

8th ESA Advanced Course on Radar Polarimetry 2026

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SAR Interferometry basics

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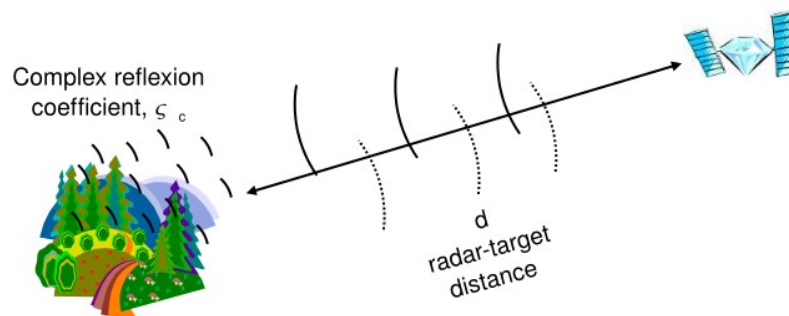
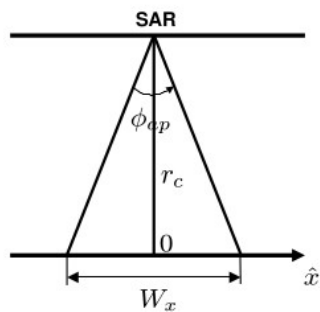
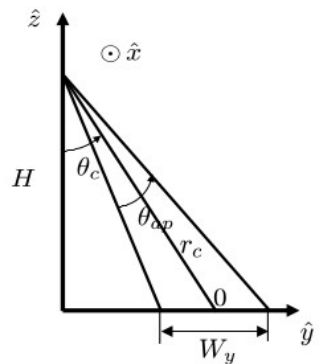
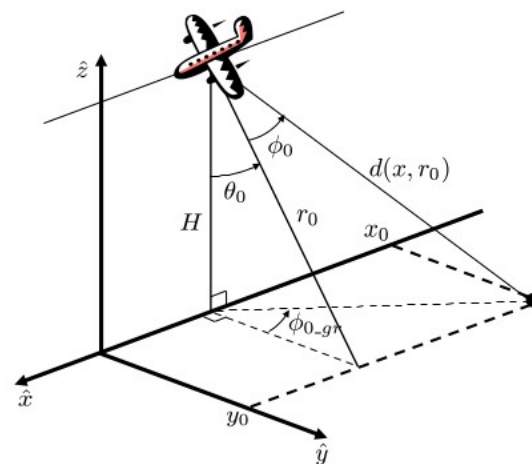
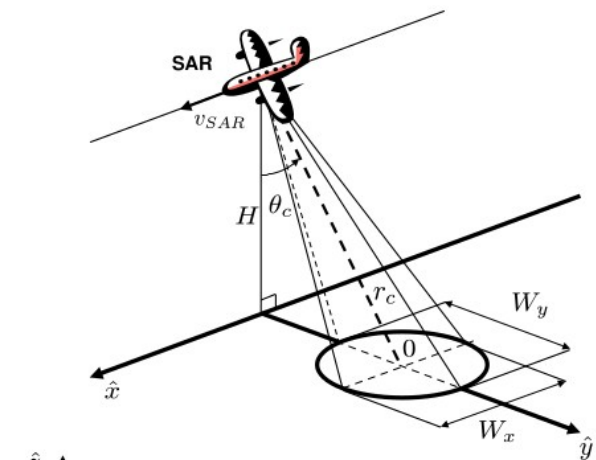
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SAR Interferometry basics

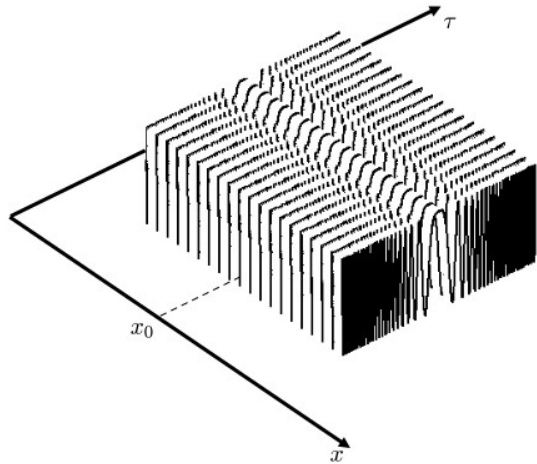
A geometrical approach

SAR acquisition geometry





SAR raw signal for a single scatterer



- Scatterer's location: (x_0, r_0)
- Reflection coefficient: a_c
- Baseband pulsed waveform: $p(\tau)$
- Migrations are neglected

