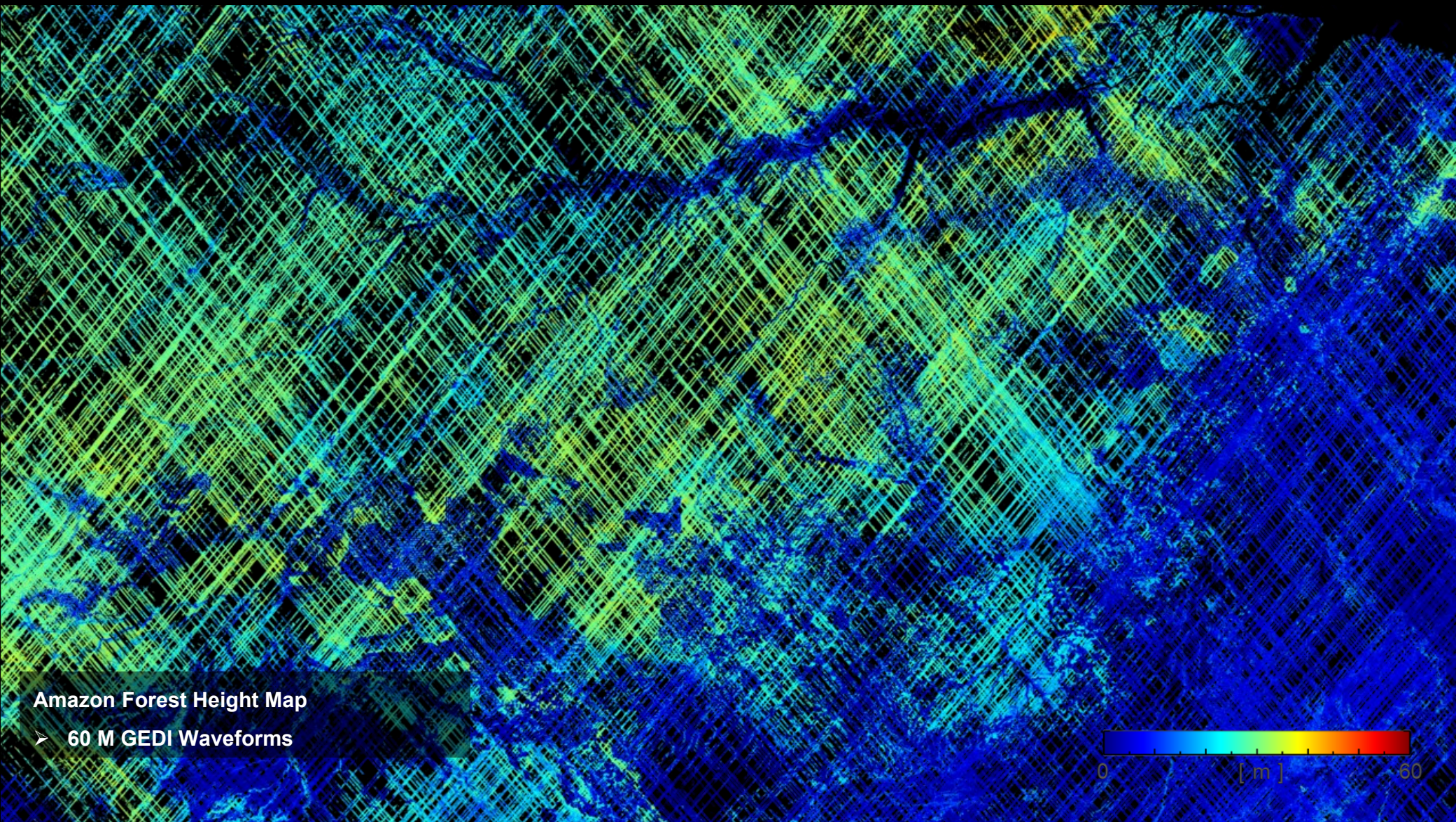
W60°

W55°

W45°

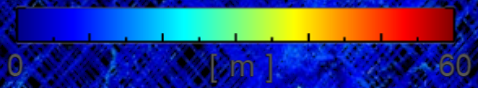
➤ 12,000 TanDEM-X CoSSC Scenes

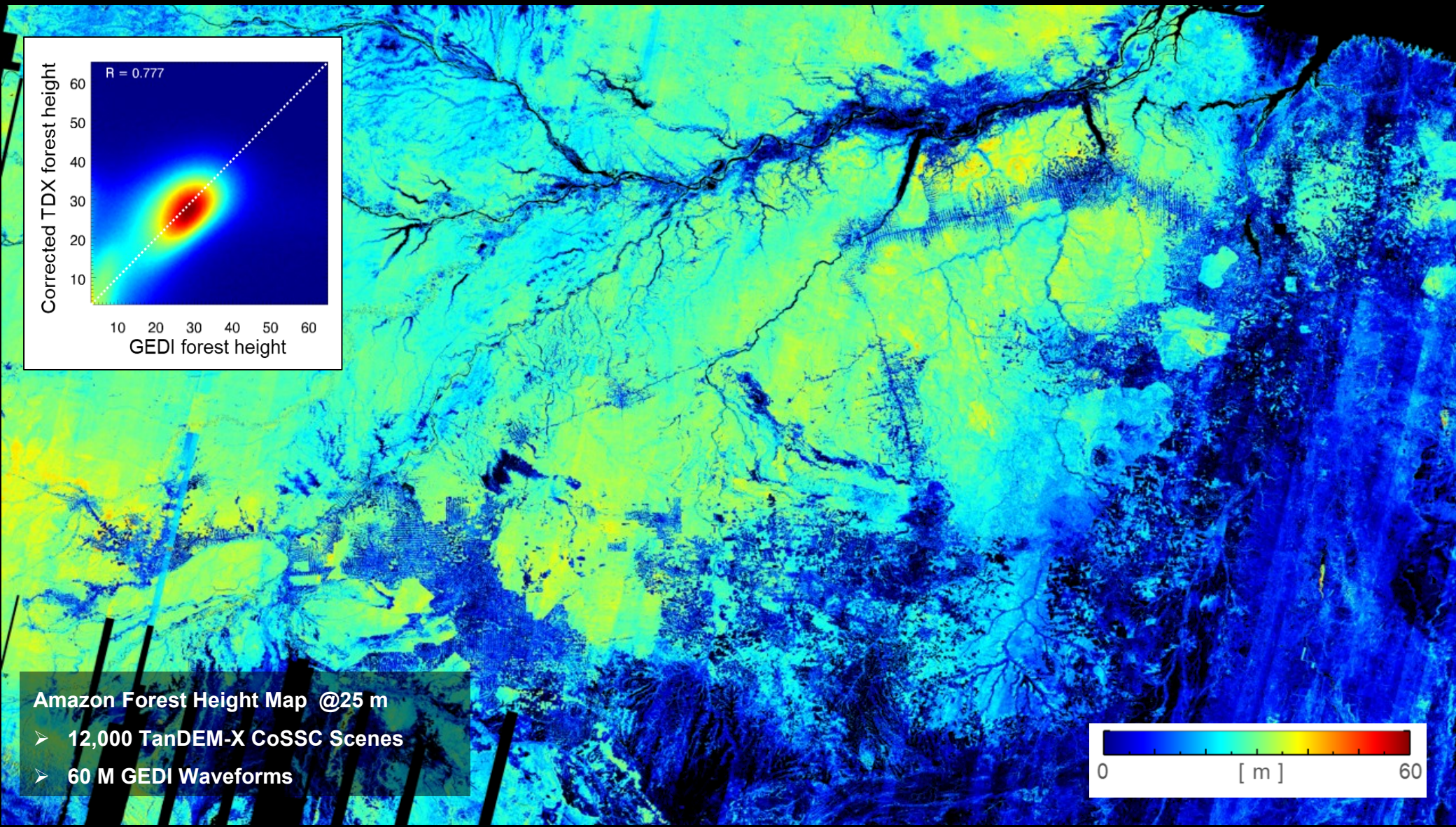
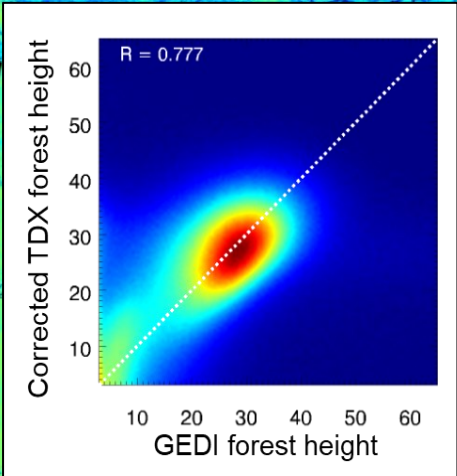
Interferometric Coherence  
0 1



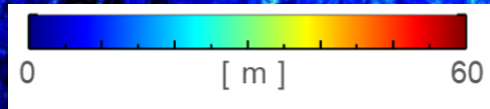
Amazon Forest Height Map

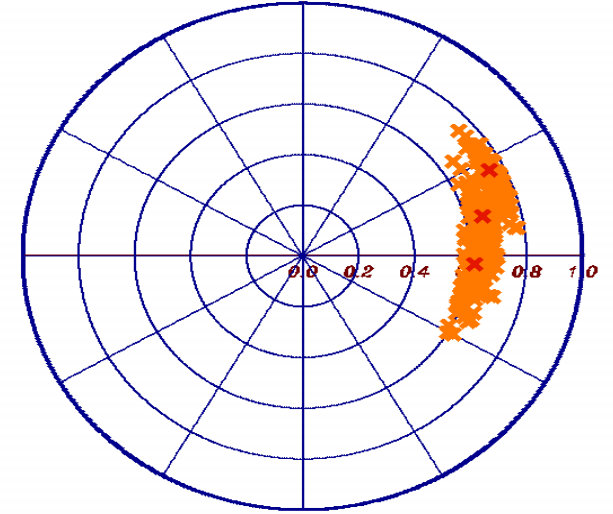
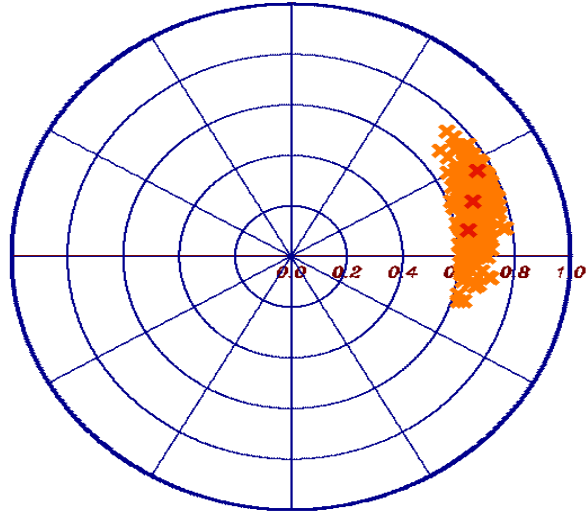
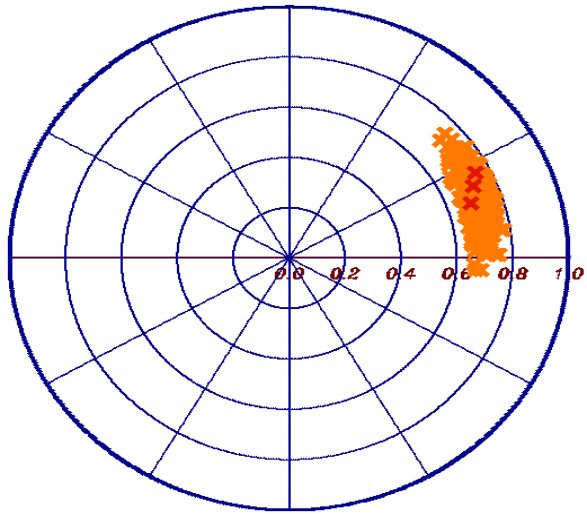
➤ 60 M GEDI Waveforms



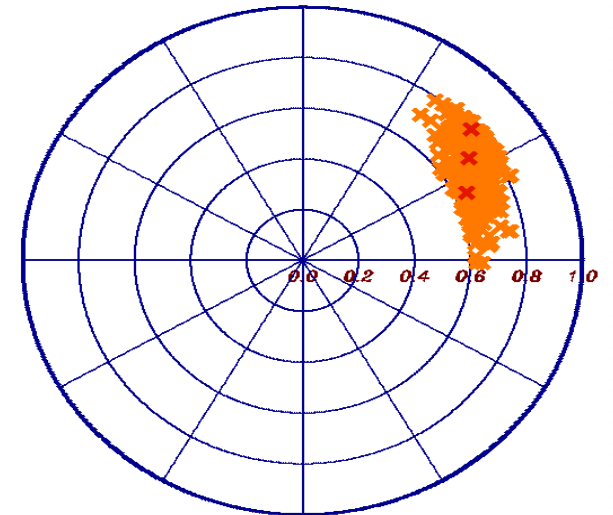
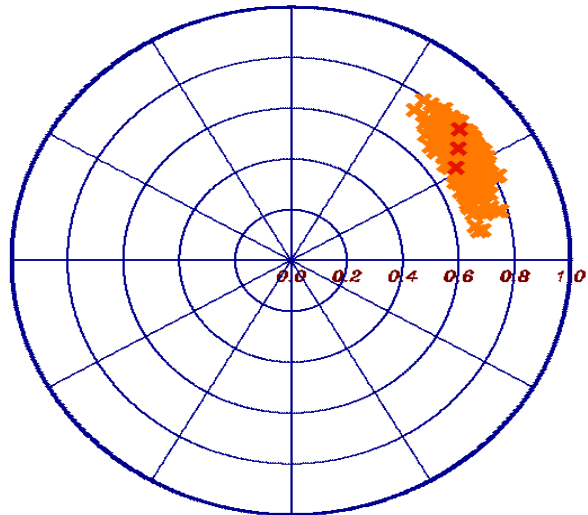
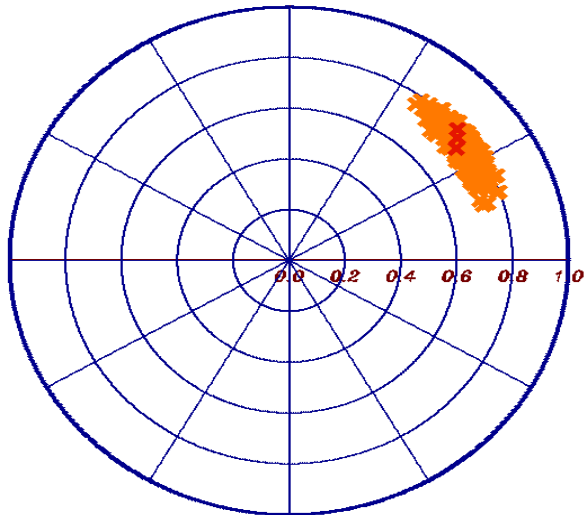


**Amazon Forest Height Map @25 m**  
➤ 12,000 TanDEM-X CoSSC Scenes  
➤ 60 M GEDI Waveforms





# Agriculture Vegetation



# Agriculture Pol-InSAR Applications

## Pol-SAR

$$[S] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$$

## Pol-InSAR

$$[S_1] = \begin{bmatrix} S_{HH}^1 & S_{HV}^1 \\ S_{VH}^1 & S_{VV}^1 \end{bmatrix}$$

$$[S_2] = \begin{bmatrix} S_{HH}^2 & S_{HV}^2 \\ S_{VH}^2 & S_{VV}^2 \end{bmatrix}$$

### Bare Surfaces: Isolated Scattering Center

- Low Entropy scatterers -> High polarimetric coherence

### Vegetated Surfaces: Volume Scatterers

- High Entropy scatterers -> Low polarimetric coherence

### Agricultural vs. Forest Vegetation

- |   |   |  |
|---|---|--|
| Orientation effects in the vegetation layer | » | Anisotropic Propagation  |
| Thinner / shorter vegetation layer          | » | Increased Importance of Ground Scattering<br>Large Spatial Baselines |
| Short crop / plant phenological cycle       | » | Short Temporal Baselines   |
| Variety of crop / plant structure           | » | Abstract Modelling   |



# Wheat

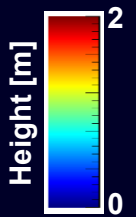
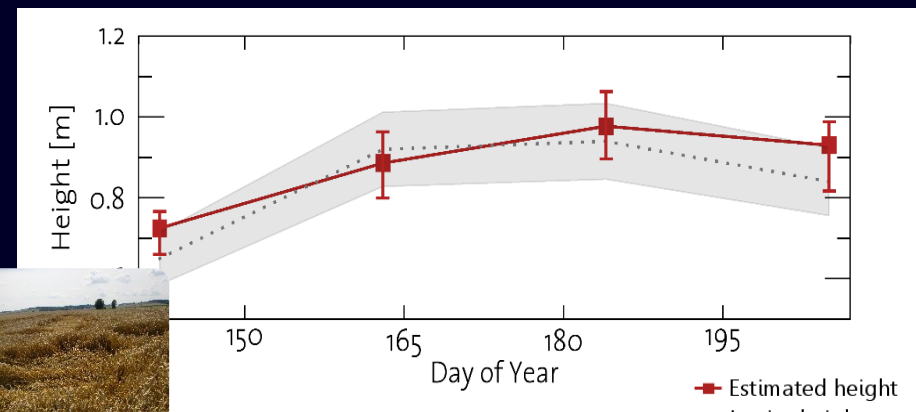
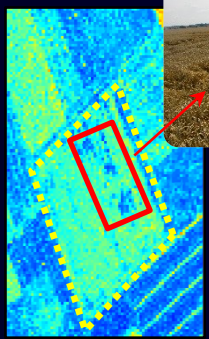
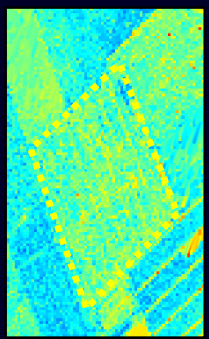
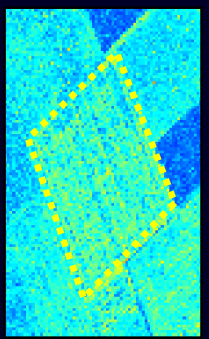
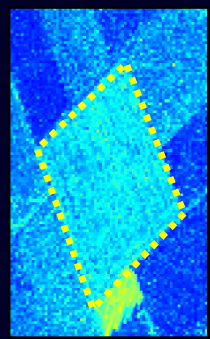


May 22

Jun 12

Jul 03

Jul 24



**Sensor:** DLR's F-SAR  
**Frequency:** C-Band ( $\approx 5$  GHz)  
**Number of spatial baselines:** 2  
**Max. temporal baseline:** 90 minutes  
**Equivalent Number of Looks:** 100

# Corn (Maize)

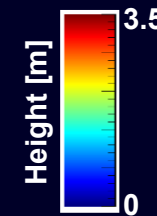
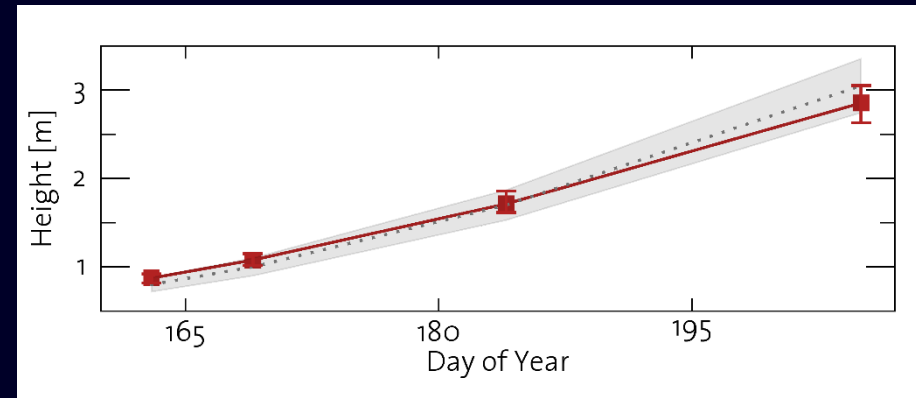
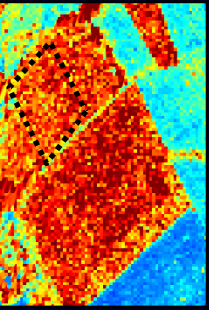
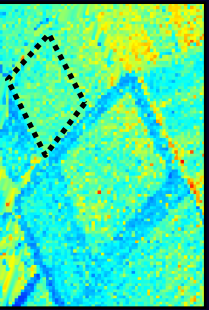
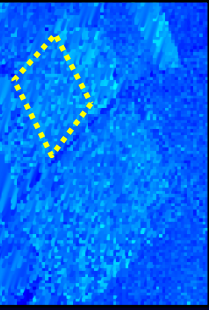
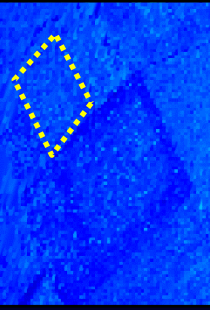


Jun 12

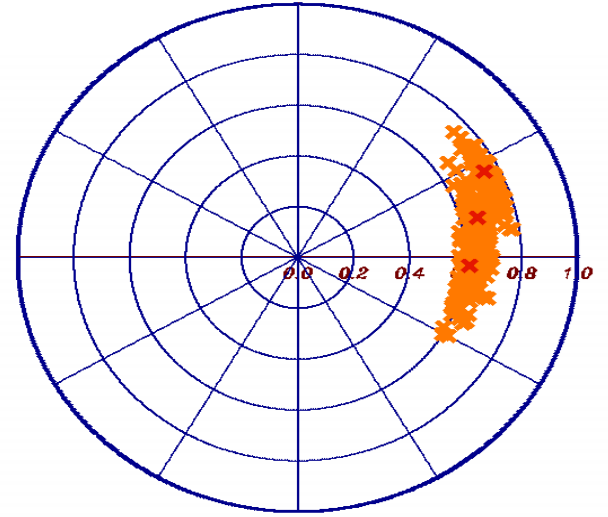
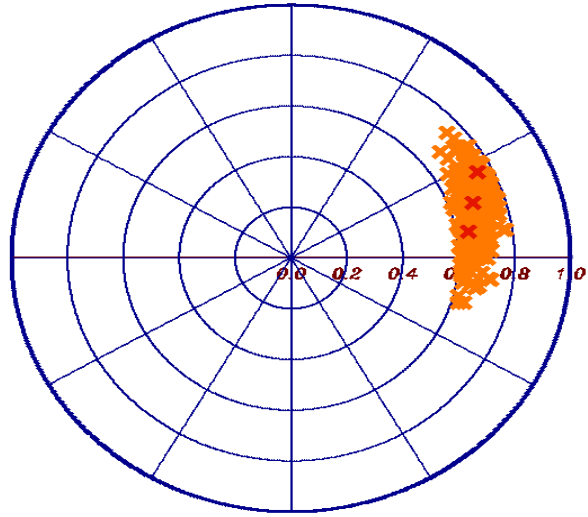
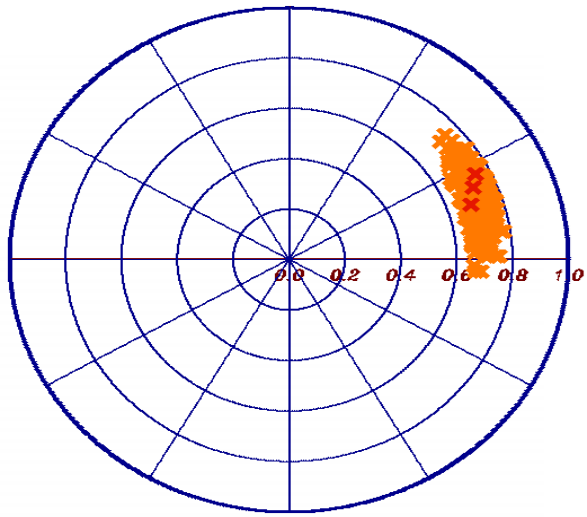
Jun 18

Jul 03

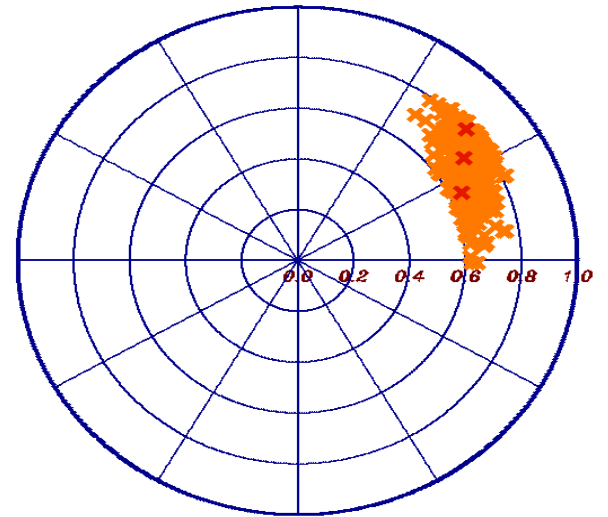
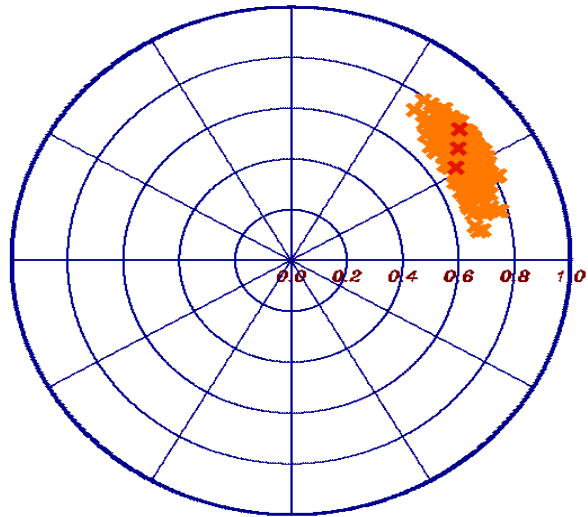
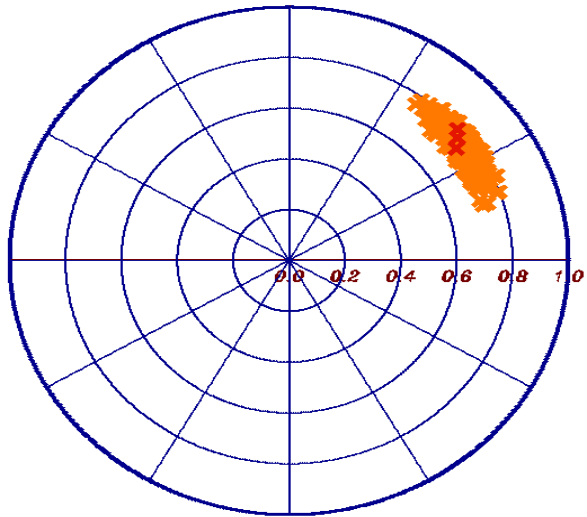
Jul 24



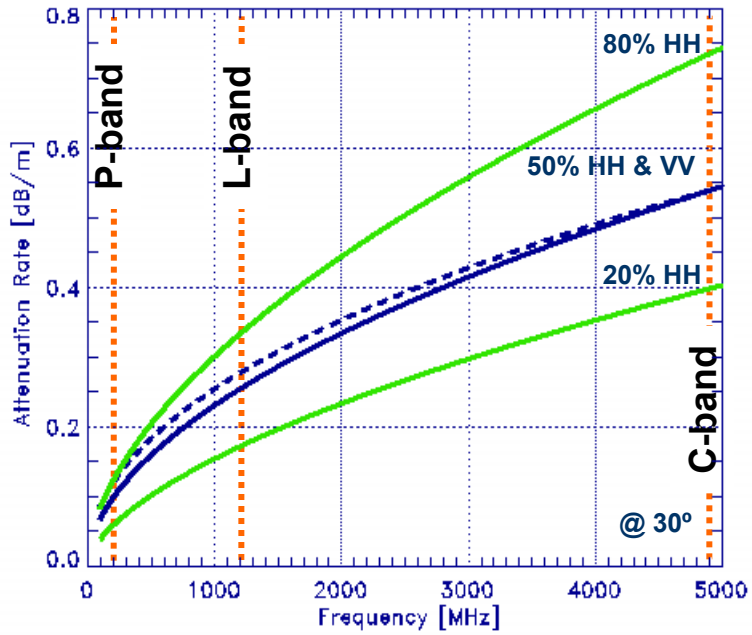
Estimated height  
In-situ height  
10% uncertainty



## Pol-InSAR: Frequency Effects

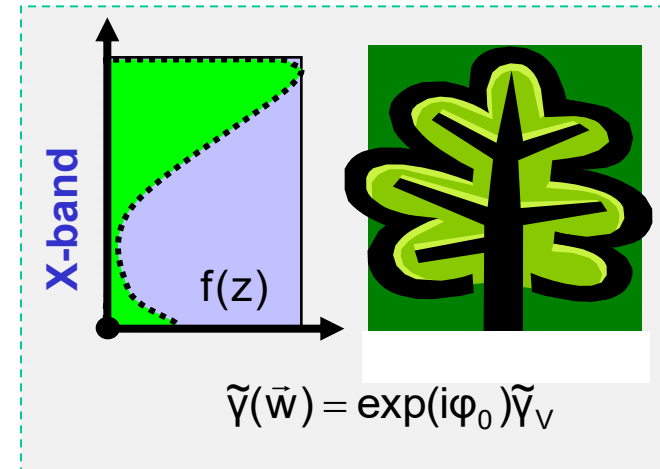
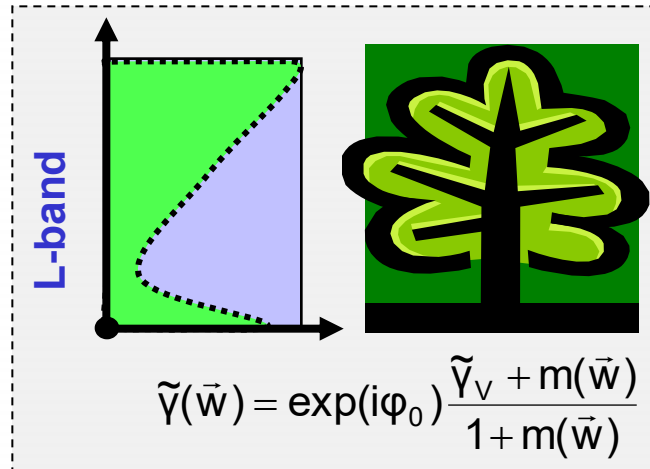
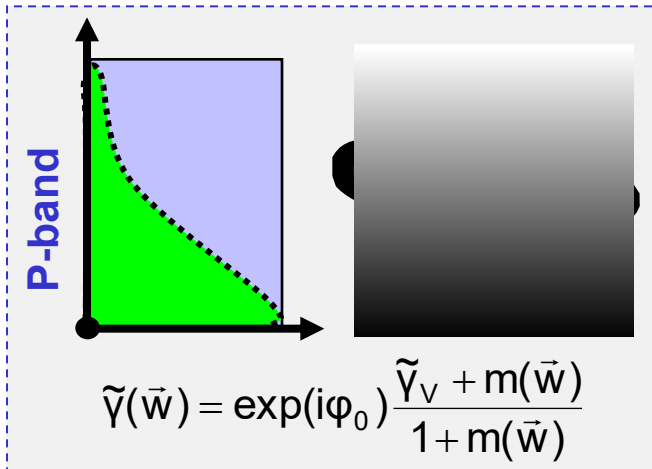


# Frequency Dependency



With decreasing frequency:

- The attenuation through the vegetation decreases;
- The relative importance of the volume decreases;
- The relative importance of the ground increases;
- The effective scatterers and their distribution changes.

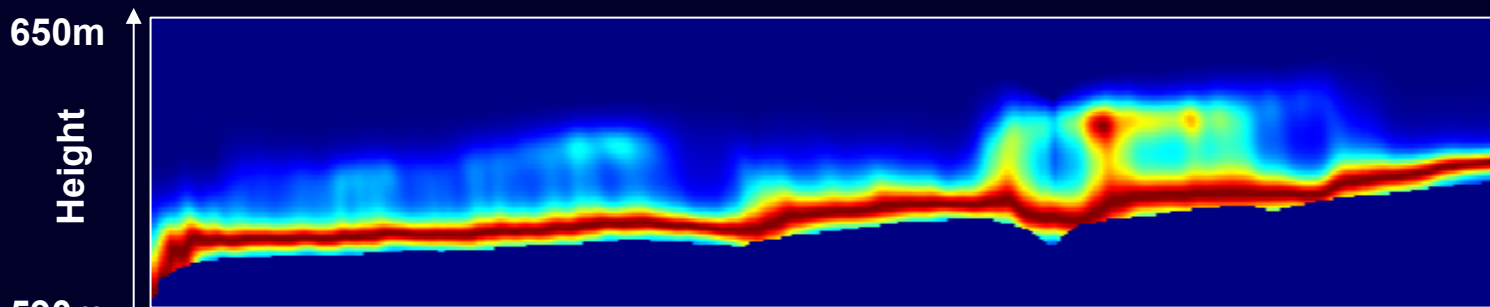


L.Bessette, S.Ayasli "Ultra Wide Band P-3 and Carabas II Foliage Attenuation and Backscatter Analysis", Proceedings of IEEE Radar Conference, 2001.

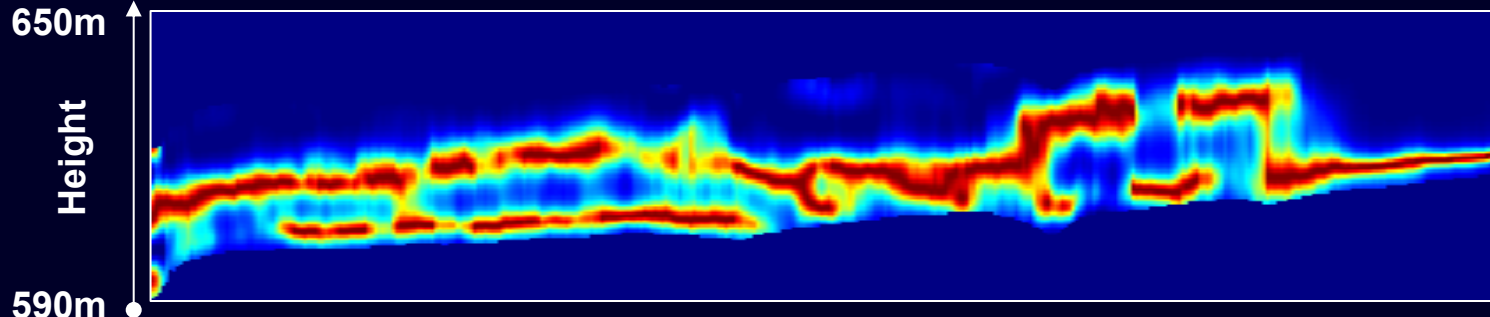


Traunstein forest (Germany) - Capon - HH

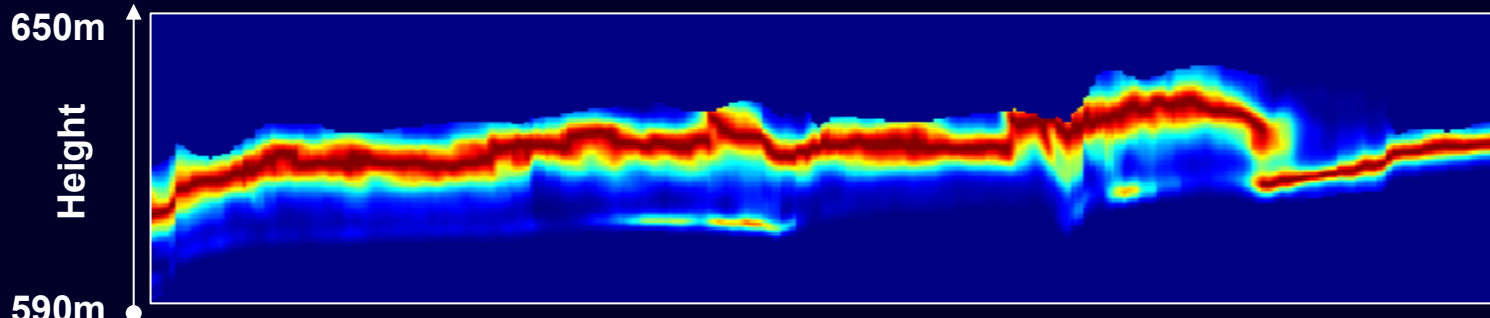
**P-band**



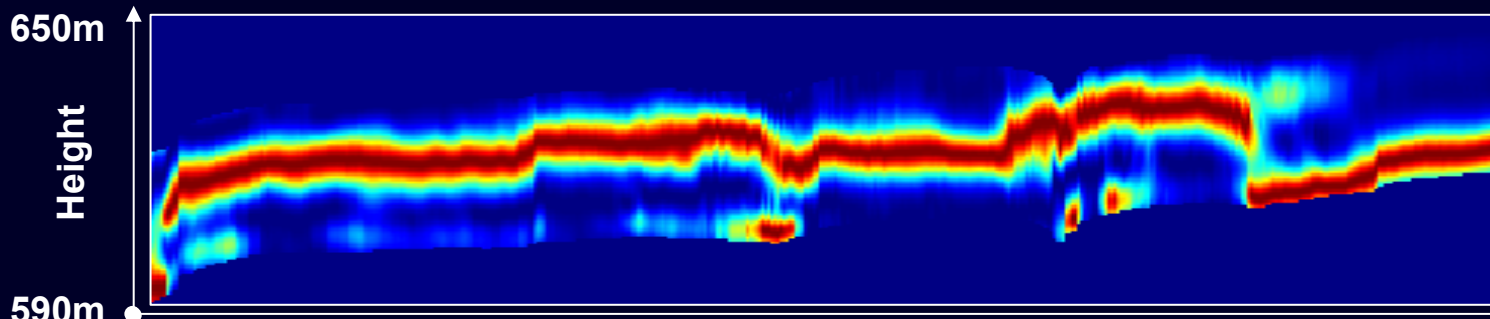
**L-band**



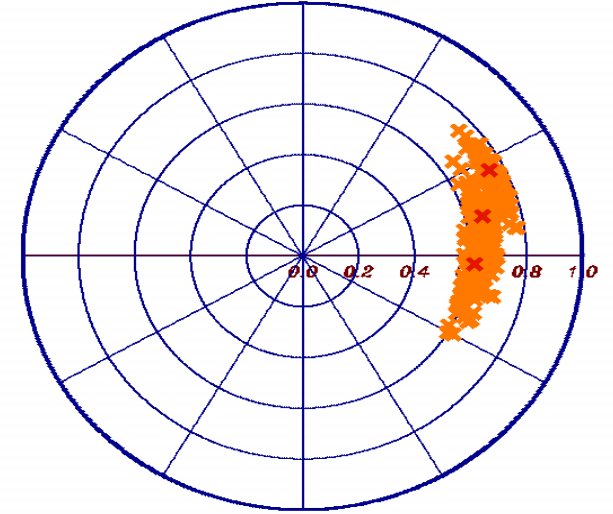
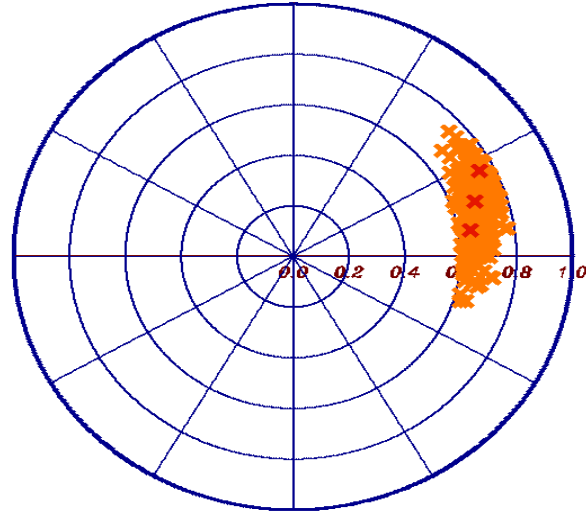
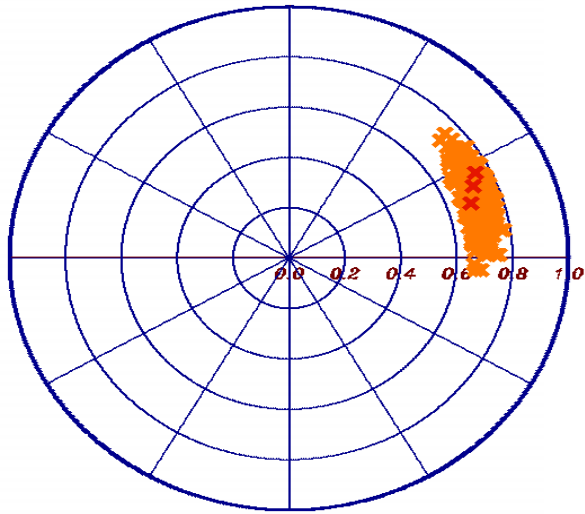
**S-band**



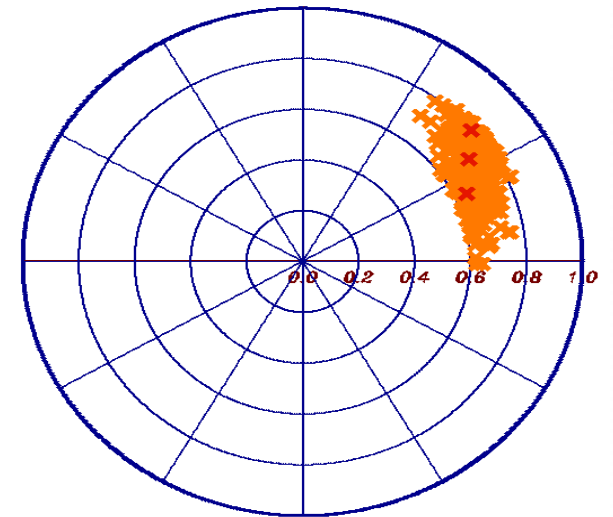
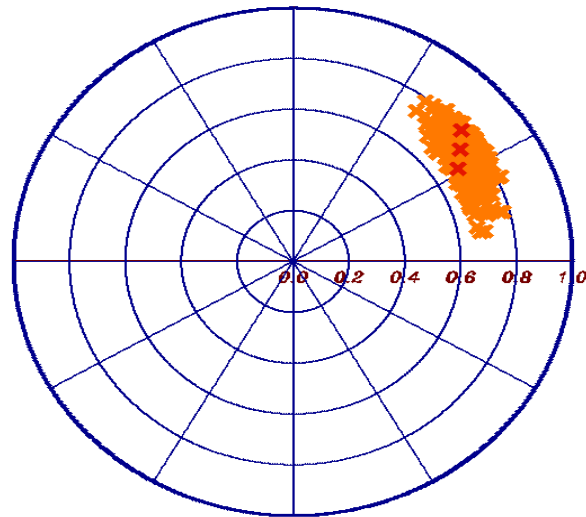
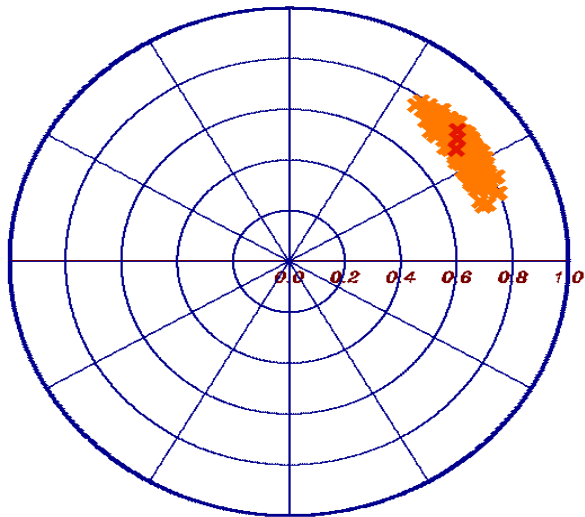
**X-band**



Slant range (0.6Km)



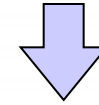
## Non-Volumetric Decorrelation Effects





## Interferometric Coherence

$$\tilde{Y}_{OBS} = \frac{\langle S_1 S_2^* \rangle}{\sqrt{\langle S_1 S_1^* \rangle \langle S_2 S_2^* \rangle}}$$



$$\tilde{Y}_{OBS} = \tilde{Y}_{TRANS} \tilde{Y}_{PROP} \tilde{Y}_{SCAT}$$

**Scatterer**

$$\tilde{Y}_{SCAT} = \tilde{Y}_{TEMP} \tilde{Y}_{BAS}$$

$\tilde{Y}_{BAS} = Y_{AZ} Y_{RG} \tilde{Y}_{VOL}$

- $\tilde{Y}_{TEMP}$  Temporal Deco
- $Y_{AZ}$  Doppler Deco (=1)
- $Y_{RG}$  Range Deco (=1)
- $\tilde{Y}_{VOL}$  Volume Deco