$$s_r(x, \tau) = a_c$$

$$p(\tau - \tau_0(x)) \quad \leftarrow \text{delayed pulse}$$

$$\exp(-j2\pi f_c \tau_0(x)) \quad \leftarrow \text{phase delay}$$

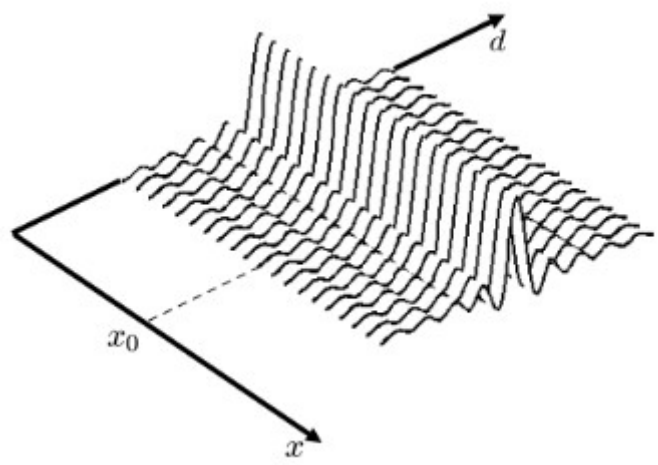
$$\text{rect}\left(\frac{x - x_0}{W_x}\right)$$

azimuth-varying delay

$$\tau_0(x) = \frac{2}{c}d_0(x) = \frac{2}{c}\sqrt{r_0^2 + (x - x_0)^2}$$



Range-focused SAR signal for a single scatterer



$$s_{rf}(x, d) = a_c$$

$$h_r(d - d_0(x)) \quad \leftarrow \text{range impulse response}$$

$$\exp(-jk_c d_0(x))$$

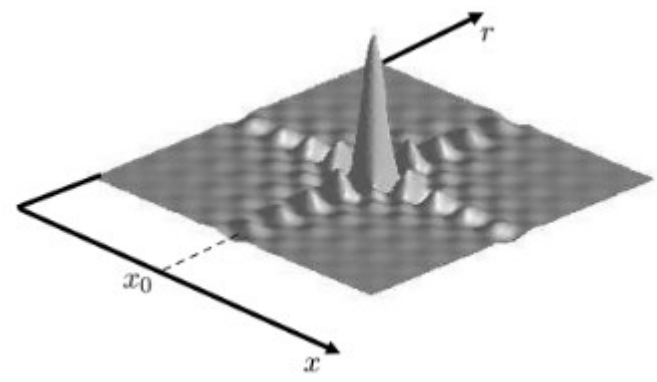
$$\text{rect}\left(\frac{x - x_0}{W_x}\right)$$

radar-scatterer distance

$$d_0(x) = \sqrt{r_0^2 + (x - x_0)^2}$$

- Scatterer's location: (x_0, r_0)
- Reflection coefficient: a_c
- Migrations are neglected

2D SAR image of a single scatterer



$$s_{rf}(x, r) = a_c$$

- $h_r(r - r_0)$ ← range impulse response
- $h_a(x - x_0)$ ← azimuth impulse response
- $\exp(-jk_c r_0)$ ← phase delay