**Propagation**

$$\tilde{Y}_{PROP} = \tilde{Y}_{ATMO} \tilde{Y}_{IONO}$$

- $\tilde{Y}_{ATMO}$  Atmospheric Deco
- $\tilde{Y}_{IONO}$  Ionospheric Deco

**System & Processing**

$$\tilde{Y}_{TRANS} = Y_{SYS} \tilde{Y}_{PRO} Y_{EST}$$

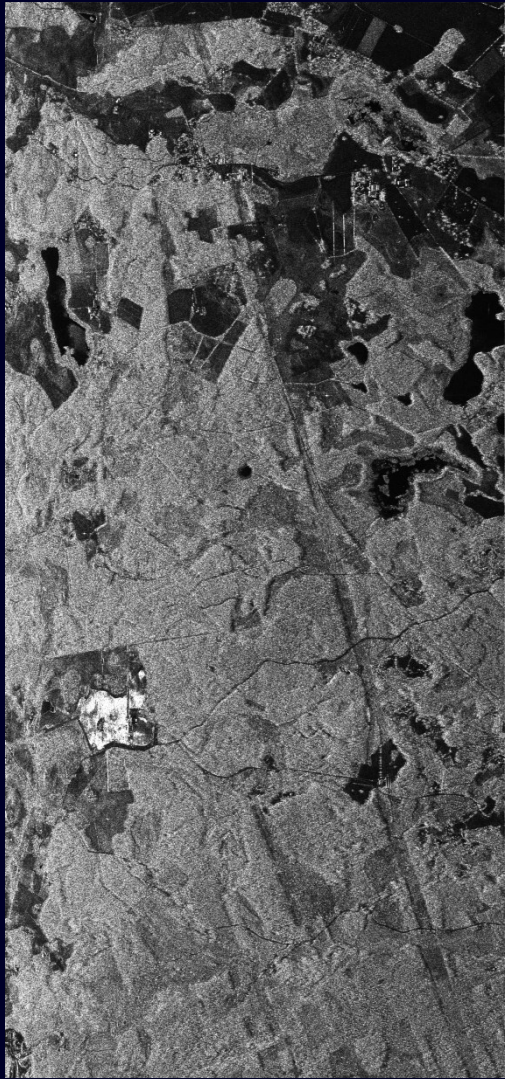
$$Y_{SYS} = Y_{QUAN} Y_{AMB} Y_{SNR}$$

$$\tilde{Y}_{PRO} = \tilde{Y}_{CAL} Y_{INTR} Y_{COR}$$

$$Y_{EST} = Y_{BIAS} Y_{TOPO}$$

- $\tilde{Y}_{SNR}$  SNR Decorrelation
- $\tilde{Y}_{QUAN}$  Quantisation Effects (0.99)
- $\tilde{Y}_{AMB}$  Ambiguities
- $\tilde{Y}_{CAL}$  Calibration Decorelation
- $Y_{INTR}$  Interpolation Effects
- $Y_{COR}$  Corregistration Effects
- $Y_{BIAS}$  Coh Estimation Bias (=1)
- $Y_{TOPO}$  Toporaphy Induced Coh Bias

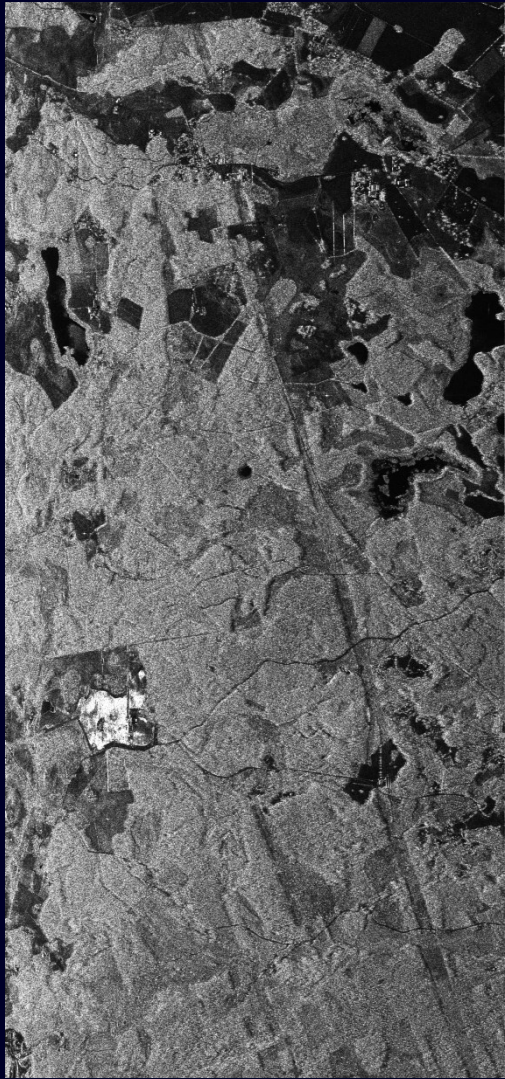
# Amplitude Image



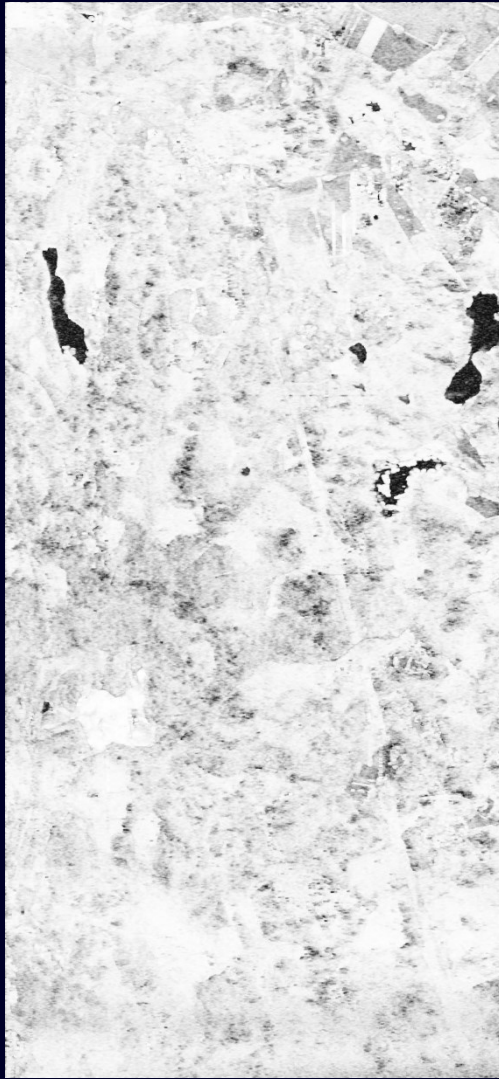
Amplitude Image HH



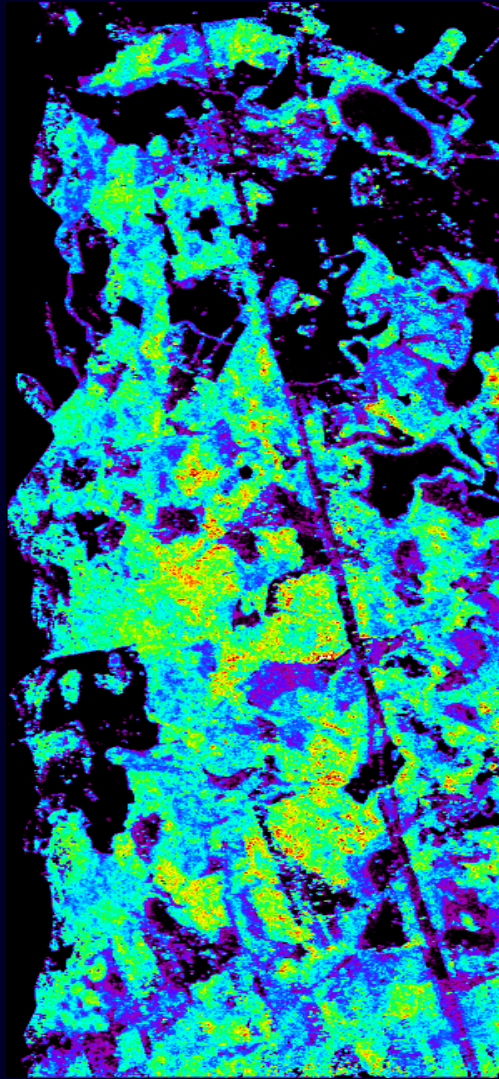
# Interferometric Coherence: Volume vs Temporal Decorrelation



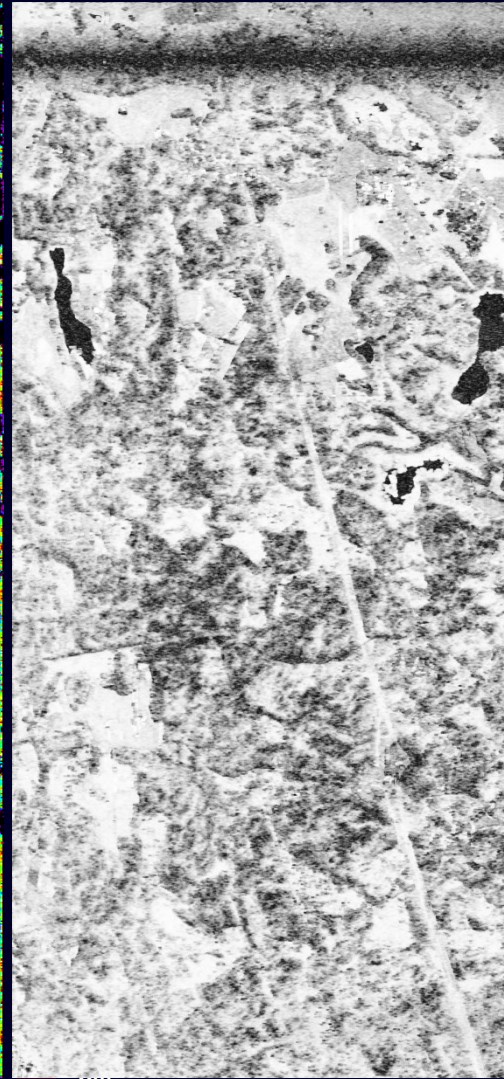
Amplitude Image HH



Volume Coherence



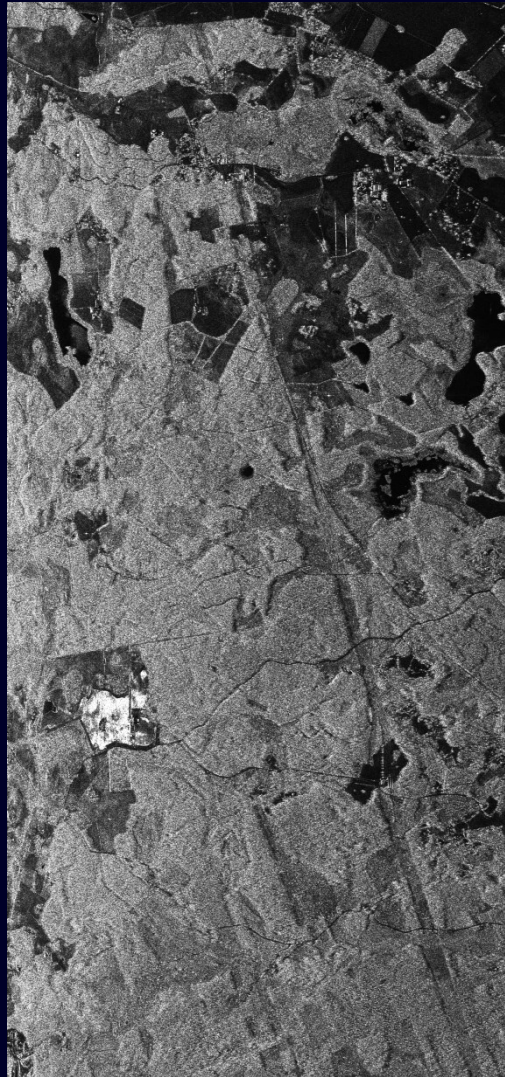
Forest Height Map



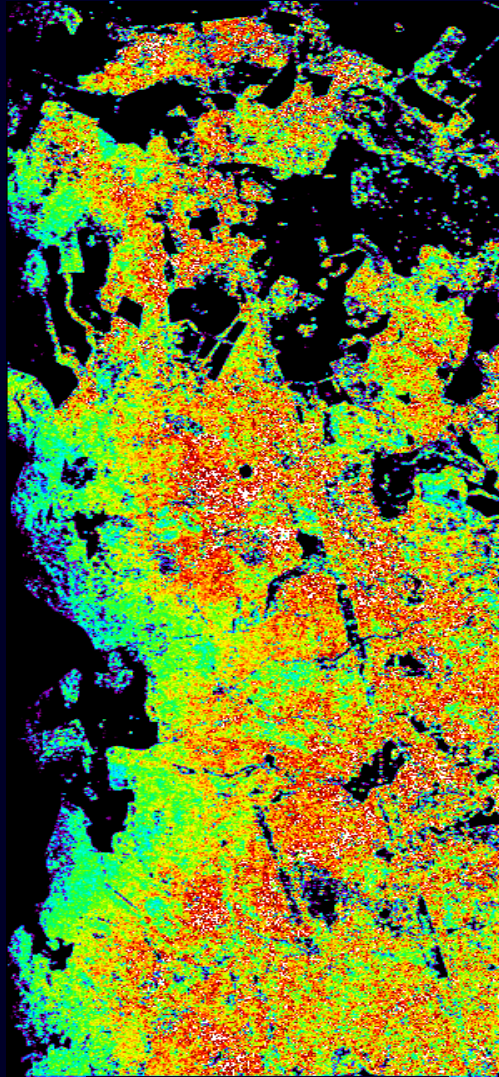
Volume + Temporal 24h



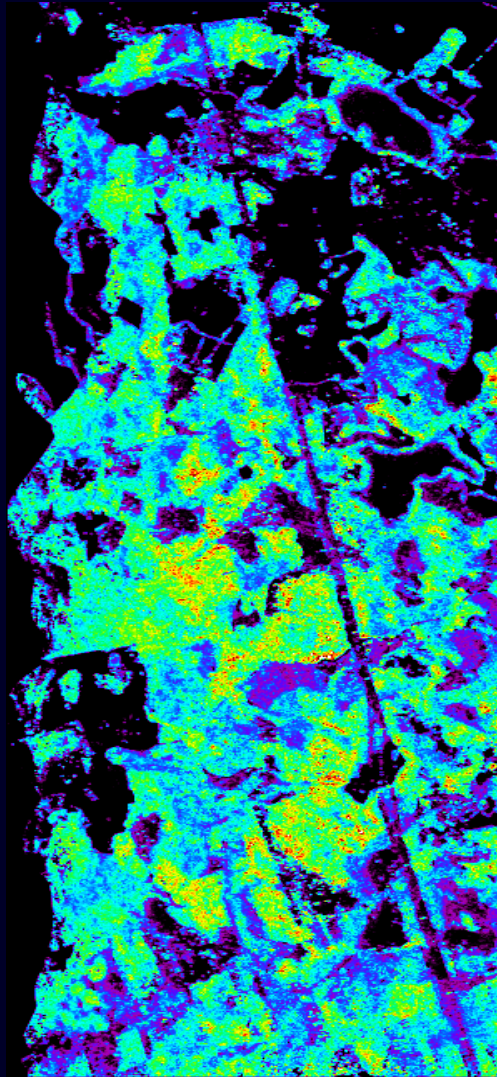
# Interferometric Coherence: Volume vs Temporal Decorrelation



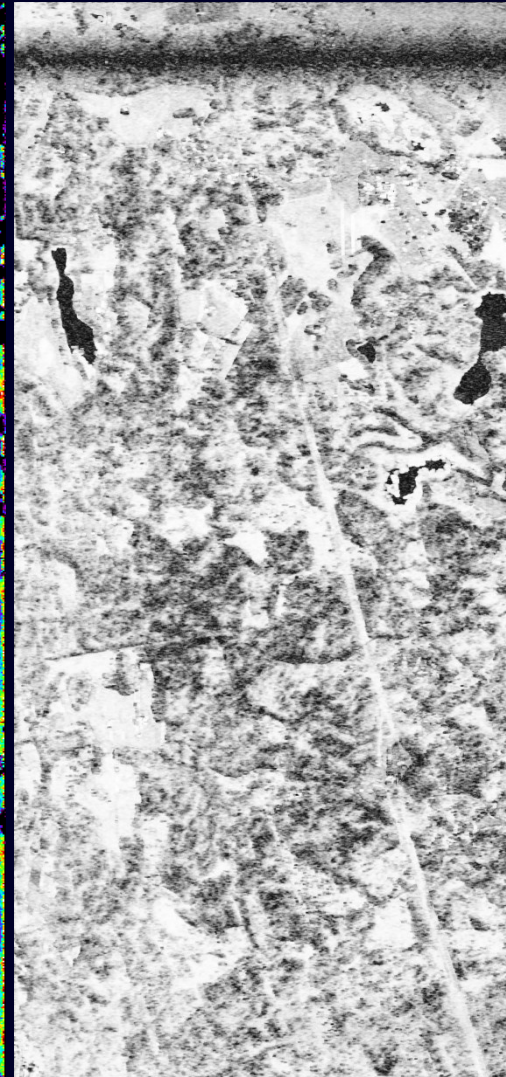
Amplitude Image HH



Forest Height Map



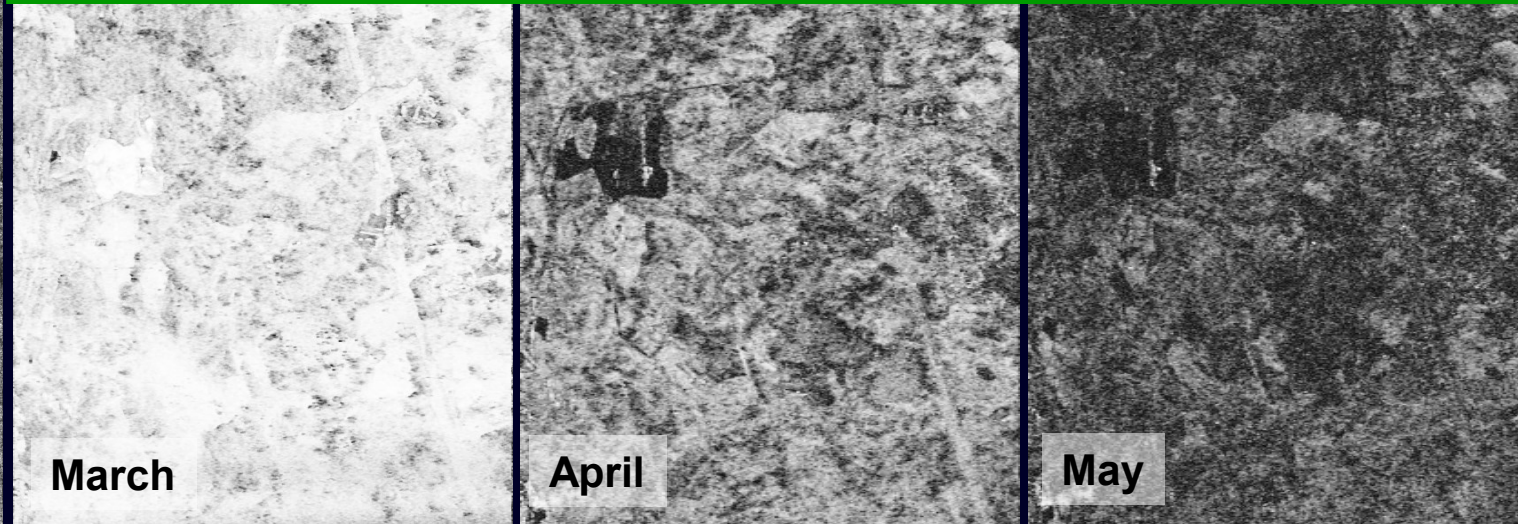
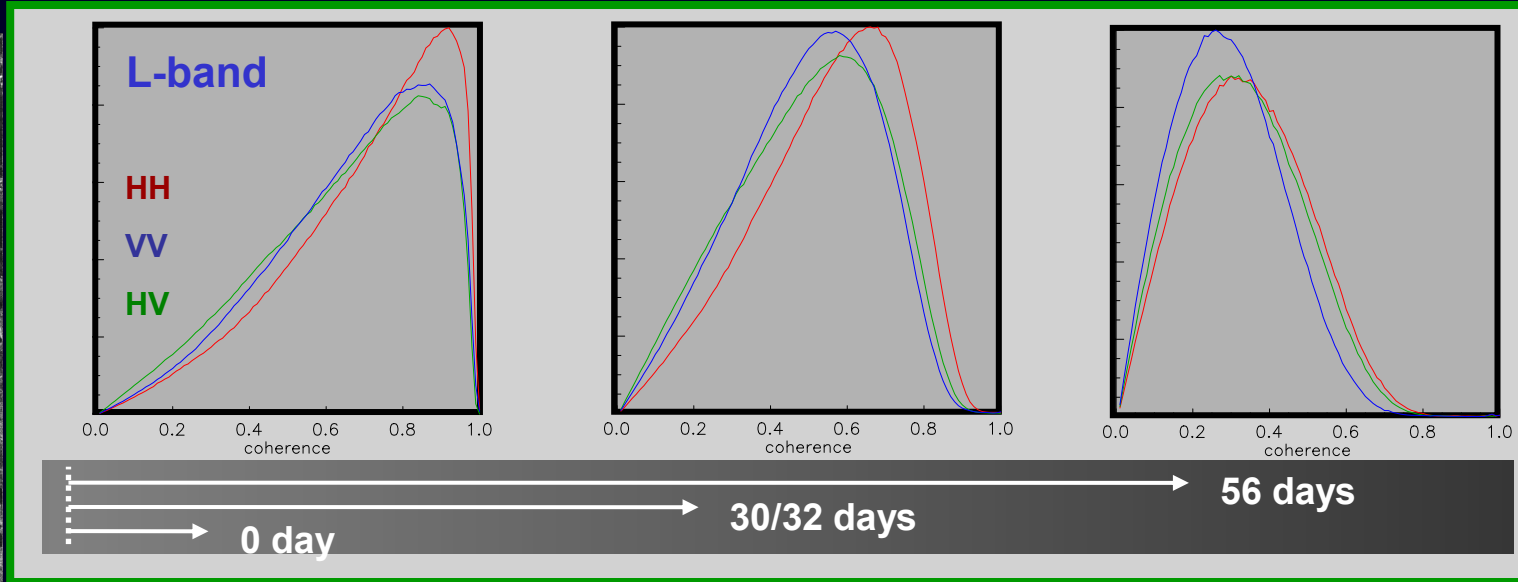
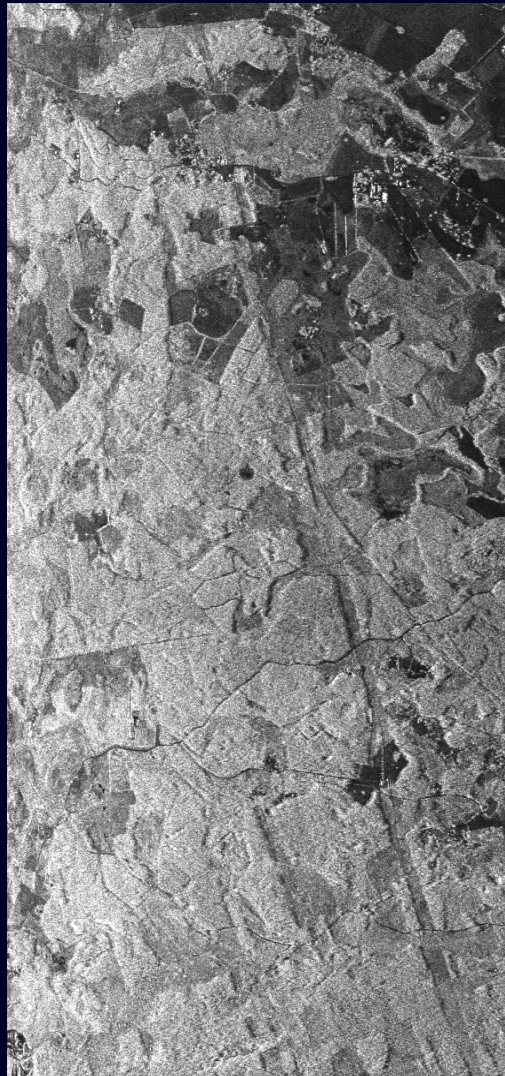
Forest Height Map



5m  
Volume + Temporal 24h



# Remningstorp Test Site: Temporal Decorrelation: L-band



HV Amplitude Image

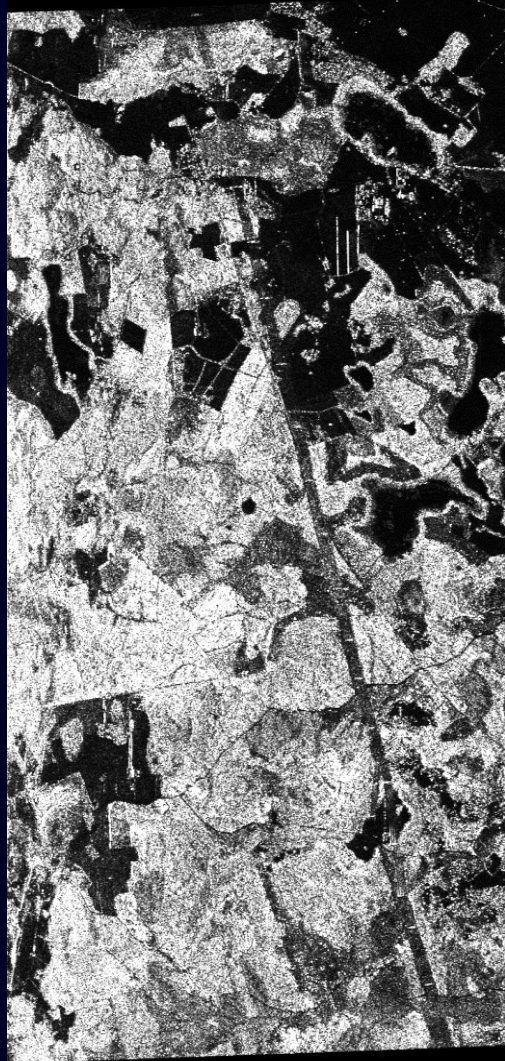
0days

30days

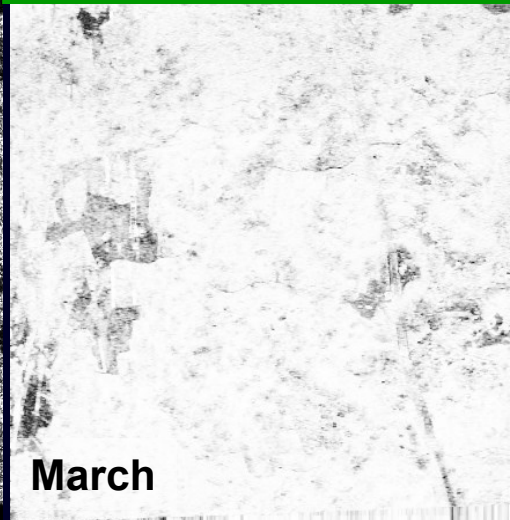
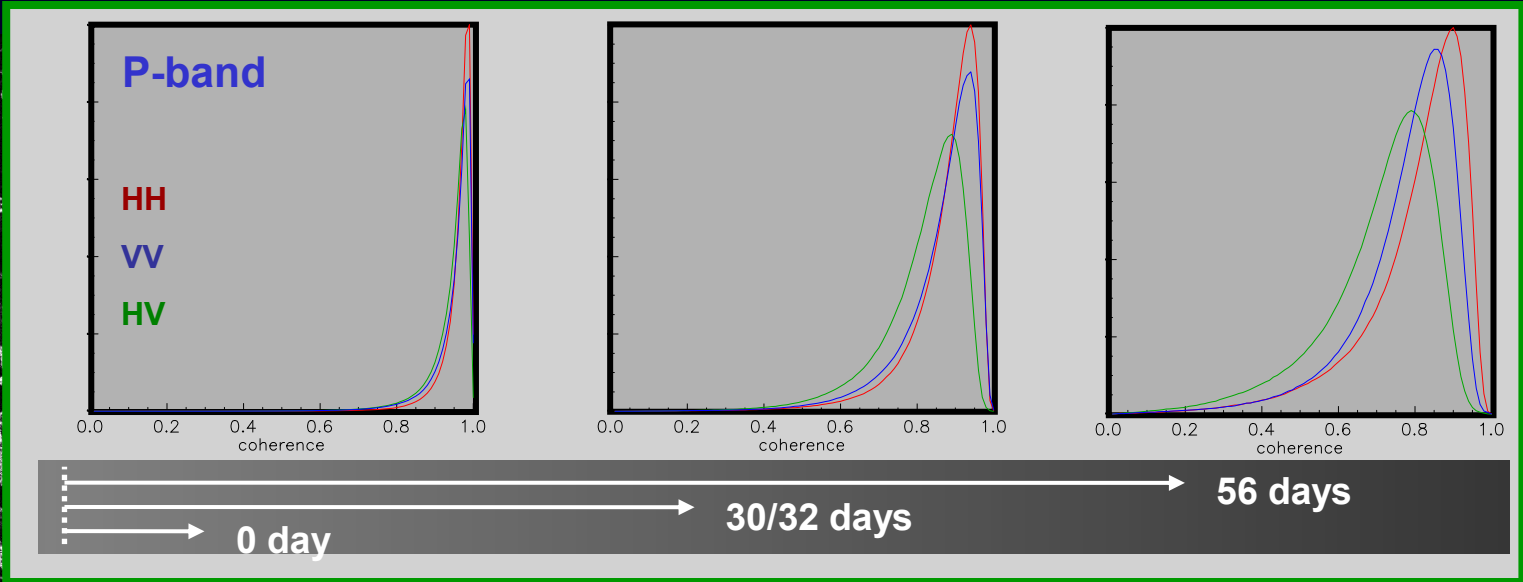
58days



# Remningstorp Test Site: Temporal Decorrelation: P-Band



HV Amplitude Image



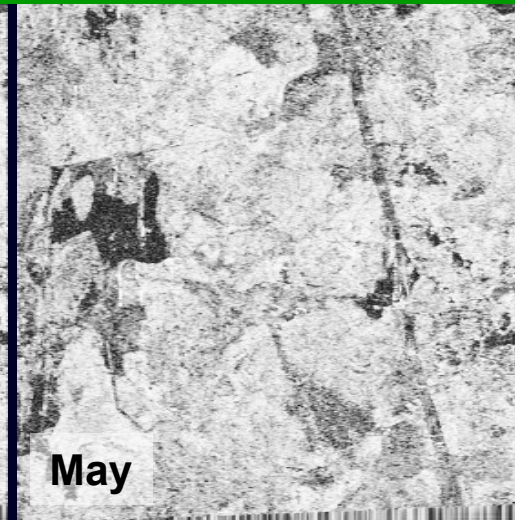
March

0days



April

30days



May

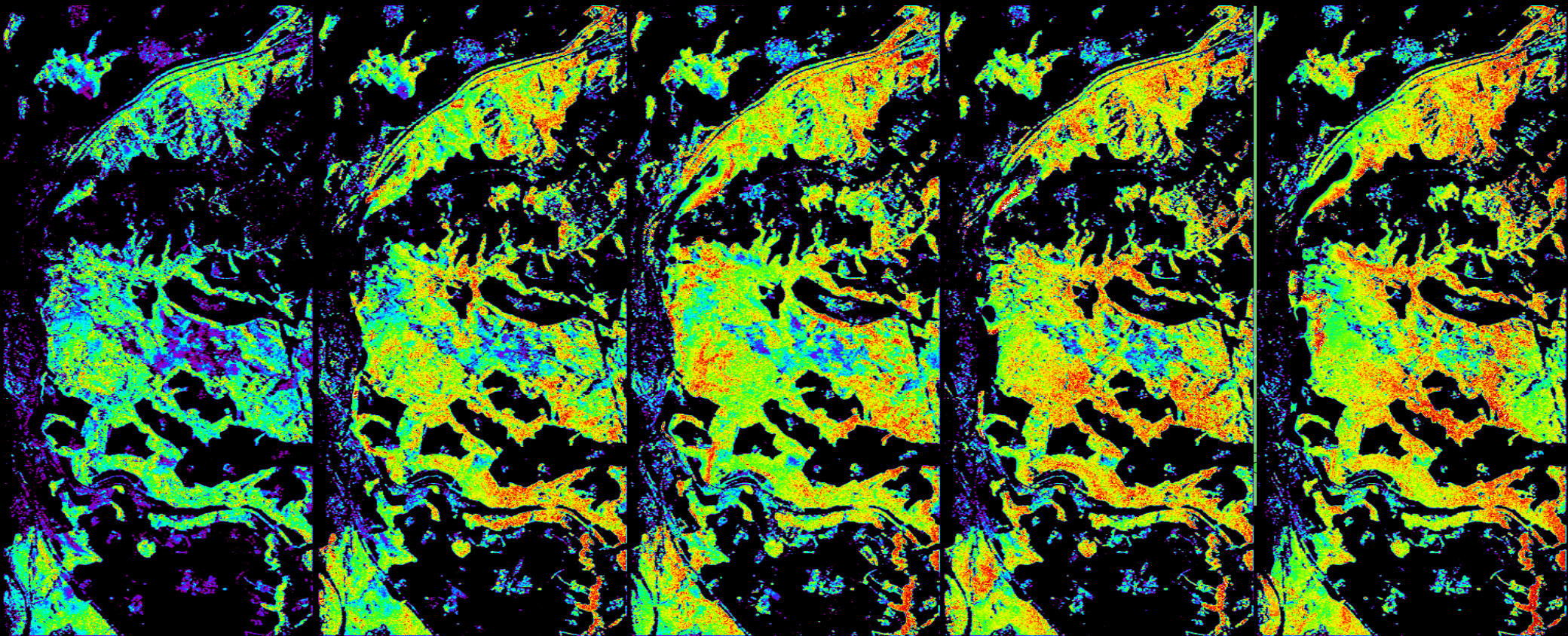
56days



# Traunstein Test Site



Forest Height Maps from different Temporal Baselines: 10min-13days



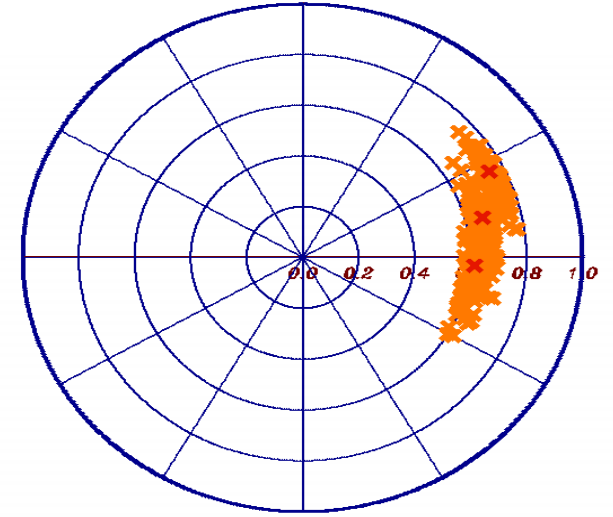
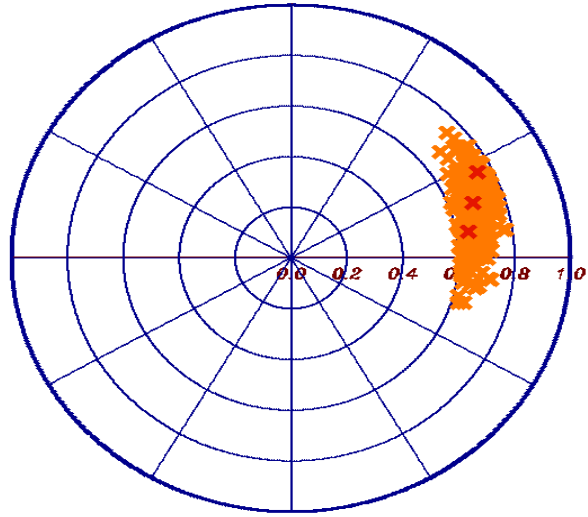
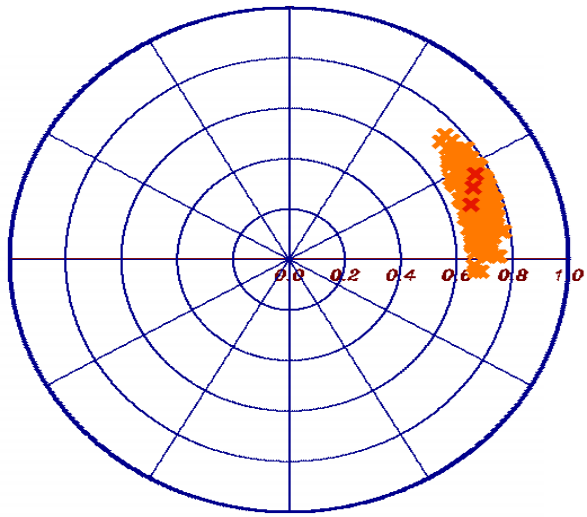
$\Delta T=10\text{min}$

$\Delta T=1\text{day}$

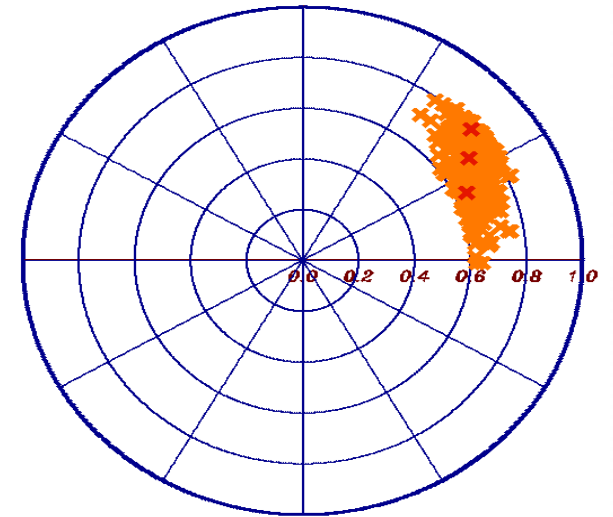
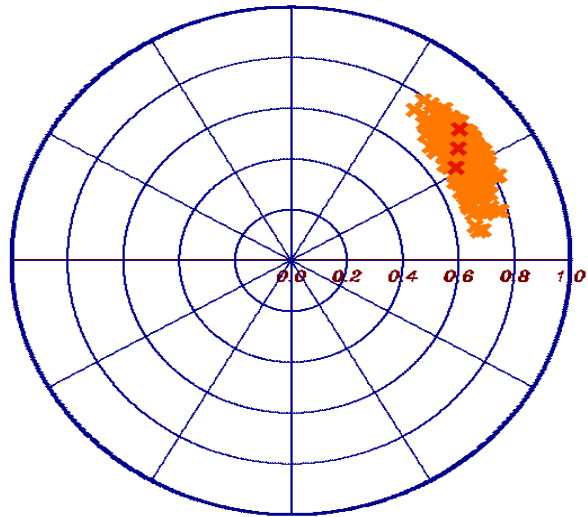
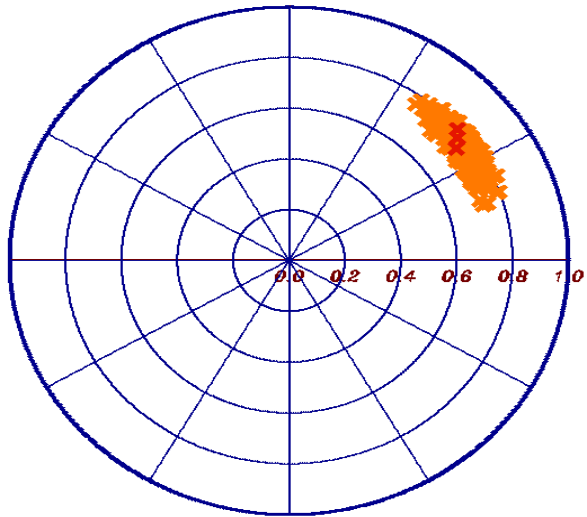
$\Delta T=5\text{days}$

$\Delta T=7\text{days}$

$\Delta T=13\text{days}$



# Polarimetric Coherence Tomography (PCT)



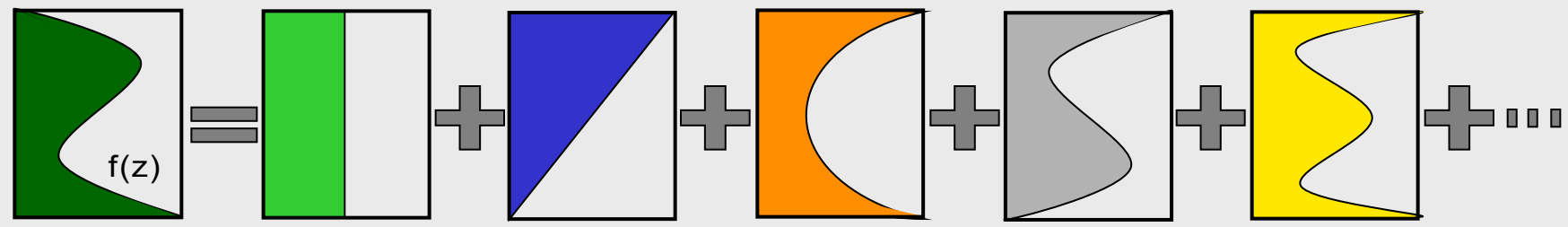
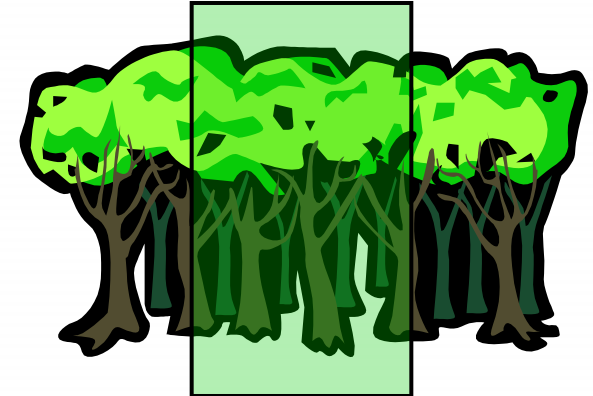
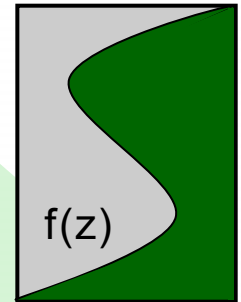
# Polarimetric Coherence Tomography (PCT)



Volume Coherence

$$\tilde{Y}_{\text{Vol}}(f(z)) = e^{ik_z z_0} \frac{\int_0^{h_v} f(z) e^{ik_z z} dz}{\int_0^{h_v} f(z) dz}$$

$f(z)$  ... vertical reflectivity function



$$\tilde{Y}_{\text{Vol}}(f(z)) = e^{ik_z z_0} \frac{\int_0^{h_v} f(z) e^{ik_z z} dz}{\int_0^{h_v} f(z) dz}$$

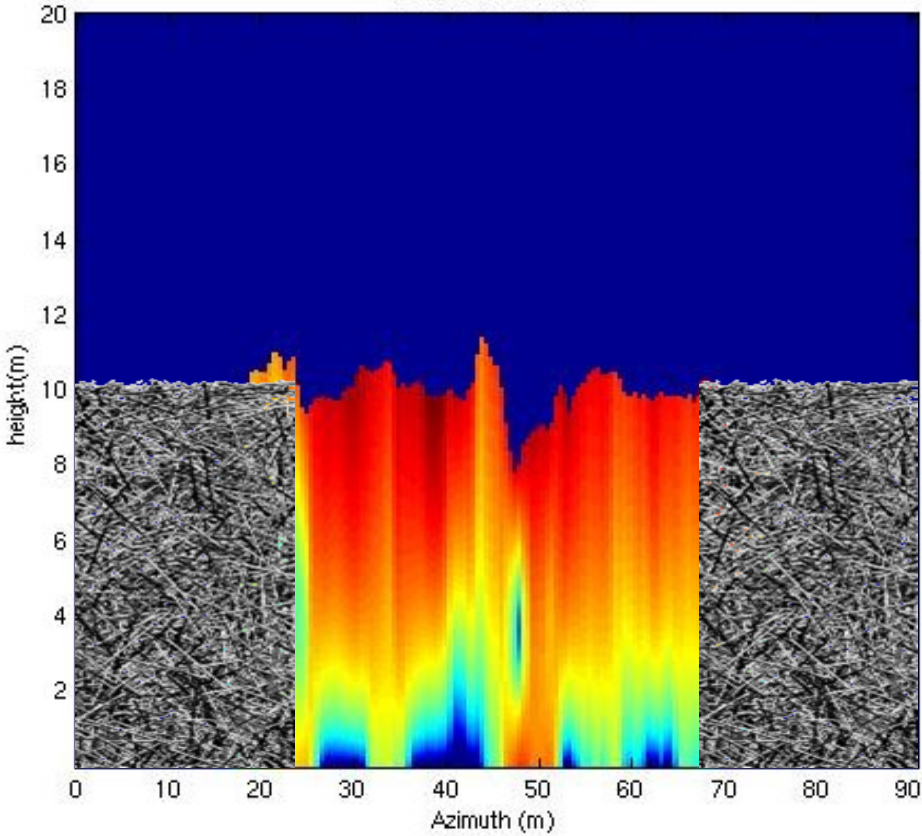
$$\int_0^{h_v} f(z) e^{ik_z z} dz = \frac{h_v}{2} e^{i \frac{k_z h_v}{2}} \int_{-1}^1 (1 + f(z')) e^{i \frac{k_z h_v}{2} z'} dz'$$

$$\int_0^{h_v} f(z) dz = \frac{h_v}{2} \int_{-1}^1 (1 + f(z')) dz'$$

Fourier Legendre Series:

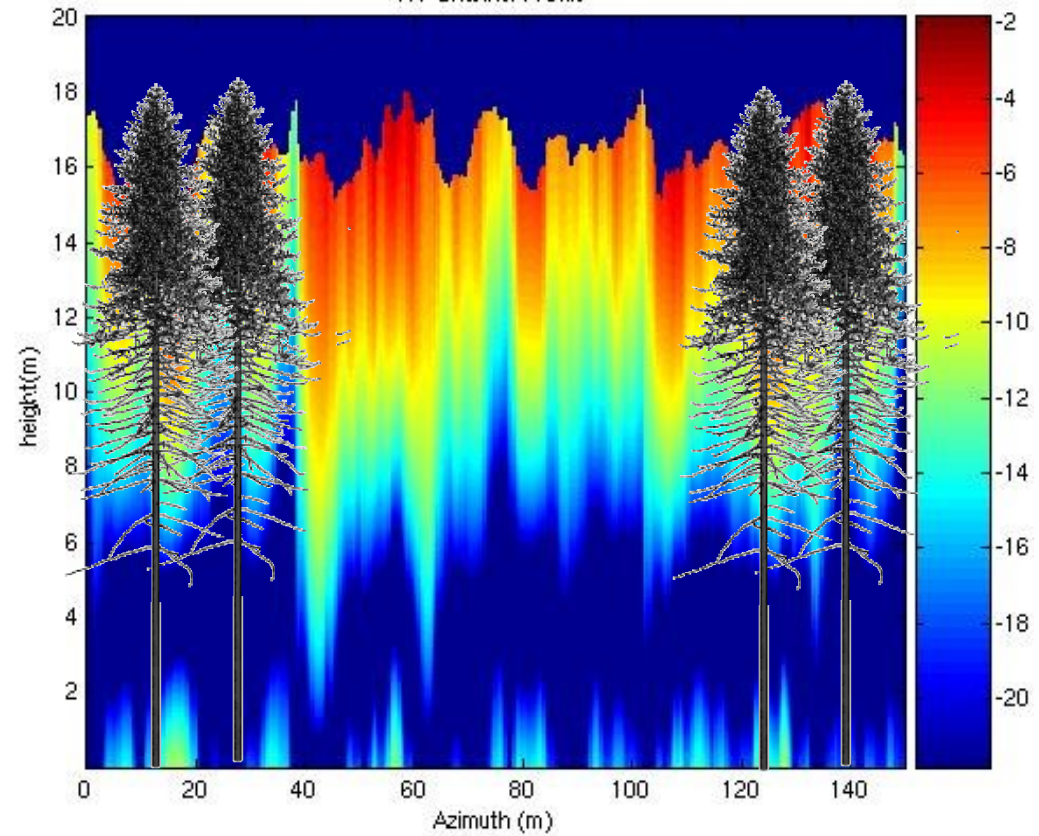
$$f(z') = \sum_n a_n P_n(z') \quad \text{where} \quad a_n = \frac{2n+1}{2} \int_{-1}^1 f(z') P_n(z') dz'$$

HV Channel Profile



10m Uniform Hedge

HV Channel Profile

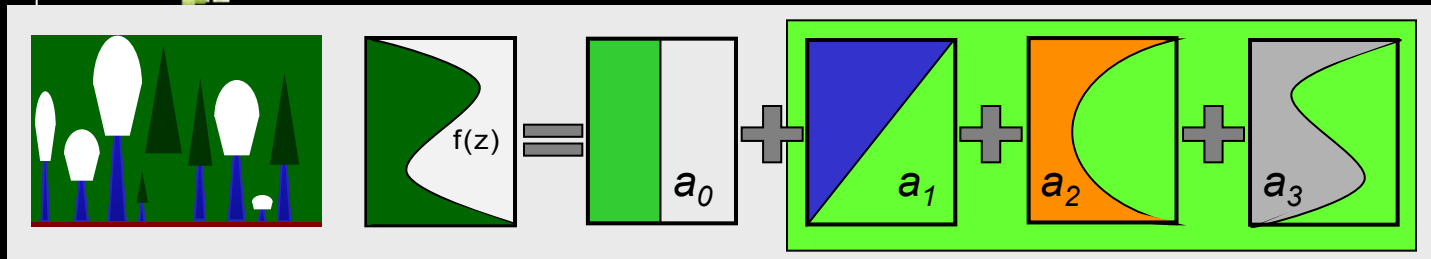
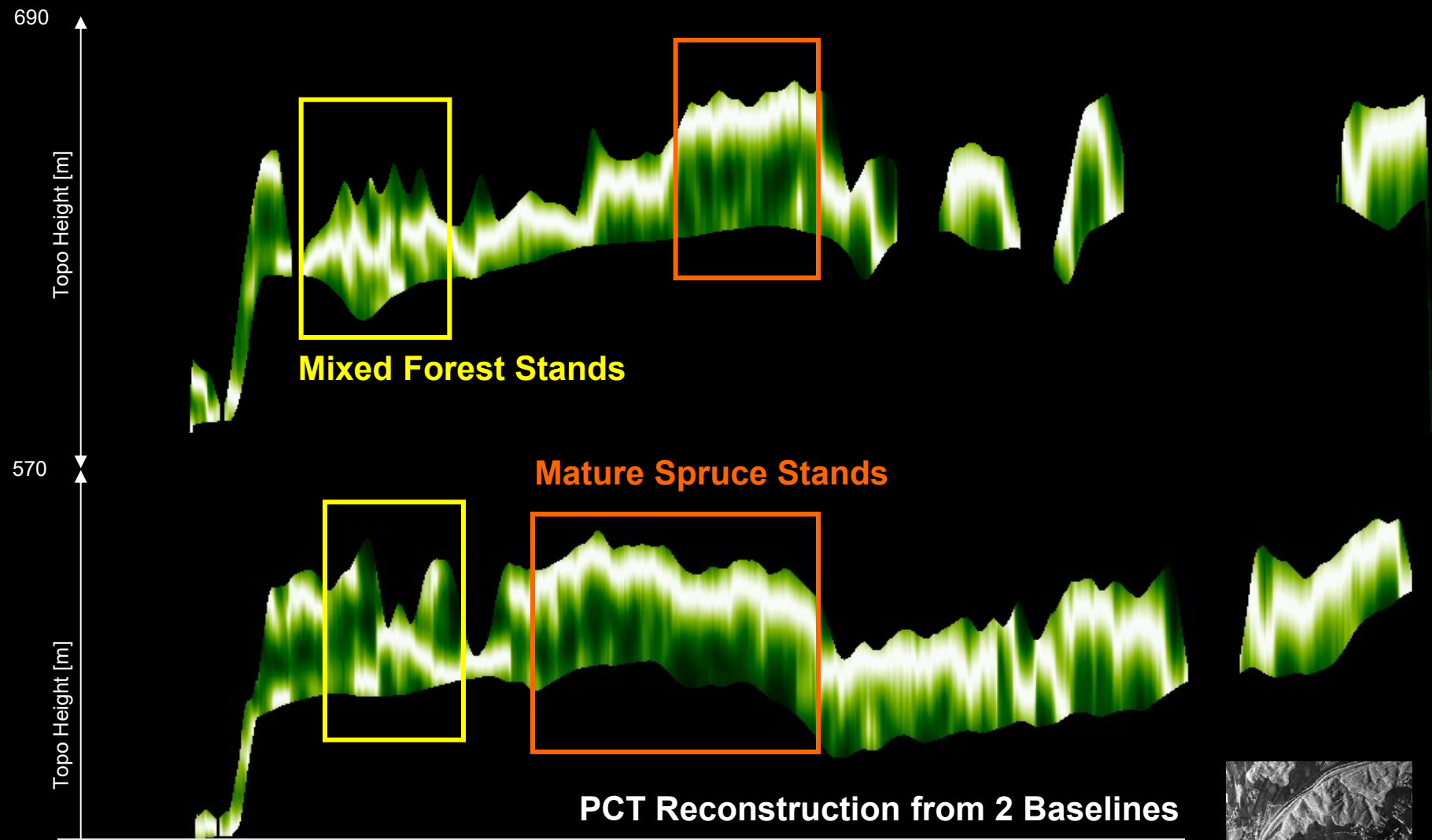


17m Scots Pine Forest

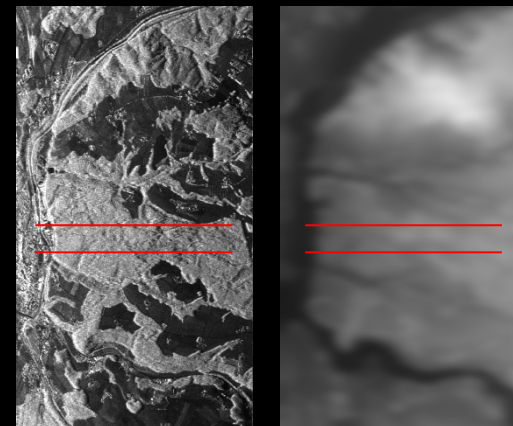
Simulations courtesy of Mark Williams:

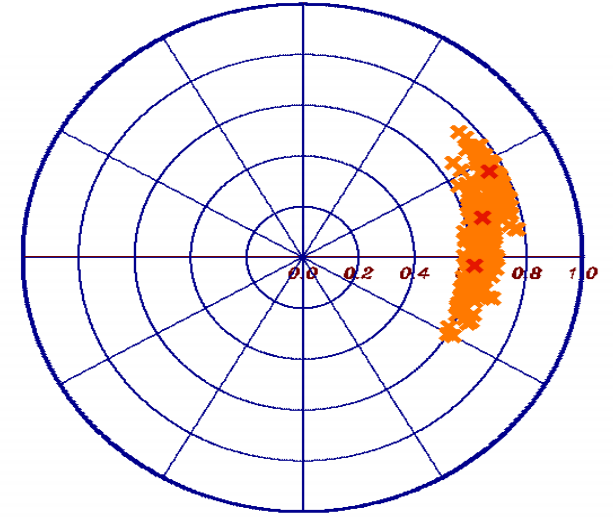
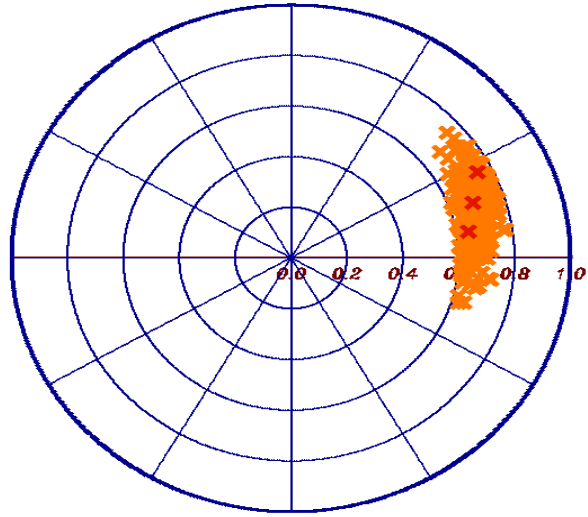
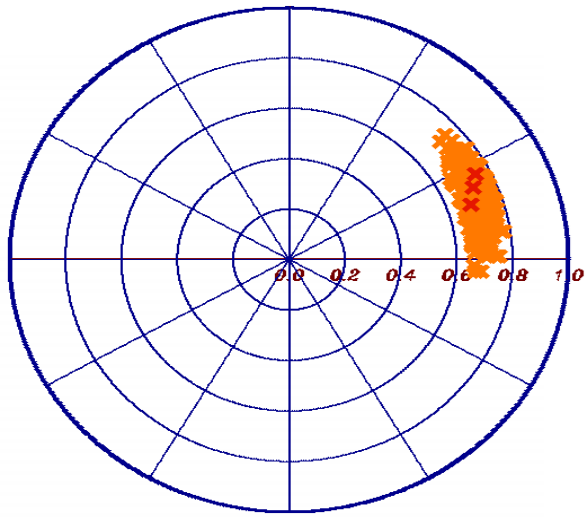
S.R. Cloude, D.G. Corr, M.L. Williams, "Target Detection Beneath Foliage Using PolInSAR", Waves in Random Media, vol. 14, pages S393 - S414., 2004



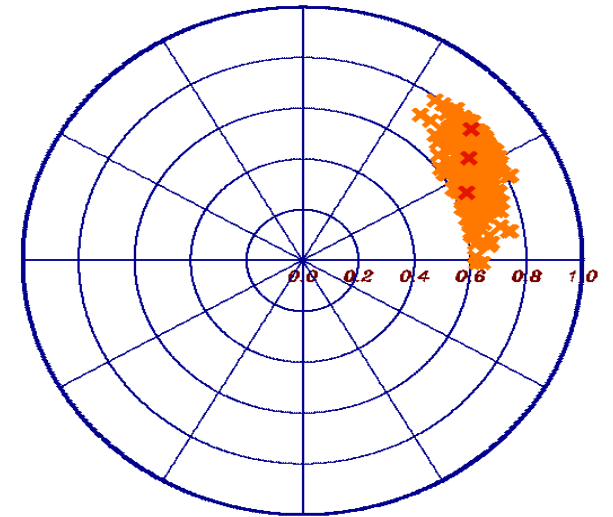
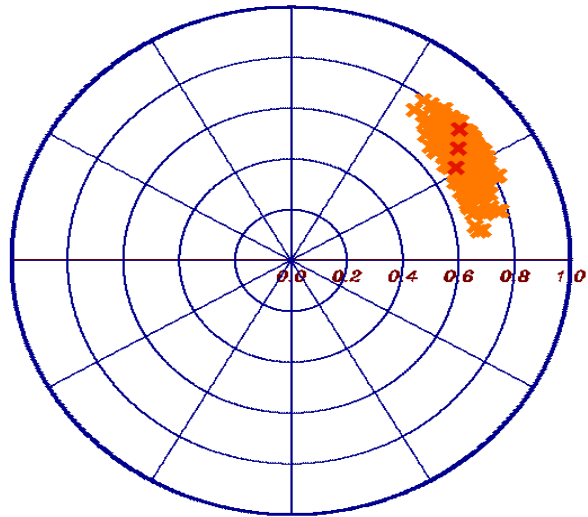
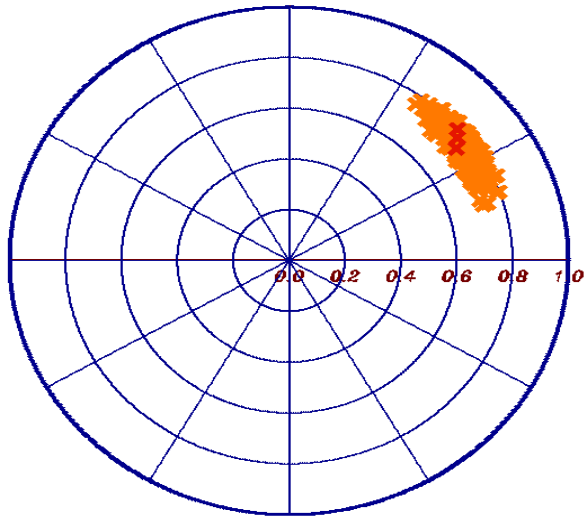


Test site: Traunstein, Germany, L-band @ HV Polarisation

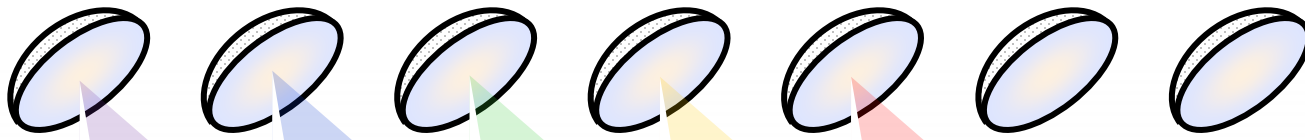




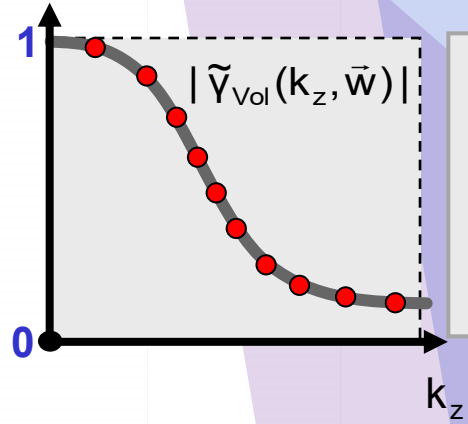
# Polarimetric SAR Tomography (TomoSAR)



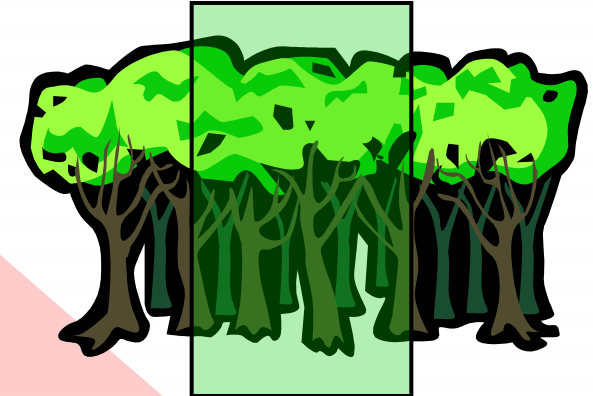
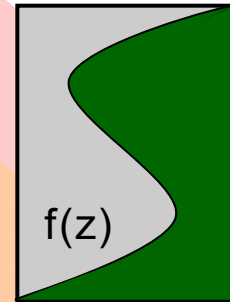
# SAR Tomography



$f(z)$  ... vertical reflectivity function

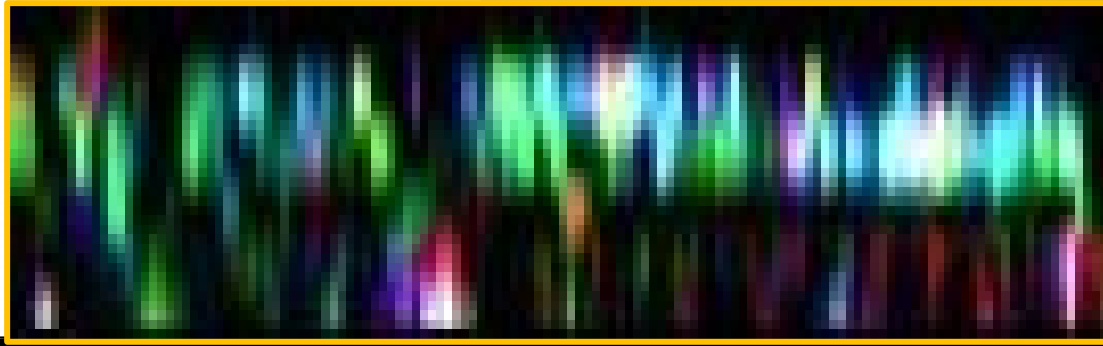


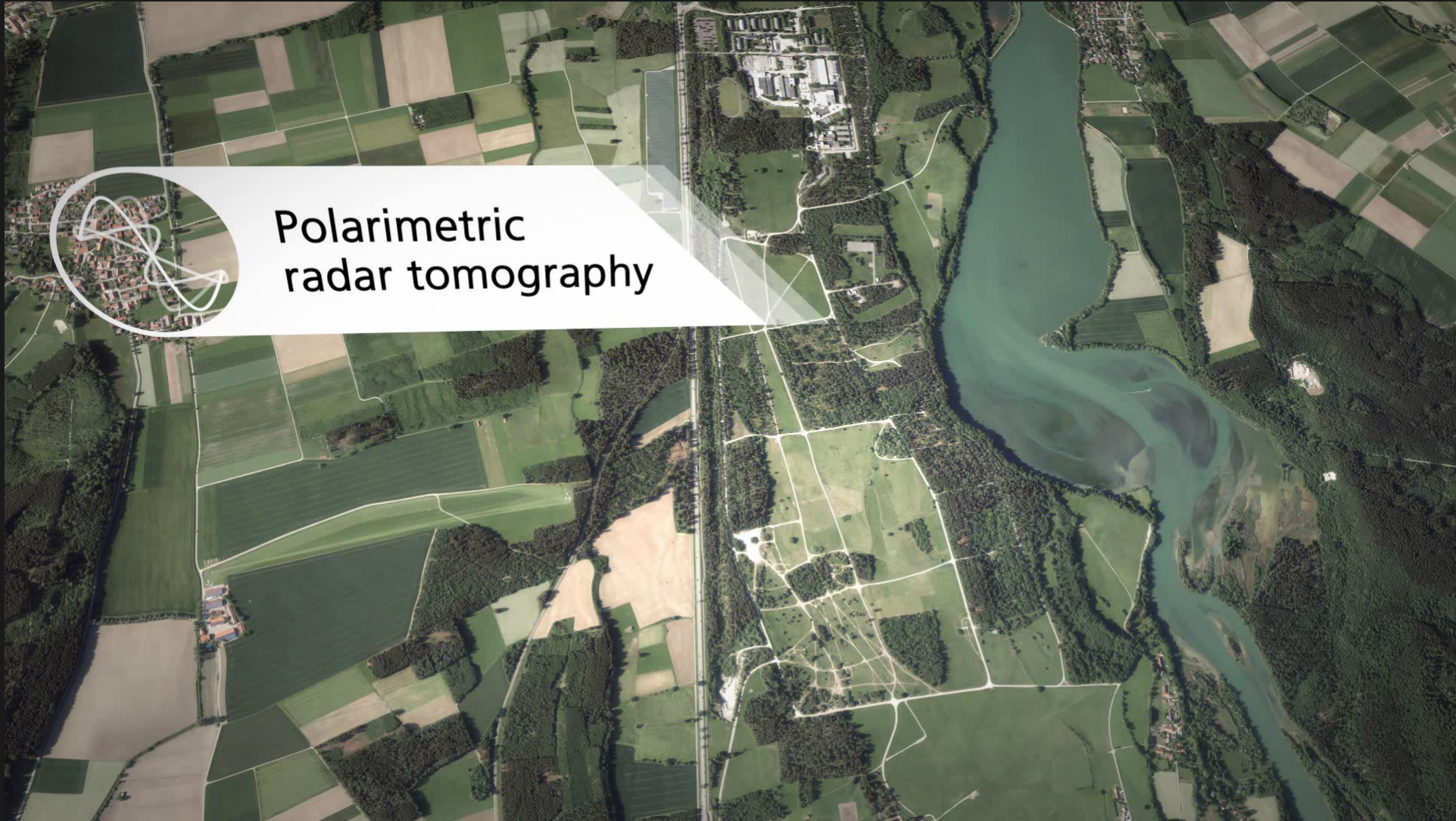
$$\tilde{Y}_{Vol}(f(z)) = e^{ik_z z_0} \frac{\int_0^{h_v} f(z) e^{ik_z z} dz}{\int_0^{h_v} f(z) dz}$$