- Scatterer's location: (x_0, r_0)
- Reflection coefficient: a_c

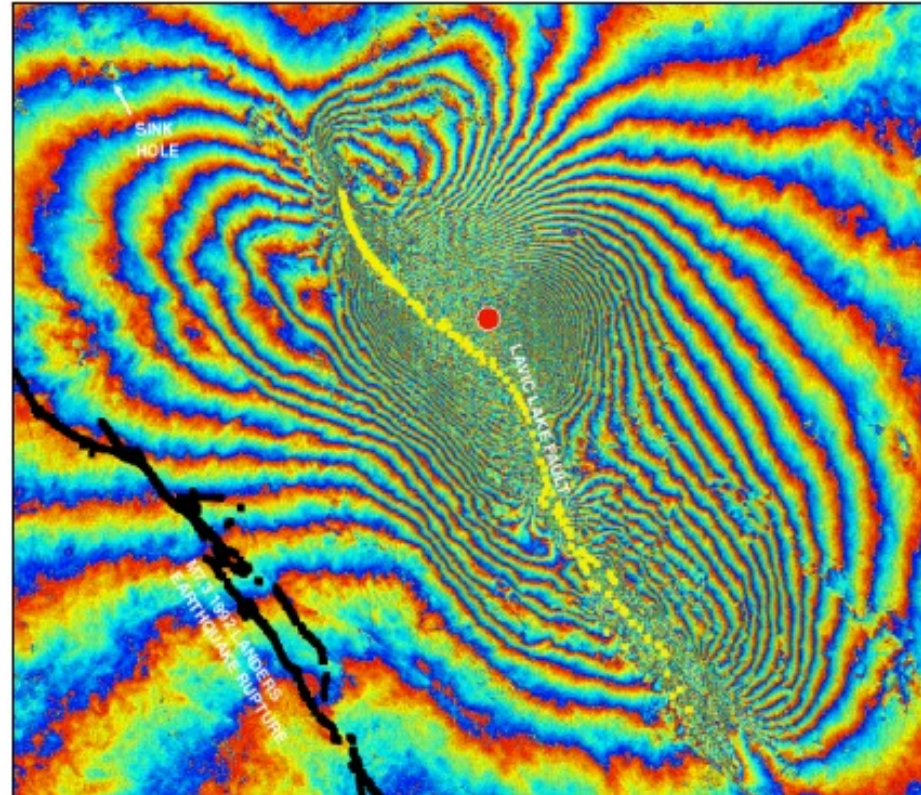
r_0 can be measured using :

- $|s_{rf}(x, r)|$: precision $\approx \delta r$
- using $\arg(\exp(jk_r r_0))$: precision $\approx \lambda_c/2$

Phase sensitivity to motion in range

Differential interferogram of the "Hector Mine" earthquake (California), 16. October 1999

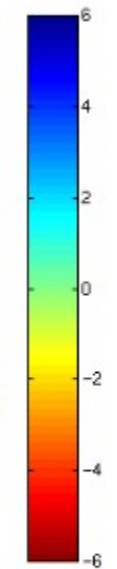
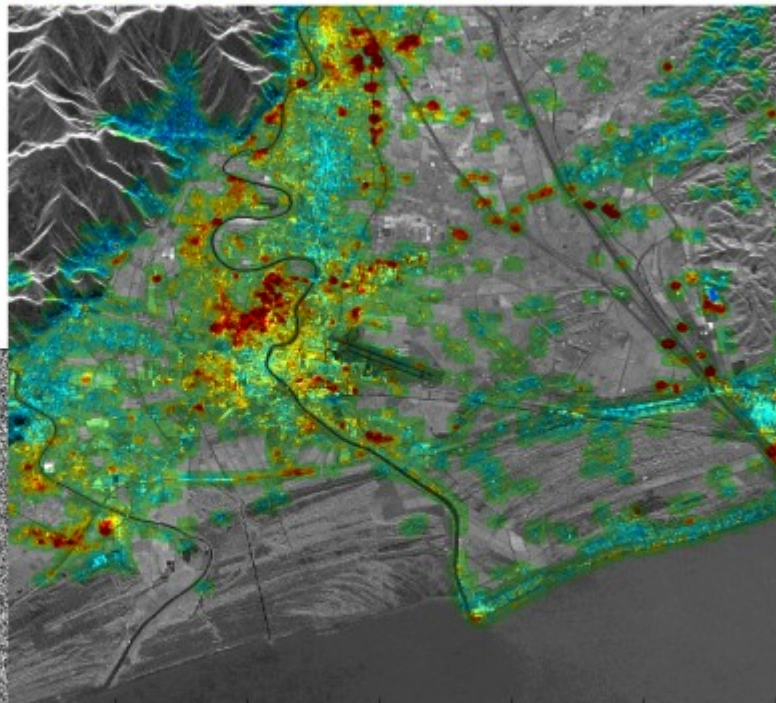
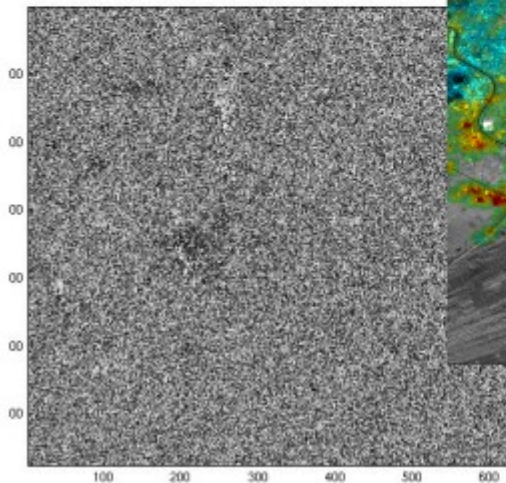
Each colour cycle represents a deformation of 2.8 cm.
temporal baseline = 35 days



Phase sensitivity to motion in range

City Deformation Mapping: Pisa

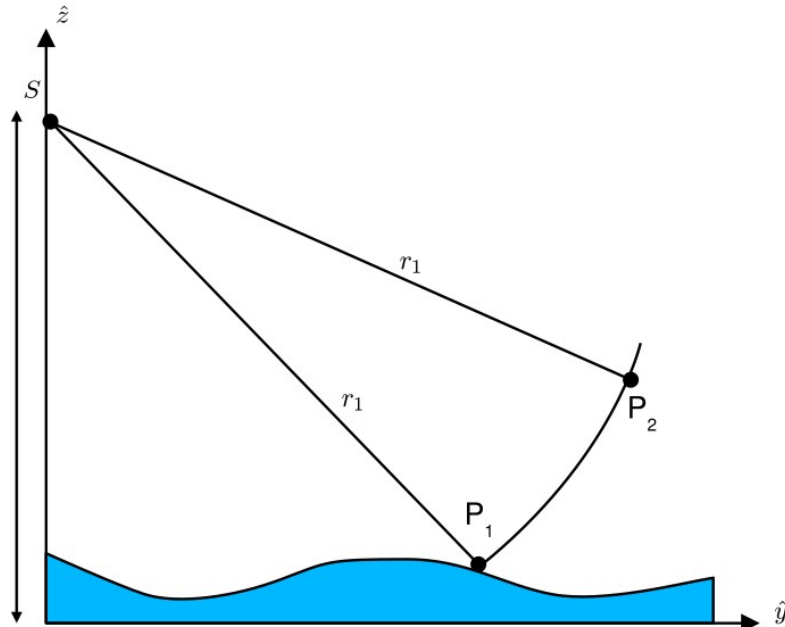
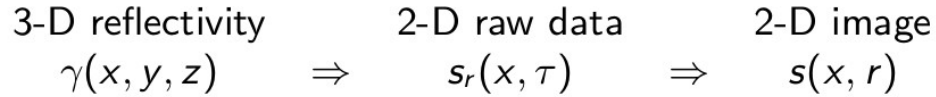
Permanent Scatterer
(analysis based on 45
ERS images):