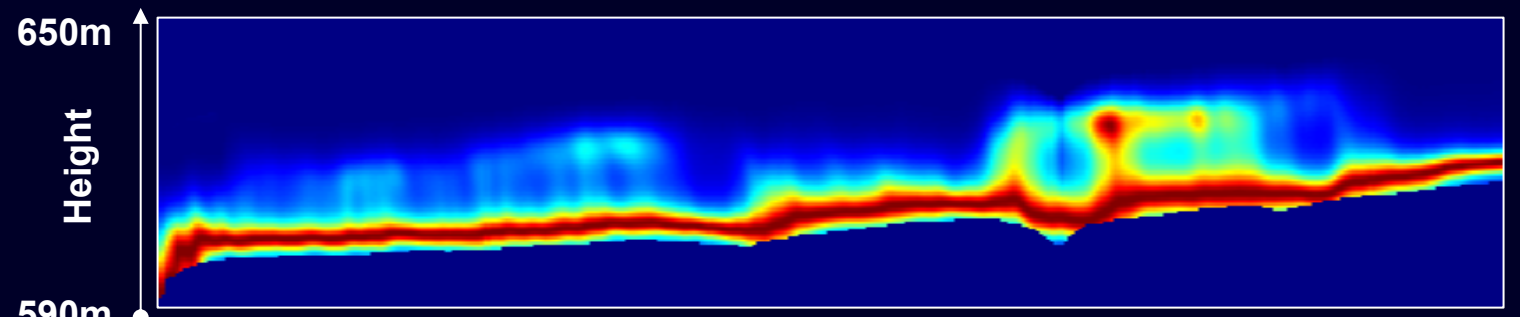
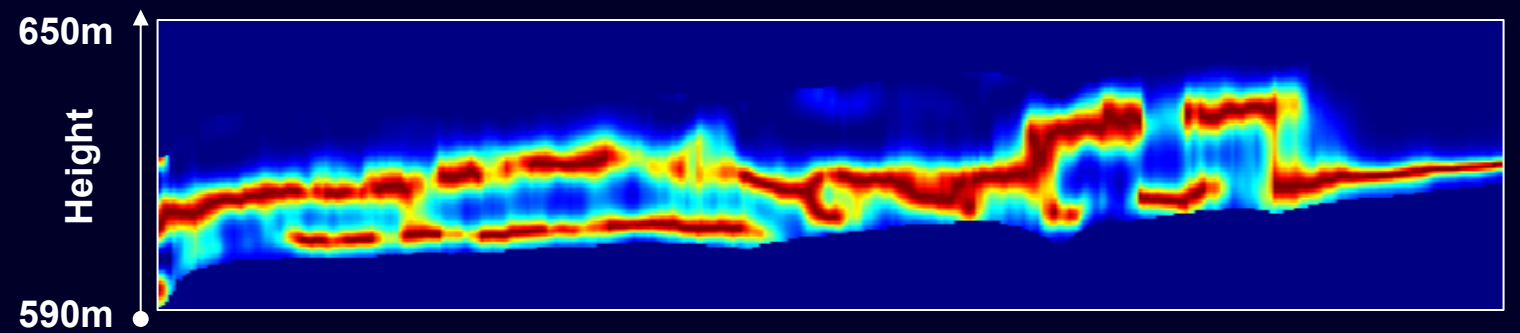
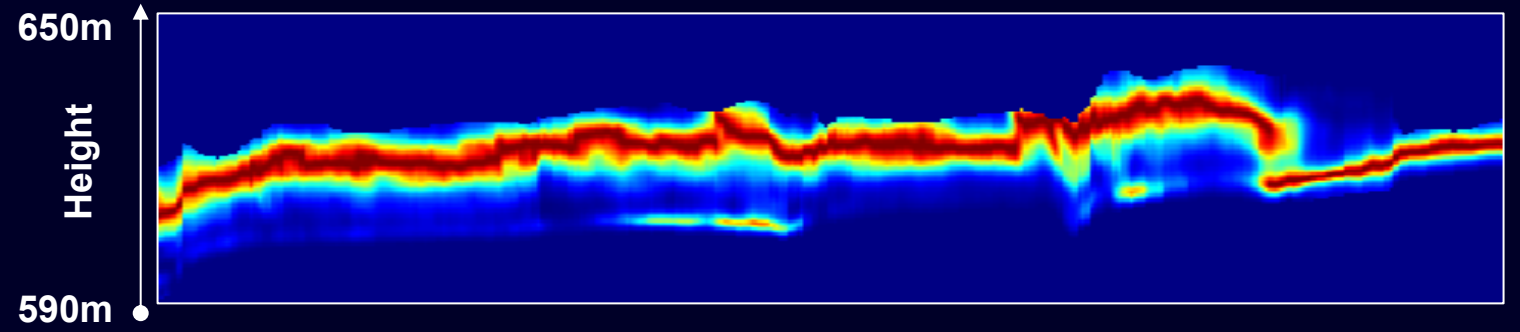
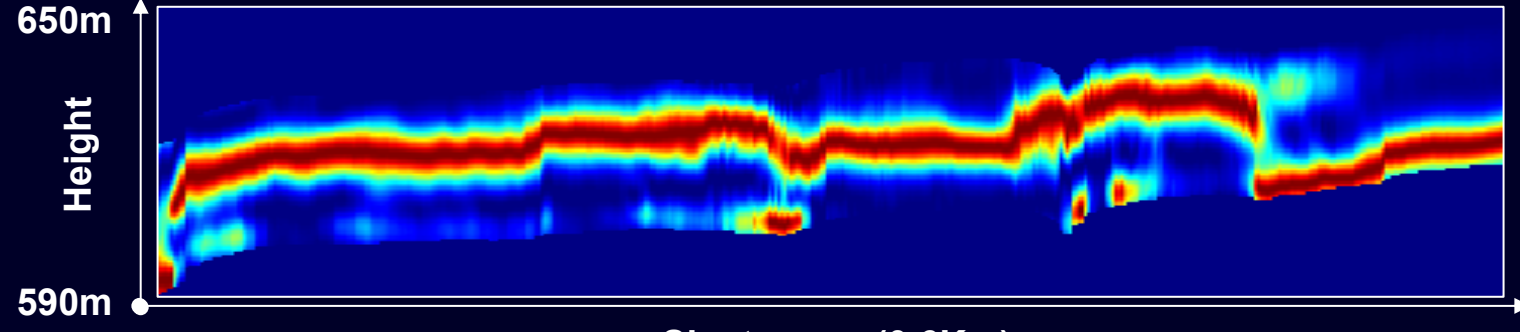
$f(z)$  ... vertical reflectivity function

Vertical Wavenumber:  $k_z = \frac{\kappa \Delta \theta}{\sin(\theta_0)}$



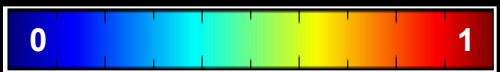


**Polarimetric  
radar tomography**

**P-band****L-band****S-band****X-band**

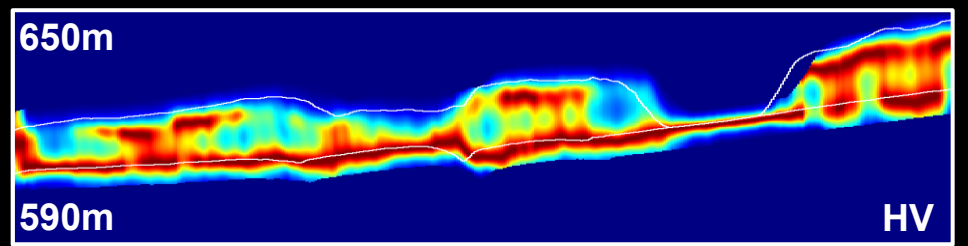
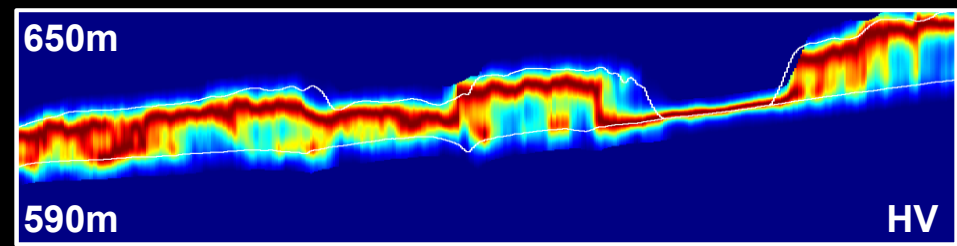
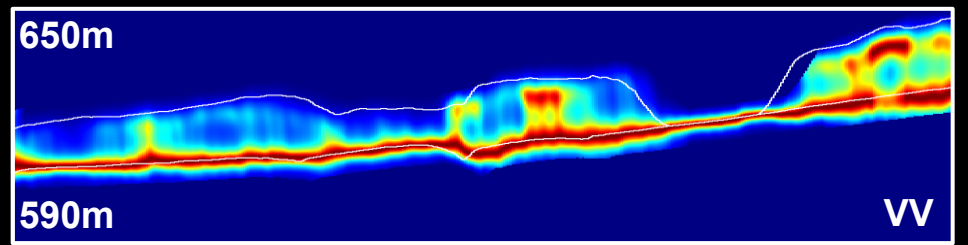
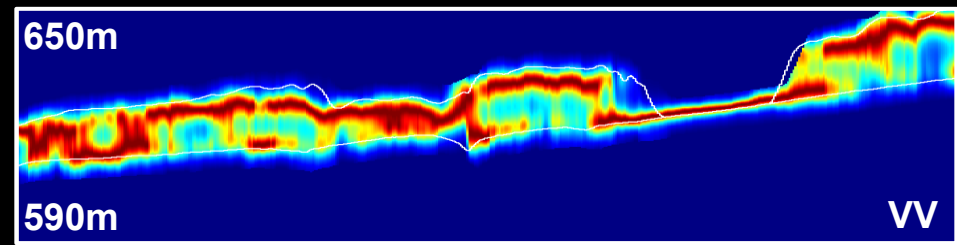
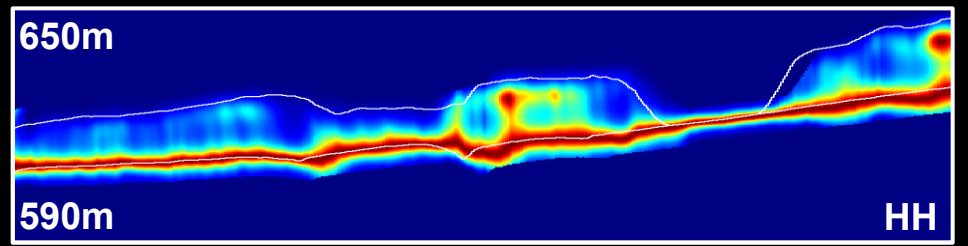
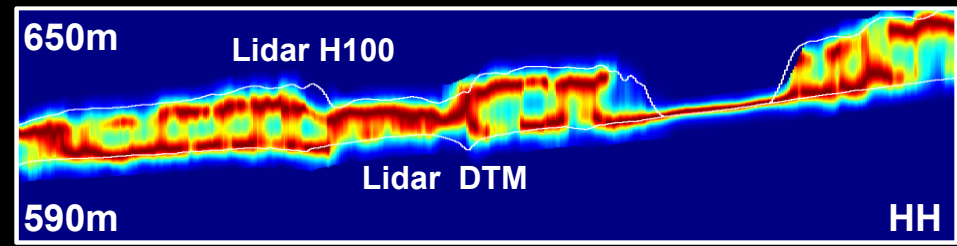
Slant range (0.6Km)

# Dependence on frequency: L- vs P-band



### L-band (23cm)

### P-band (80cm)

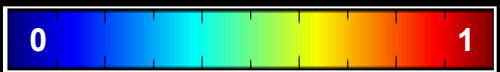


Slant range (~1Km)

Slant range (~1Km)

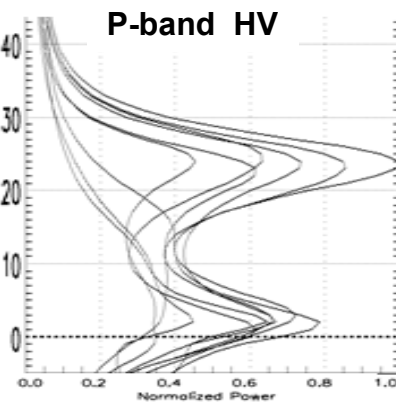
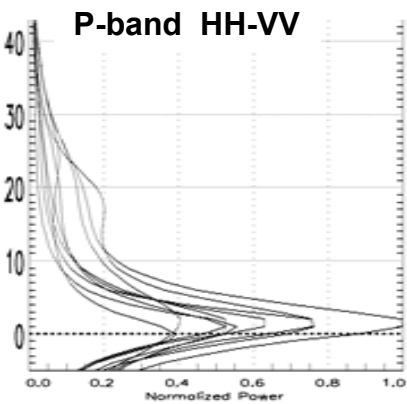
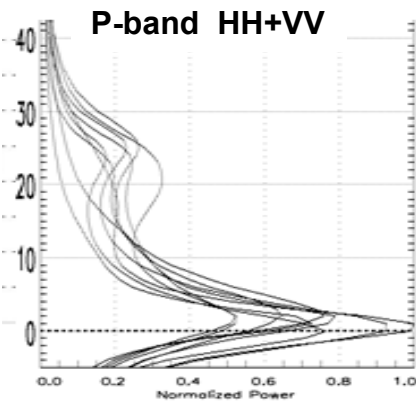
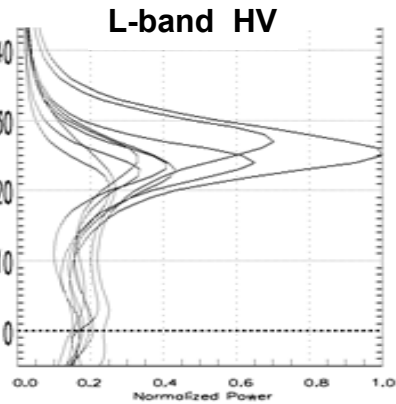
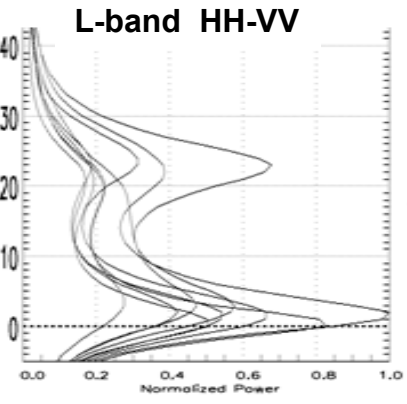
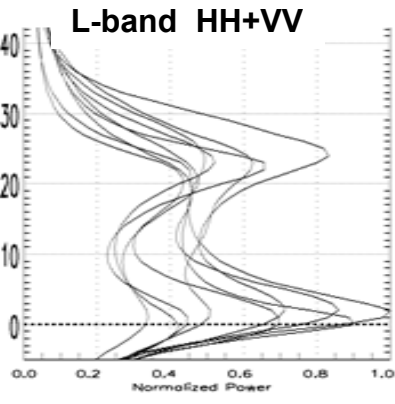
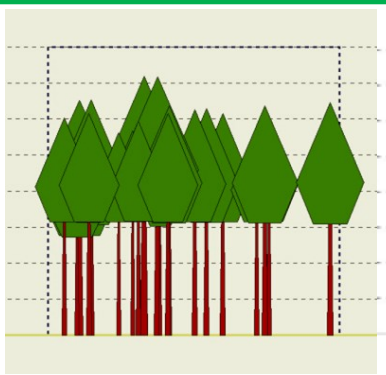
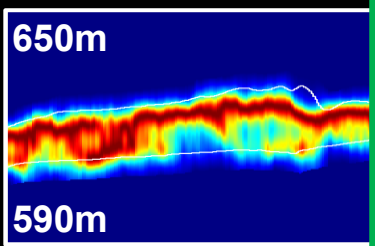
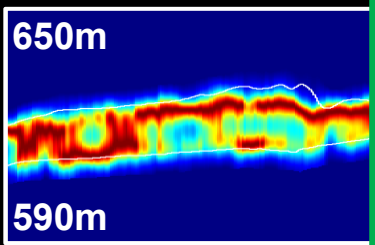
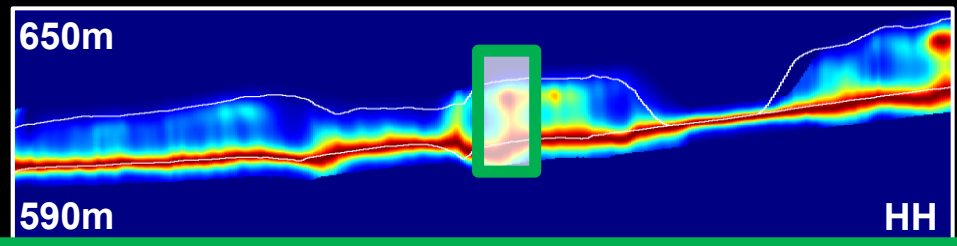
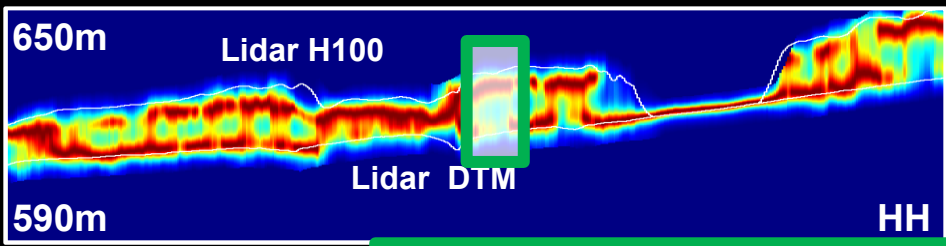


# Dependence on frequency: L- vs P-band



L-band (23cm)

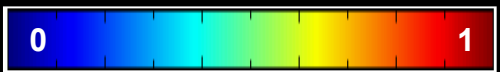
P-band (80cm)



Slant

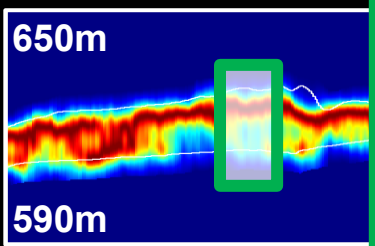
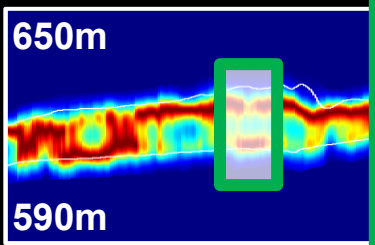
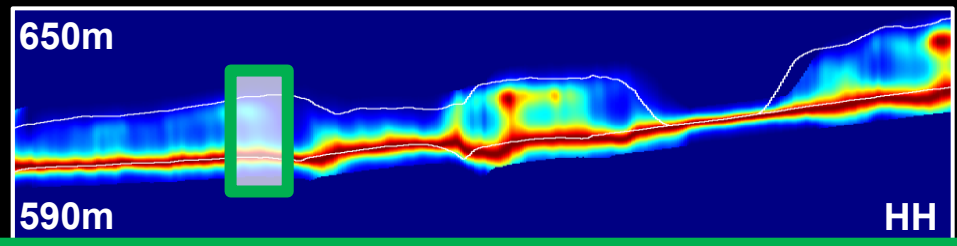
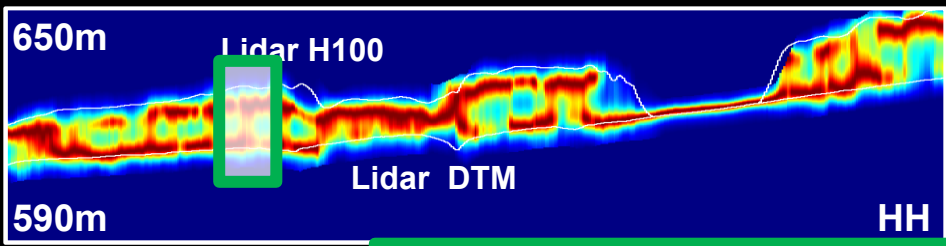


# Dependence on frequency: L- vs P-band



L-band (23cm)

P-band (80cm)