Classical
differential interferogram

3D mapping using SAR

Cylindrical ambiguity of 2D SAR imaging



Mapping of $\gamma(x_0, y_0, z_0)$

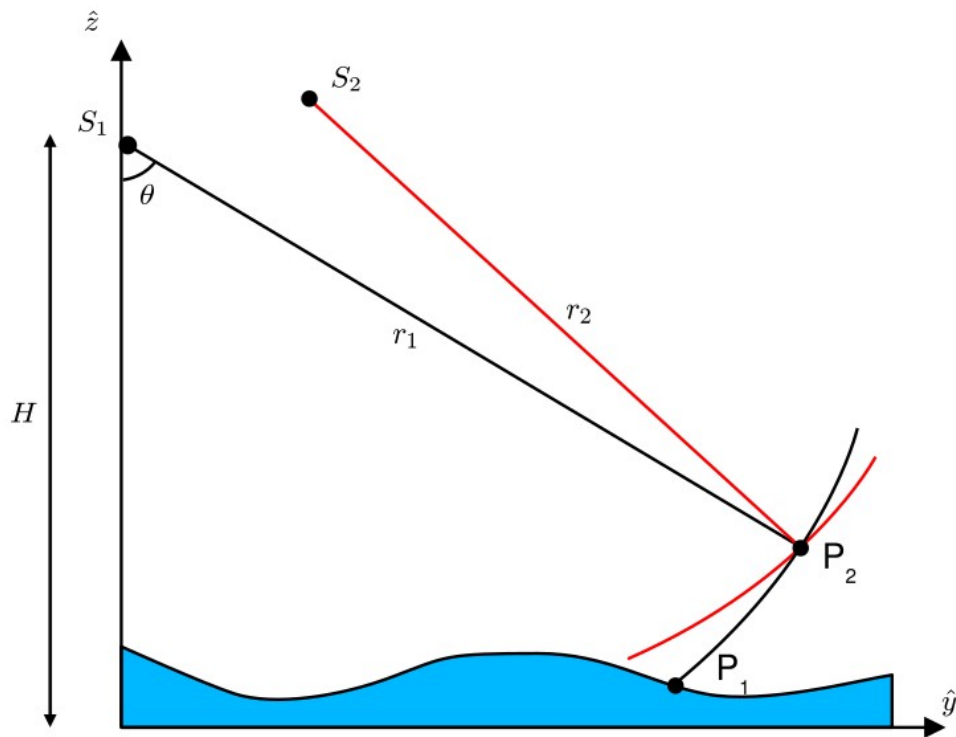
- ▶ good localization in azimuth, x_0 (accuracy : δ_{az})
- ▶ circular ambiguity in the (\hat{y}, \hat{z}) plane

$$y_i^2 + (H - z_i)^2 = r_0^2$$

\Rightarrow multiple solutions (y_i, z_i)

3D mapping using SAR

A solution: use of space diversity



3-step unambiguous estimation

- ▶ Slant range distance from S_1

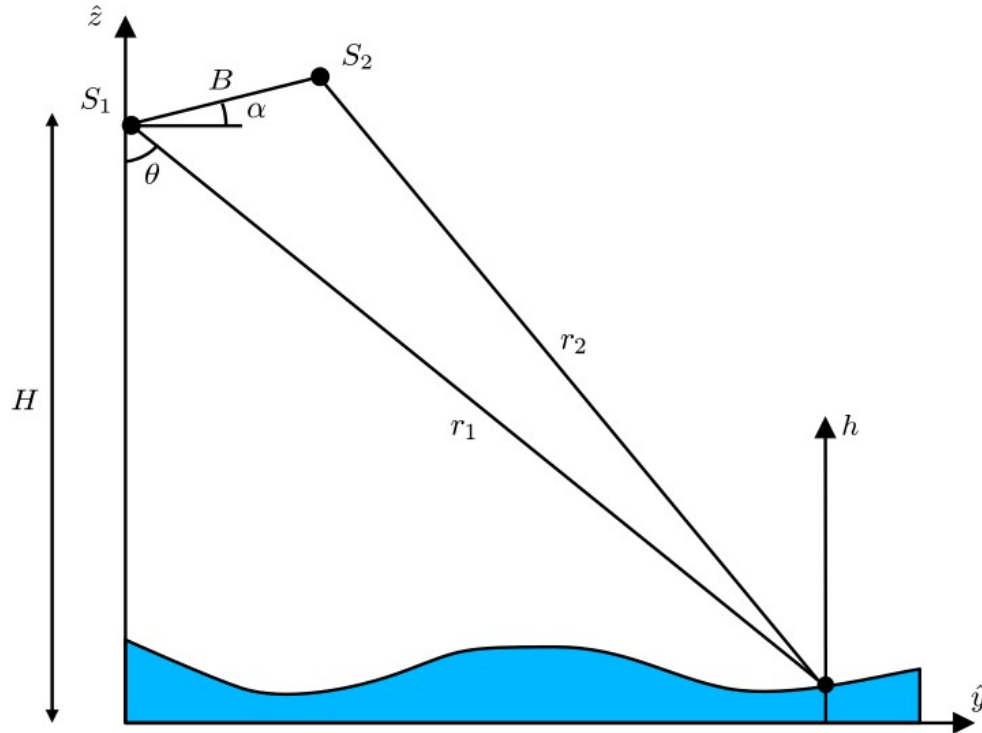
$$r_1 = \|\overrightarrow{S_1 P_2}\| = \|\overrightarrow{S_1 P_1}\|$$

- ▶ Slant range distance from S_2

$$r_2 = \|\overrightarrow{S_2 P_2}\|$$

- ▶ h_2 from r_1, r_2 and the geometrical configuration

Acquisition geometry

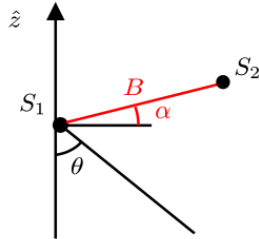


Acquisition geometry descriptors

- ▶ $S_{1,2}$: sensor positions
- ▶ B : baseline
- ▶ α : baseline inclination
- ▶ $r_{1,2}$: slant range distances
- ▶ h : elevation position
- ▶ H : altitude above reference
- ▶ θ : off-nadir incidence angle
- ▶ y : ground range position

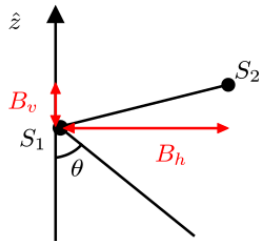
3D mapping using SAR

Baseline conventions



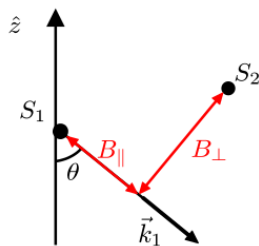
Polar convention

- ▶ B : baseline length
- ▶ α : baseline inclination



Cartesian convention

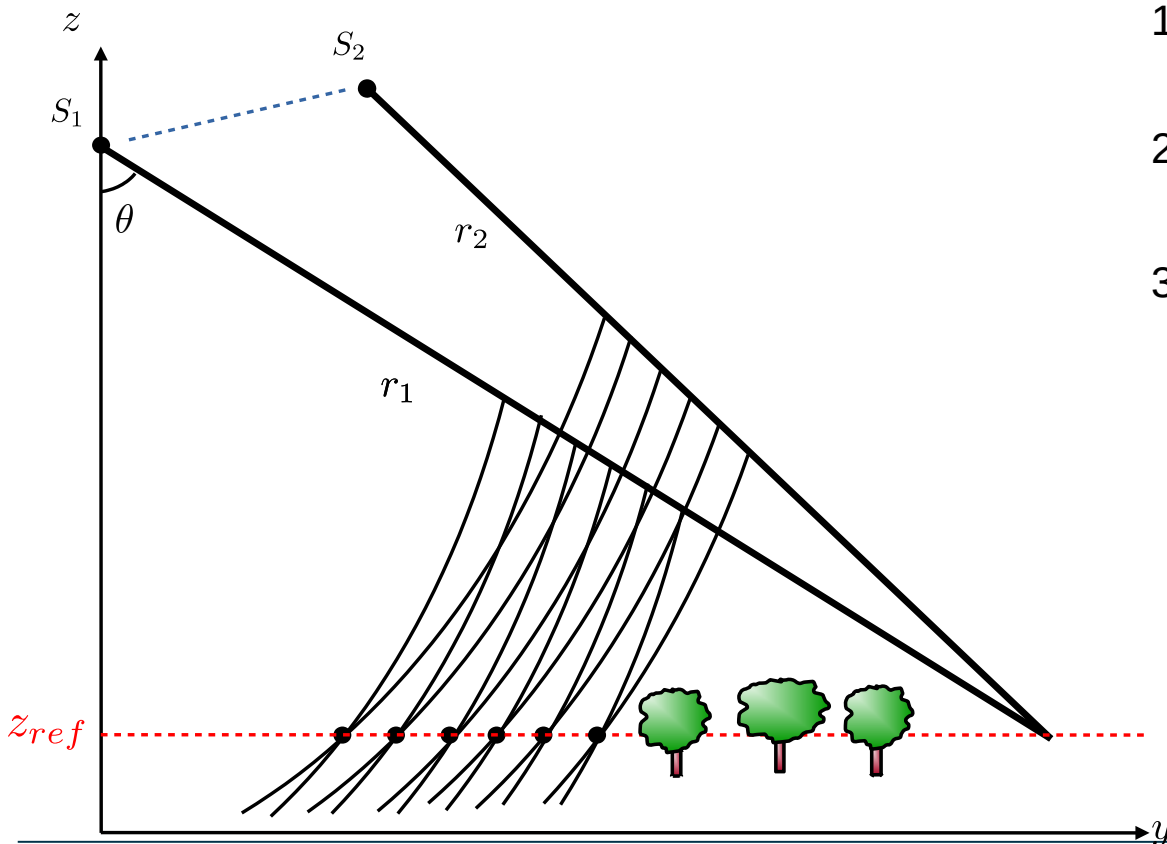
- ▶ $B_h = B \cos \alpha$: horizontal baseline
- ▶ $B_v = B \sin \alpha$: vertical baseline