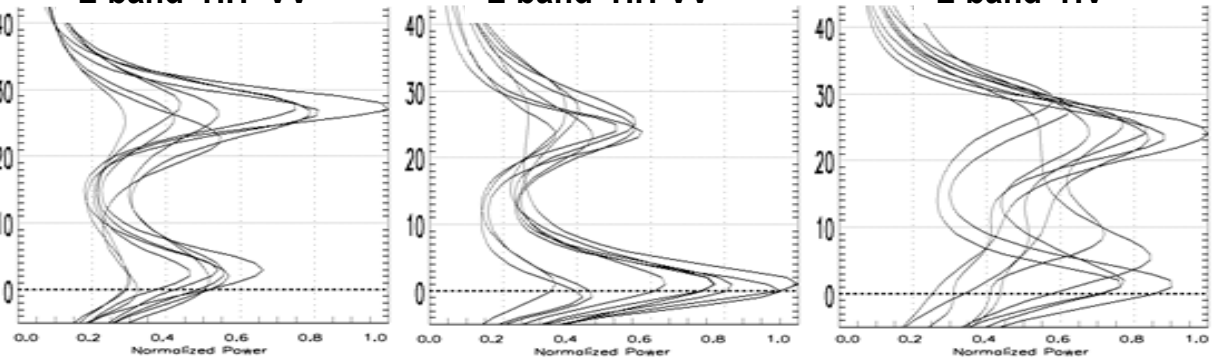


Slant

L-band HH+VV

L-band HH-VV

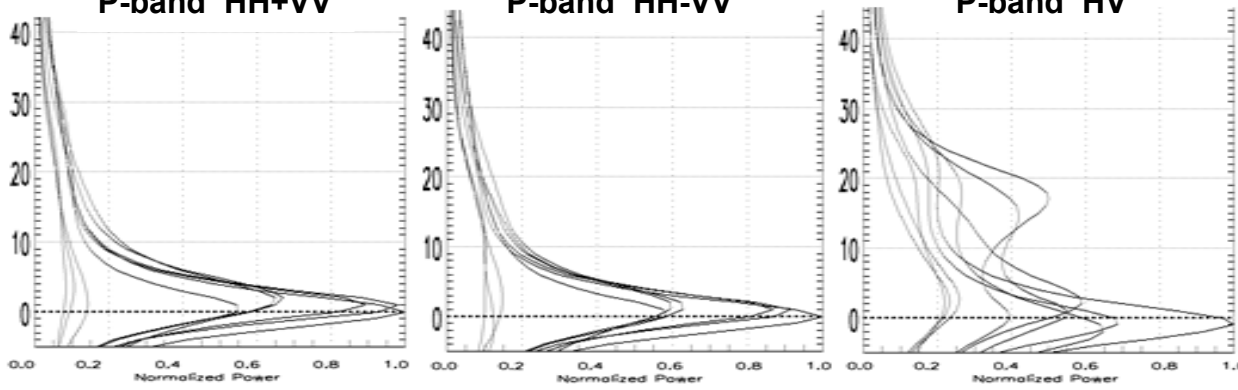
L-band HV



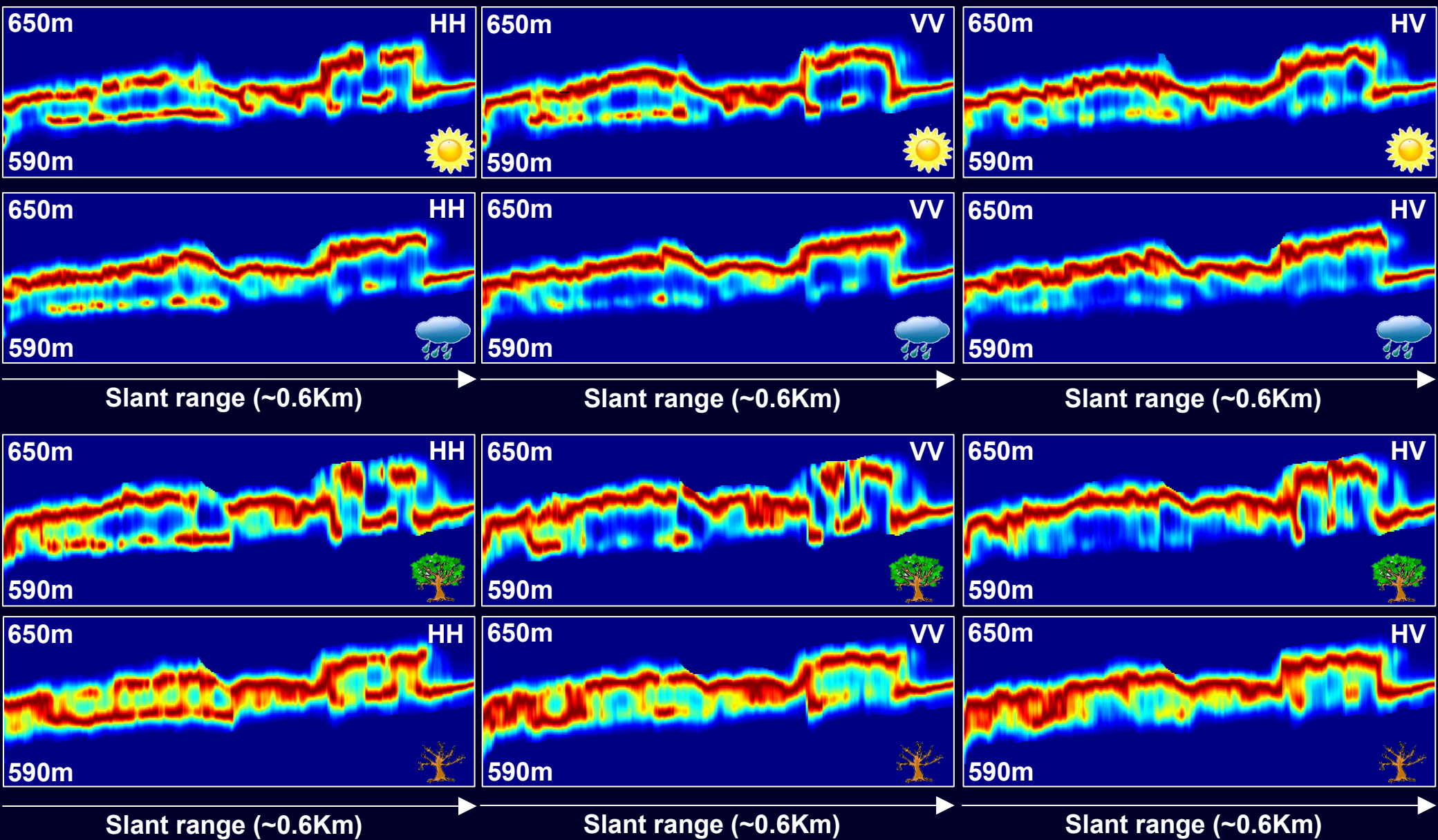
P-band HH+VV

P-band HH-VV

P-band HV



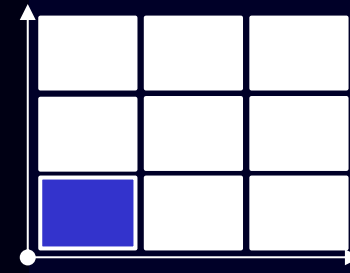
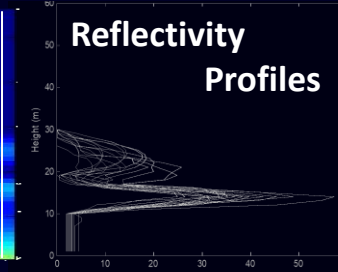
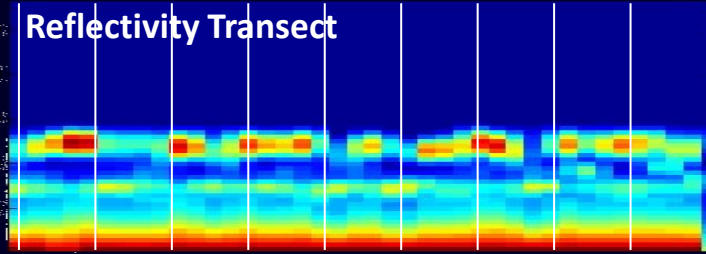
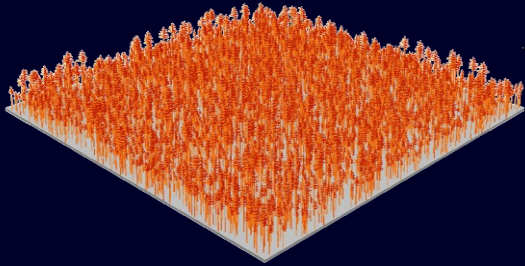
# Temporal variations at L-band (Capon)



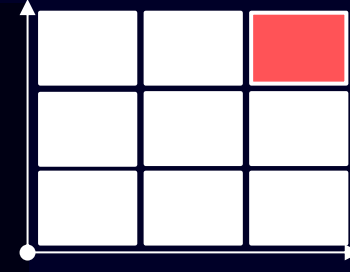
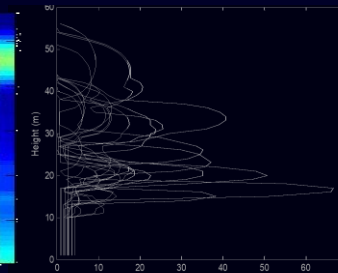
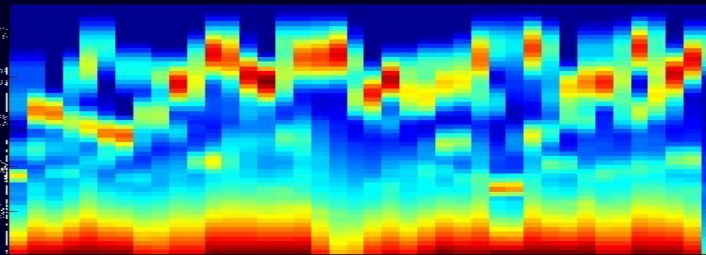
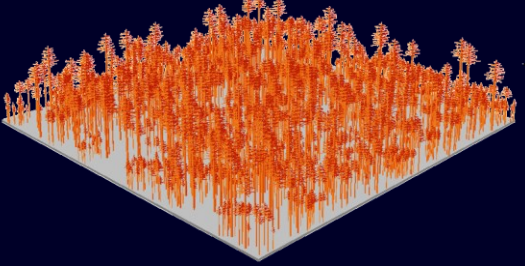
# Forest Structure Characterisation



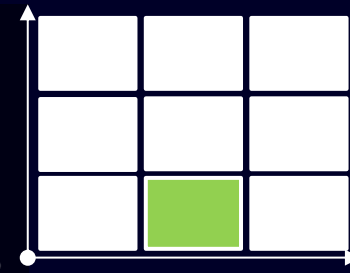
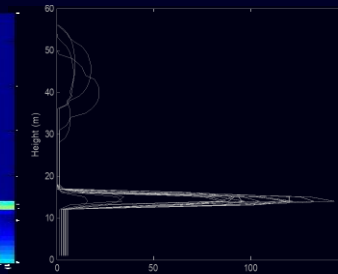
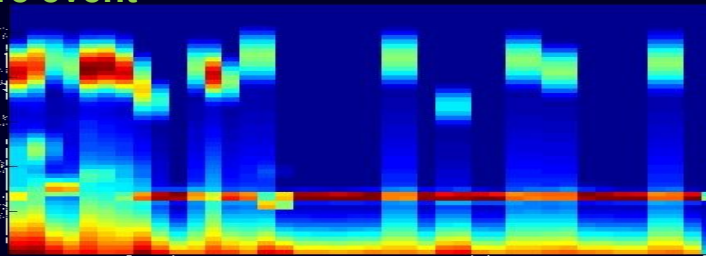
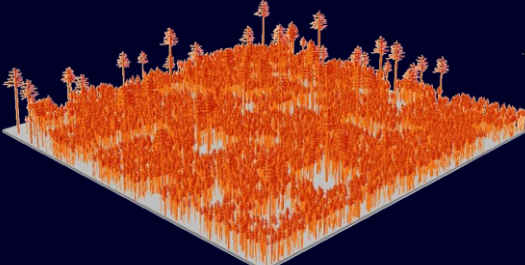
## ▶ Young forest, 50 years old



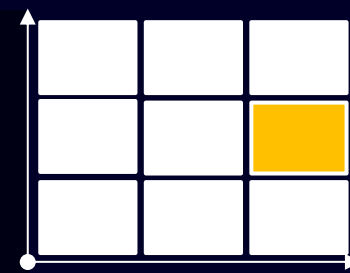
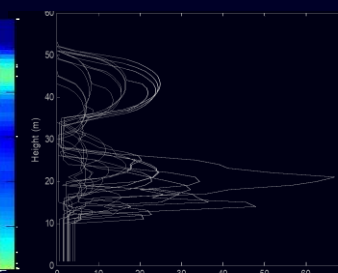
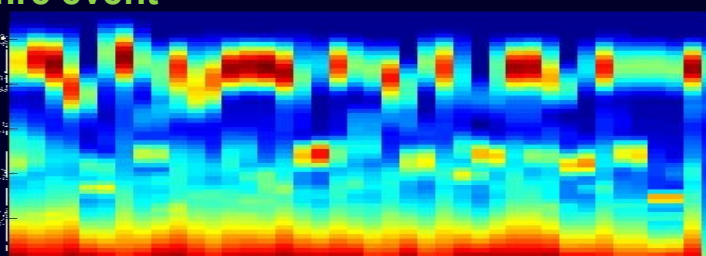
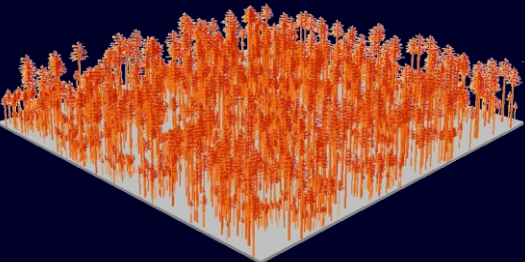
## ▶ Old forest, 500 years old

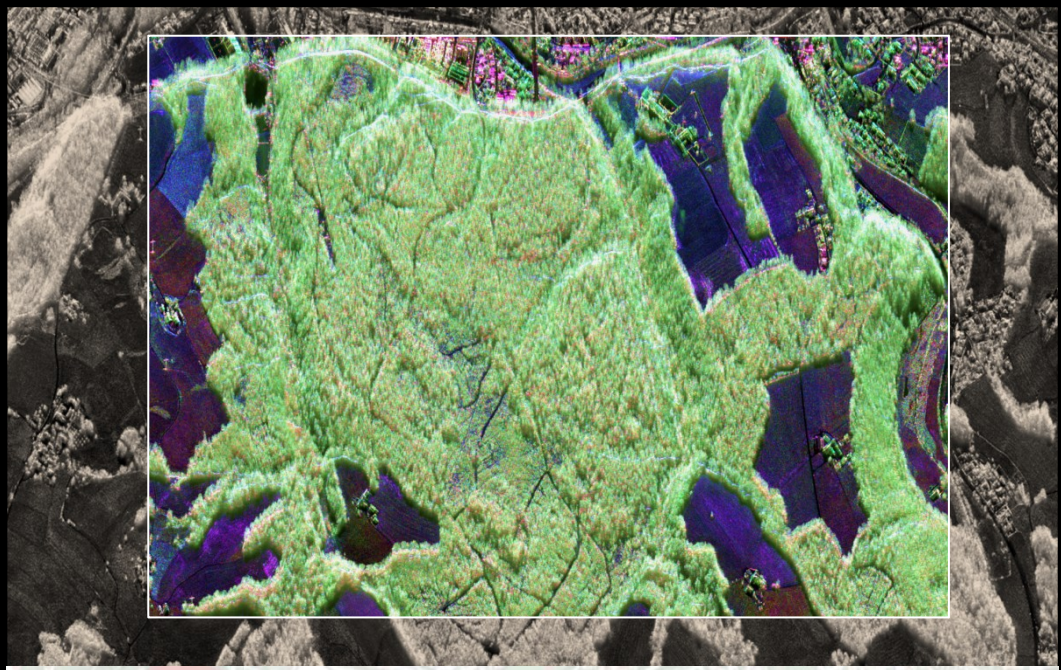


## ▶ Old forest, 10 years after a fire event

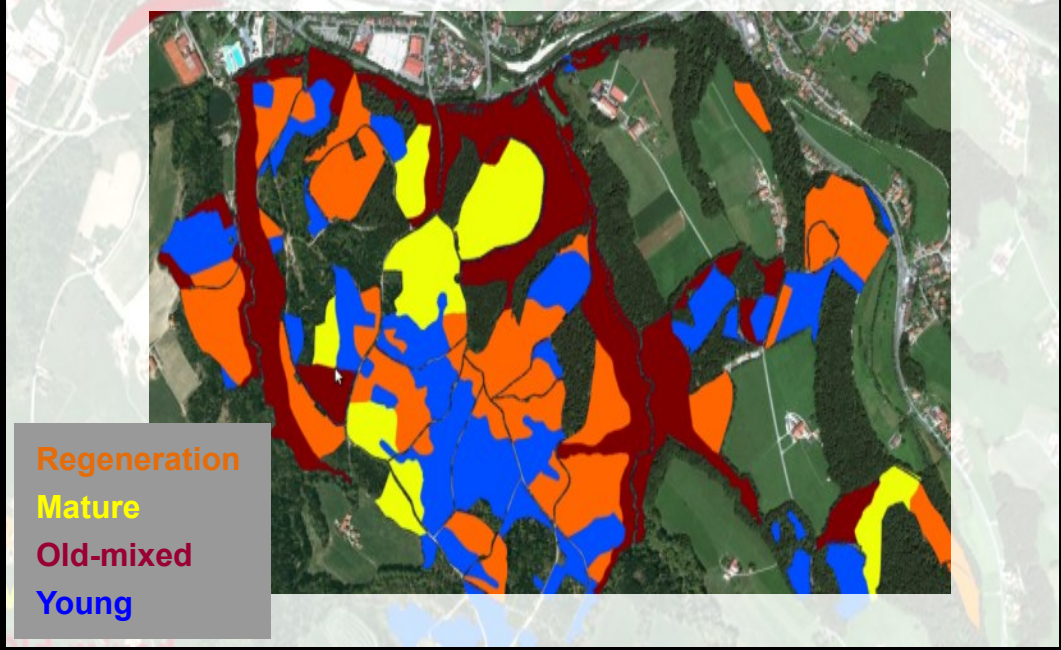
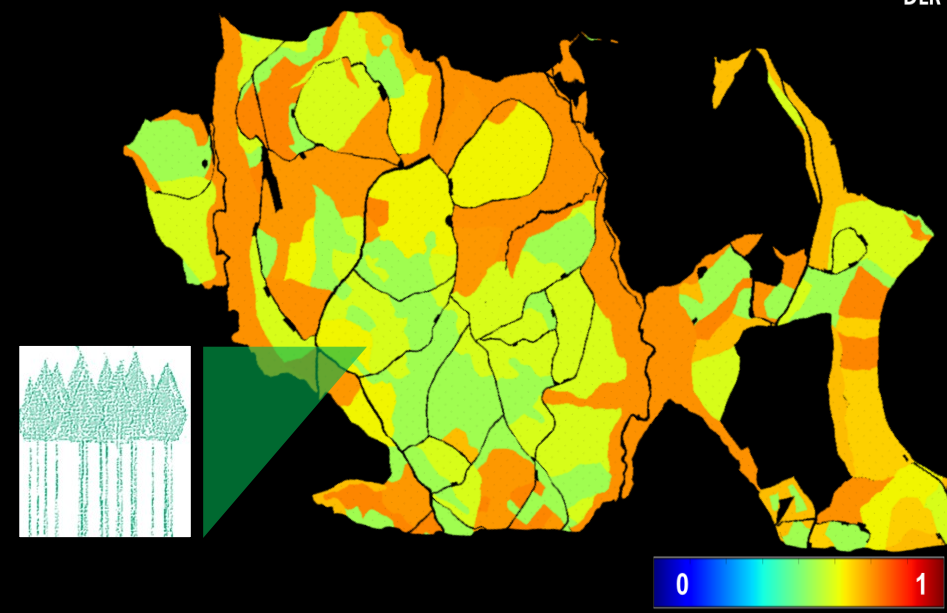


## ▶ Old forest, 200 years after a fire event

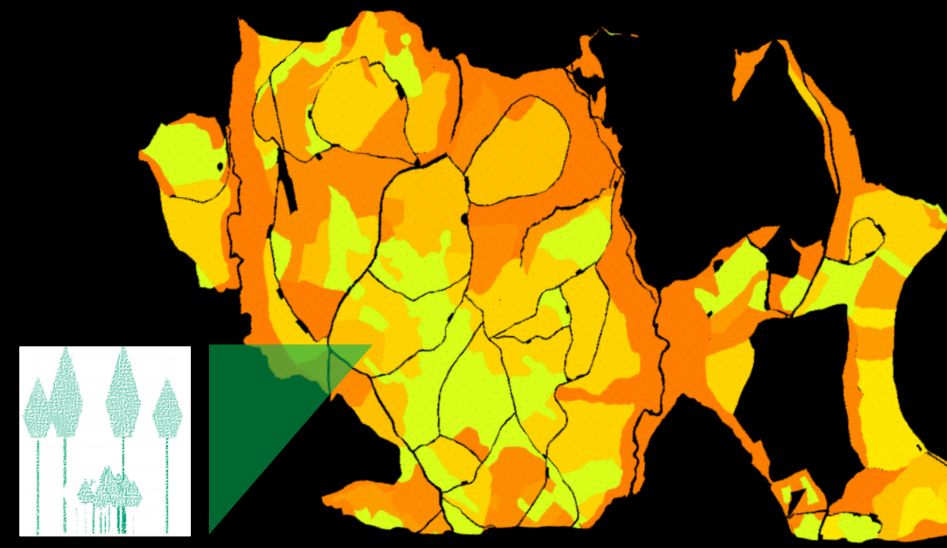




Vertical structure CM (Radar 2008)

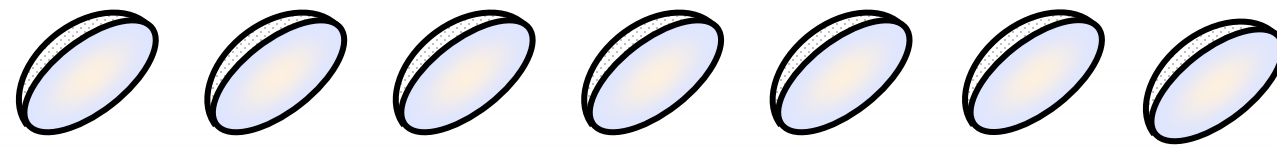


Vertical structure CM (Radar 2012)



Forest Structure Classification (25x25 m): Traunstein, Germany, 2008 / 2012

# SAR Tomography ... beyond profiles



Ground Layer Volume Layer

Single-pol coherences

$$f(z) = P_G f_G(z) + P_V f_V(z) \quad \Rightarrow \quad R = P_G \Gamma_G + P_V \Gamma_V$$

Assumption :  $f_G(z) = \delta(z - z_G)$

$P_G, P_V$  : backscattering powers (single-pol)

$\Gamma_G, \Gamma_V$  : multi-baseline coherence matrices

From single-pol to full-pol, Random volume assumption:

$$R_P = C_G \otimes \Gamma_G + C_V \otimes \Gamma_V \quad \text{Sum of Kronecker Products (SKP)}$$

$C_G, C_V$  : polarimetric covariance matrices

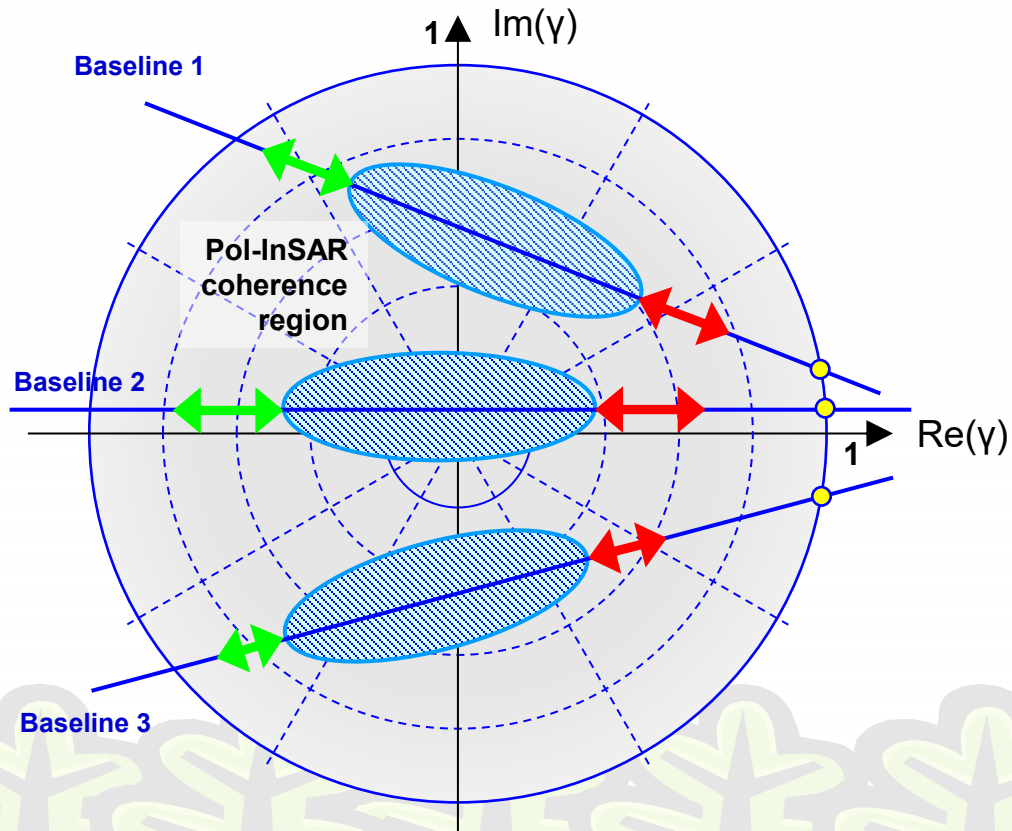
In both single and multi polarimetric cases, even under an RVoG assumption and independently on the number of baselines, the separation of ground / volume InSAR coherences and polarimetric covariances does not admit a unique solution !!