Wave convention

- ▶ $B_{\parallel} = B \sin(\theta - \alpha)$: baseline parallel to \vec{k}_1
- ▶ $B_{\perp} = B \cos(\theta - \alpha)$: baseline perpendicular to \vec{k}_1

Coregistration of a pair of images



- 1) Set a reference common geometry
 $\{y_{ref}, z_{ref}\}$ plane, DTM...
- 2) Compute reference range coordinates
 $\{y_{ref}, z_{ref}\} \rightarrow \{r_{1ref}, r_{2ref}\}$
- 3) Co-register (resample) images to the reference geometry

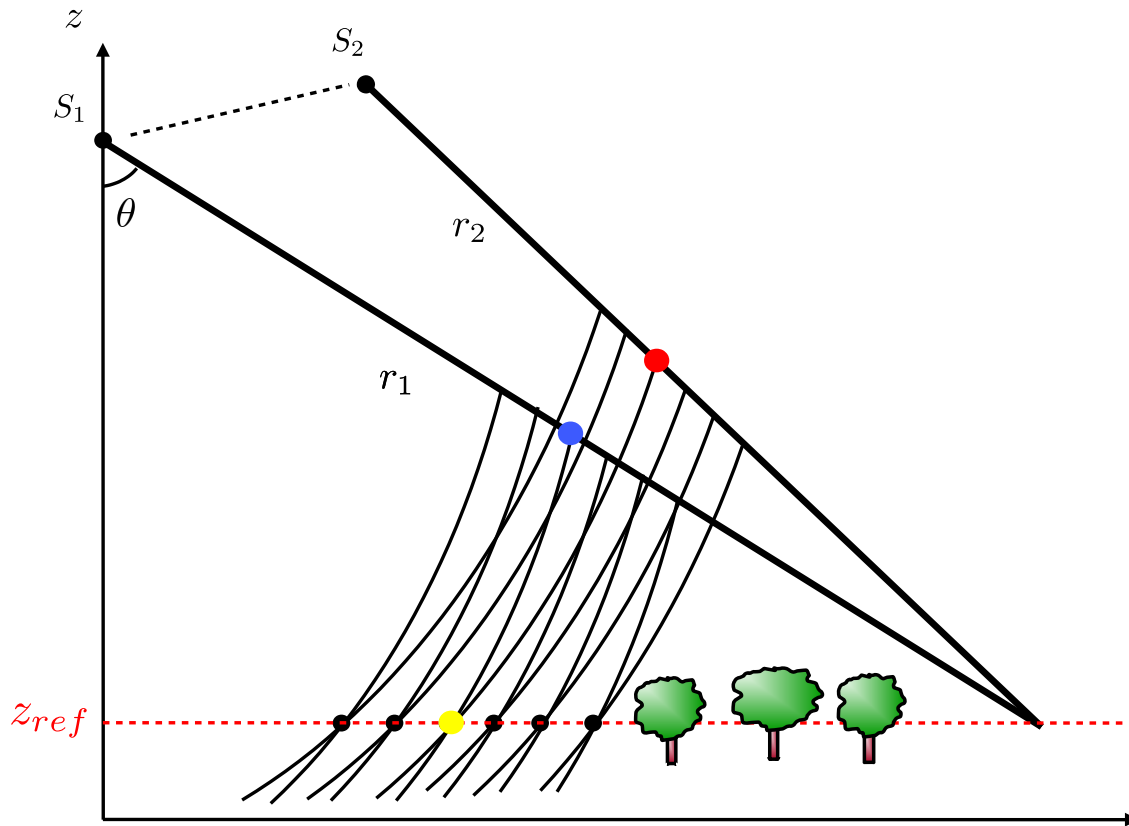