[T. Marzetta, IEEE-Proc. 1983 – S. Tebaldini, IEEE-TGARS 2009]



# The Sum-of-Kronecker-Products

- ▶ It extends Pol-InSAR concepts to TomoSAR
- ▶ Based on simple algebraic tools



$$\mathbf{R}_P = \mathbf{C}_G \otimes \Gamma_G + \mathbf{C}_V \otimes \Gamma_V \quad \text{Sum of Kronecker Products (SKP)}$$

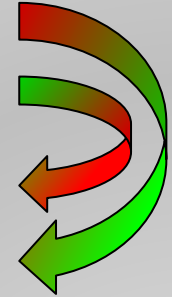
$$\hat{\Gamma}_{G,V}, \hat{\mathbf{C}}_{G,V} = \arg \min_{\Gamma_{G,V}, \mathbf{C}_{G,V}} \|\mathbf{R}_P - [\mathbf{C}_G \otimes \Gamma_G + \mathbf{C}_V \otimes \Gamma_V]\|^2$$

$$\Gamma_G = a \mathbf{R}_1 + (1 - a) \mathbf{R}_2$$

$$\Gamma_V = b \mathbf{R}_1 + (1 - b) \mathbf{R}_2$$

$$\mathbf{C}_G = [(1 - b) \mathbf{C}_1 - b \mathbf{C}_2] / (a - b)$$

$$\mathbf{C}_V = [-(1 - a) \mathbf{C}_1 + a \mathbf{C}_2] / (a - b)$$



$\mathbf{R}_1, \mathbf{R}_2$  from SVD of a permutation of  $\mathbf{R}_P$

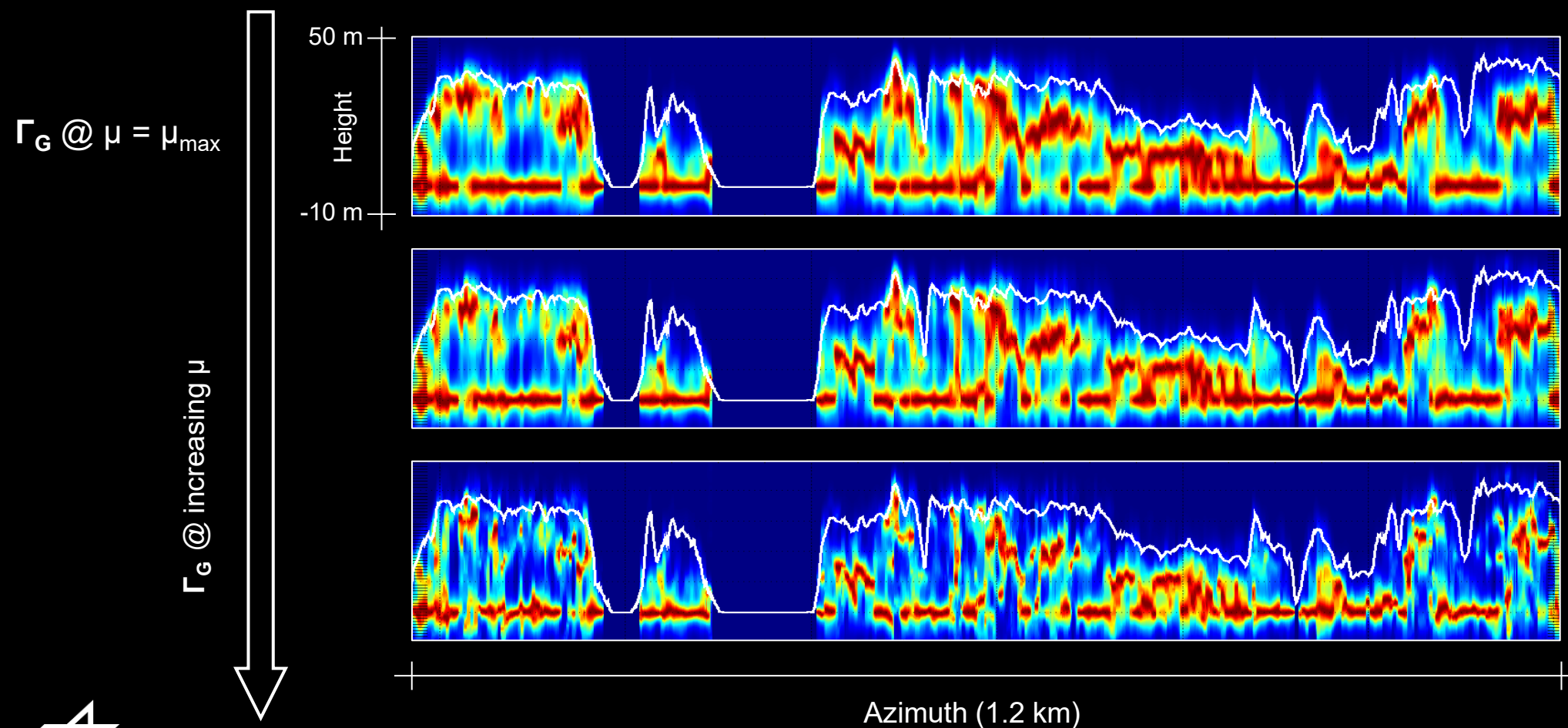
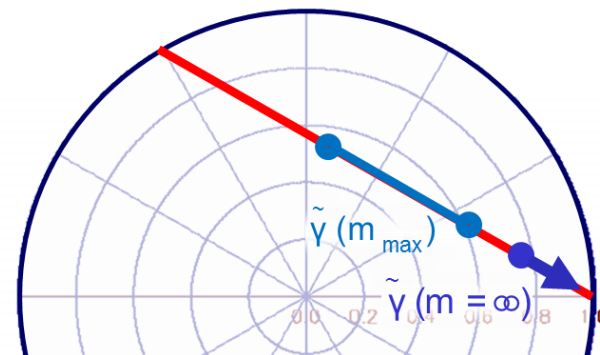
(a,b) are free parameters, bounded in order to provide positive (semi-)definite matrices

- ▶ It is an unconstrained Least Squares fitting, with no additional external knowledge
- ▶ The separation ambiguity is transferred to two unknowns scalar parameters



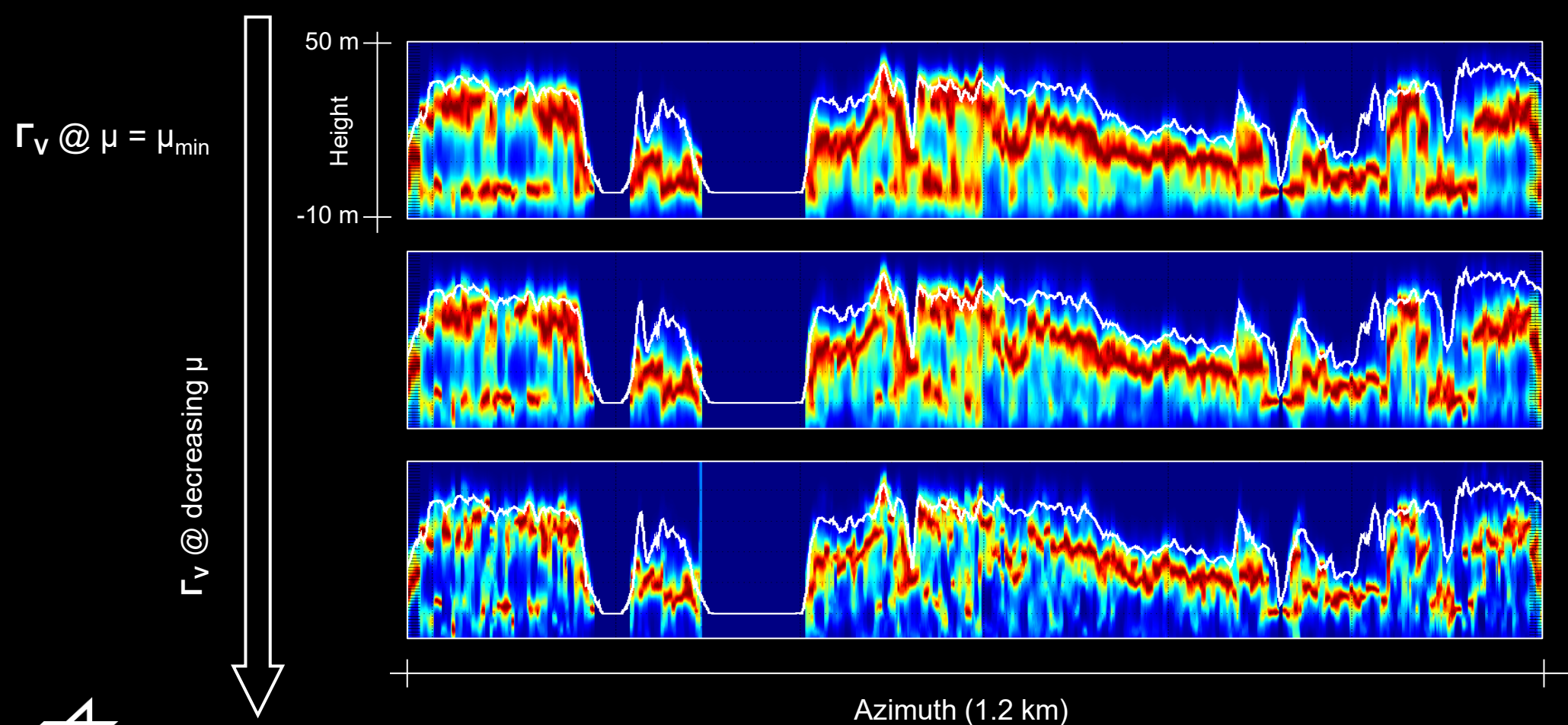
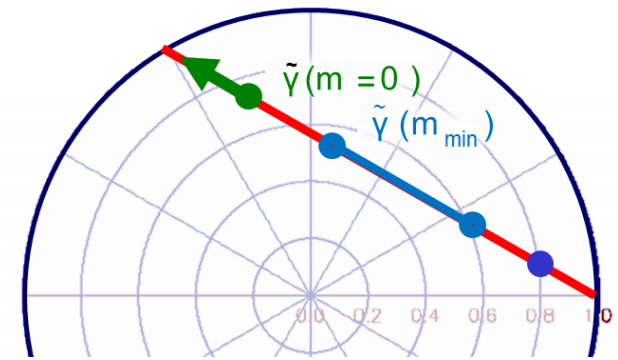
# Ground solutions

All the feasible ground coherence matrices are on the Pol-InSAR line segments outside the coherence region, under the positive (semi-)definiteness constraint.

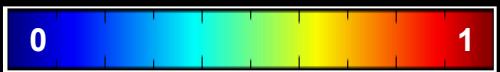


# Volume solutions

All the feasible volume coherence matrices are on the Pol-InSAR line segments outside the coherence region, under the positive (semi-)definiteness constraint.

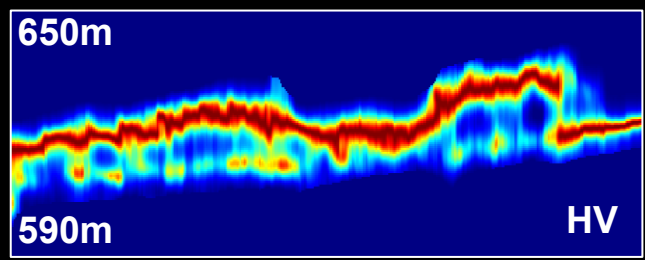
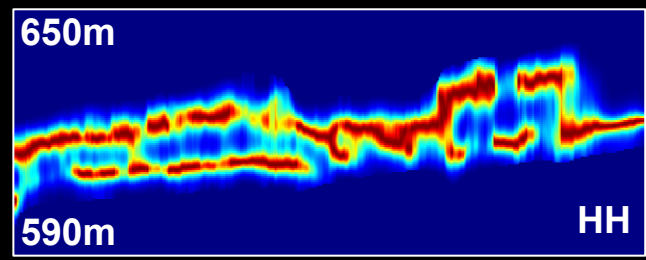


# Dependence on polarization & frequency

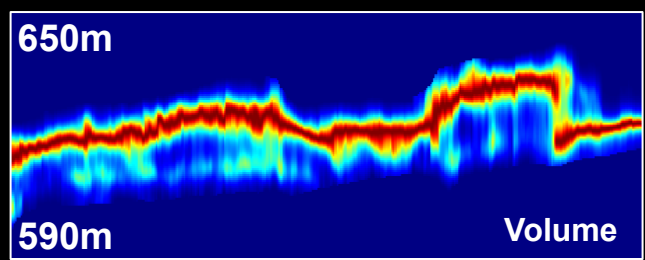
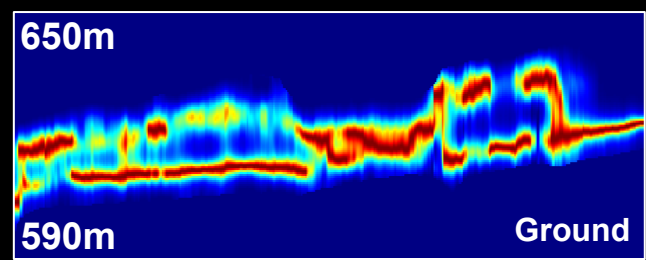


L-band

Lexicographic basis  
HH vs HV



Maximum ground vs  
Maximum volume

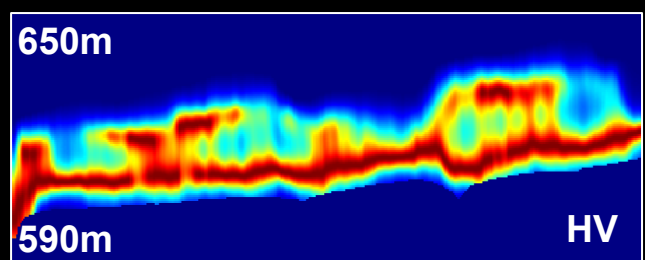
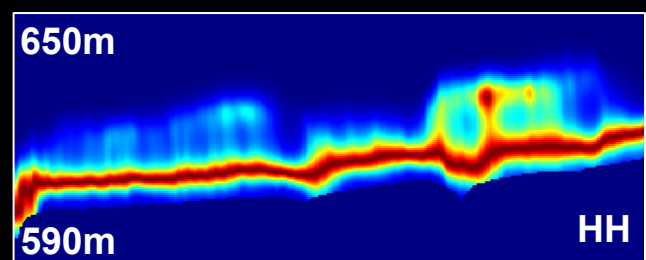


Slant range (~0.6Km)

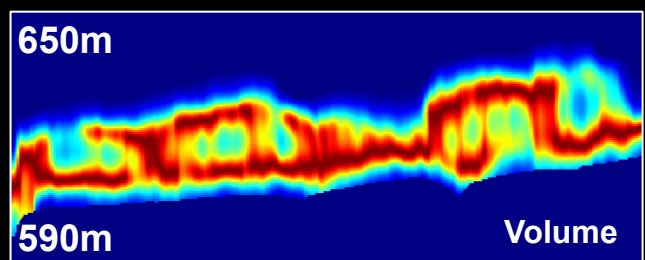
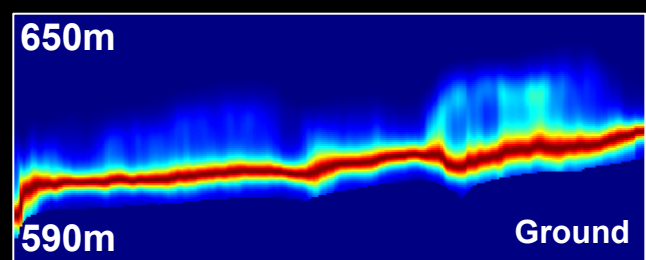
Slant range (~0.6Km)

P-band

Lexicographic basis  
HH vs HV



Maximum ground vs  
Maximum volume



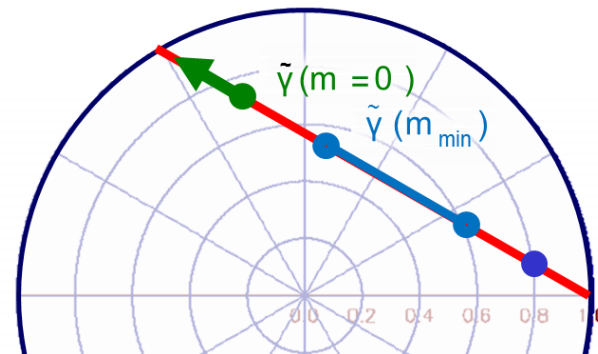
Slant range (~0.6Km)

Slant range (~0.6Km)

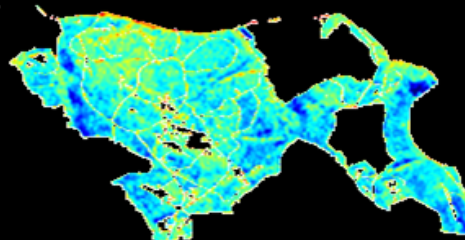


# Ground powers / polarimetry solutions

All the feasible ground covariance matrices are on the Pol-InSAR line segments outside the coherence region, under the positive (semi-)definiteness constraint.



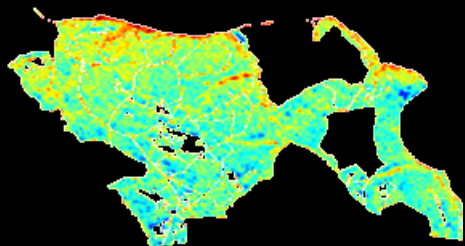
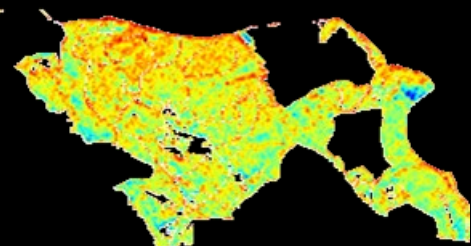
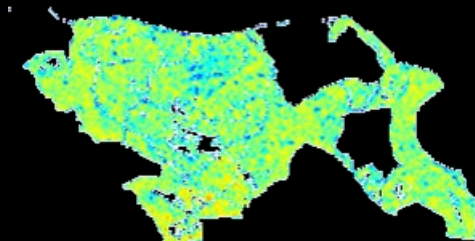
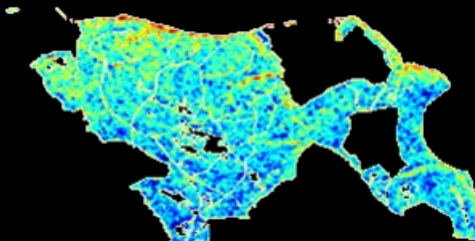
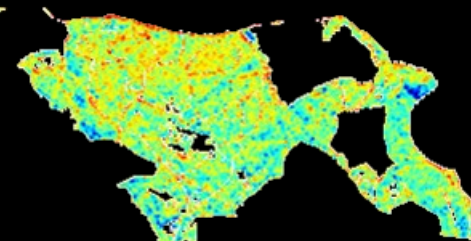
Capon estimates  
With known ground



Ground power, HH

Ground power, HV

Ground entropy



$C_G$  @ decreasing  $\mu$

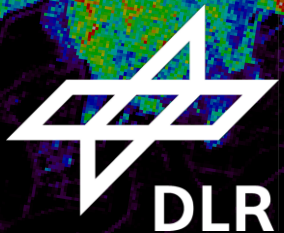


# Polarimetric SAR Interferometry

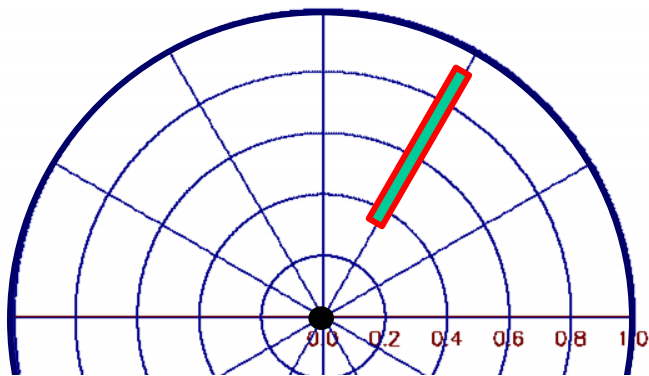
**Konstantinos P. Papathanassiou, Matteo Pardini**

German Aerospace Center (DLR)  
Microwaves and Radar Institute (DLR-HR)

[kostas.papathanassiou@dlr.de](mailto:kostas.papathanassiou@dlr.de)  
[matteo.pardini@dlr.de](mailto:matteo.pardini@dlr.de)

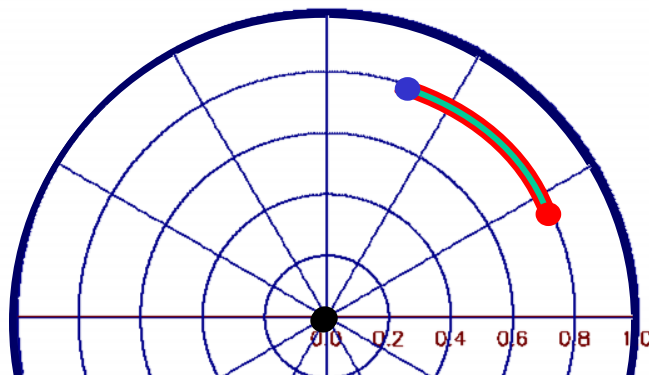


# Coherence Region (CR) Interpretation



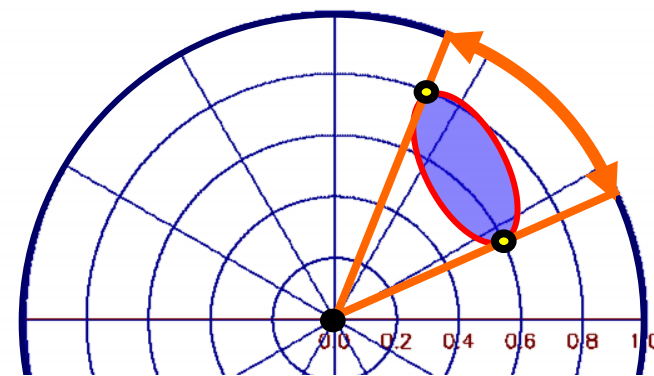
## Radial Shaped CR

i.e. InSAR coherence amplitude changes with polarisation but not the location of the phase center.



## Arc Shaped CR

i.e. InSAR phase center location changes with polarisation but not the absolute value of the coherence amplitude.



## Elliptical Shaped CR

i.e. InSAR coherence magnitude and phase center location changes with polarisation.

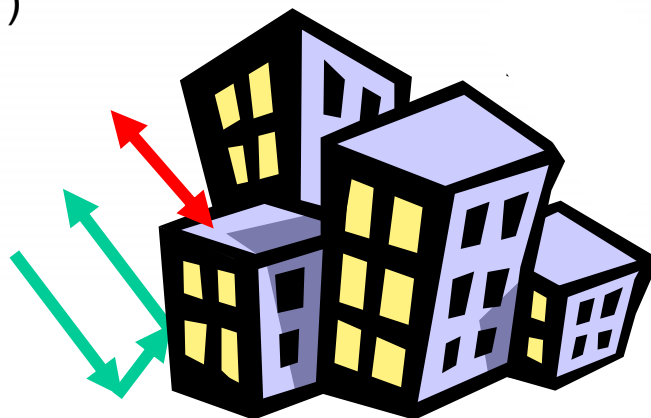
## Surface Scattering

$$\tilde{\gamma}(\vec{w}) = Y_{\text{SNR}}(\vec{w}) \tilde{\gamma}_{\text{Vol}}^{\tilde{\gamma}_{\text{Vol}}:=1} = Y_{\text{SNR}}(\vec{w})$$



## (Polarised) Coherent scatterers

at different heights



## (Depolarising) Scatterers

at different heights