$$s_i(x, r_{i_{ref}}) = a_c$$

$$h_r(r_{i_{ref}} - r_{i_0})$$

$$h_a(x - x_0)$$

$$\exp(-jk_c r_{i_0})$$

Retrieving topography from co-registered images

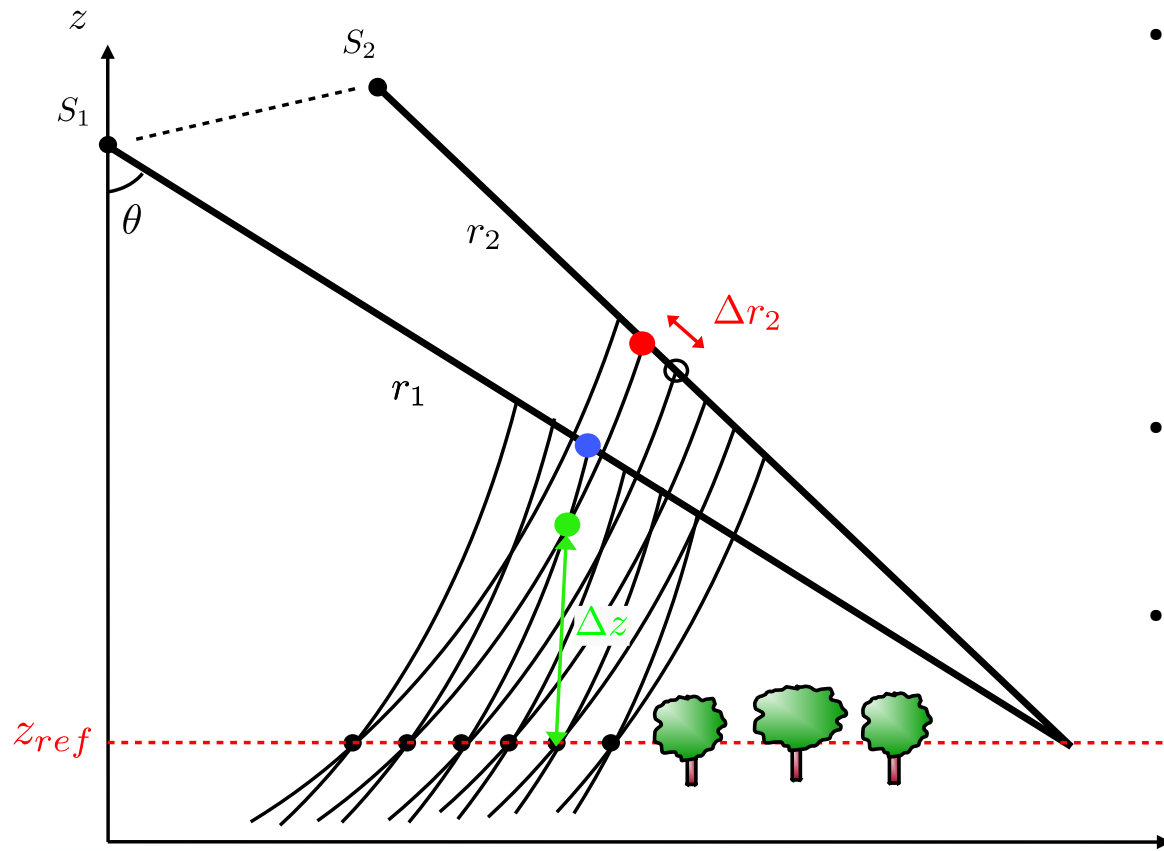


- Case 1:
 - the scatterer lies on the reference geometry
 - same coordinates in the co-registered images

$$r_{1ref} = r_1 \Rightarrow r_{2ref} = r_2$$

Spatial diversity modes

Retrieving topography from co-registered images



- Case 2

- very large vertical offset Δz

$$r_{1ref} = r_1 \not\Rightarrow r_{2ref} = r_2$$

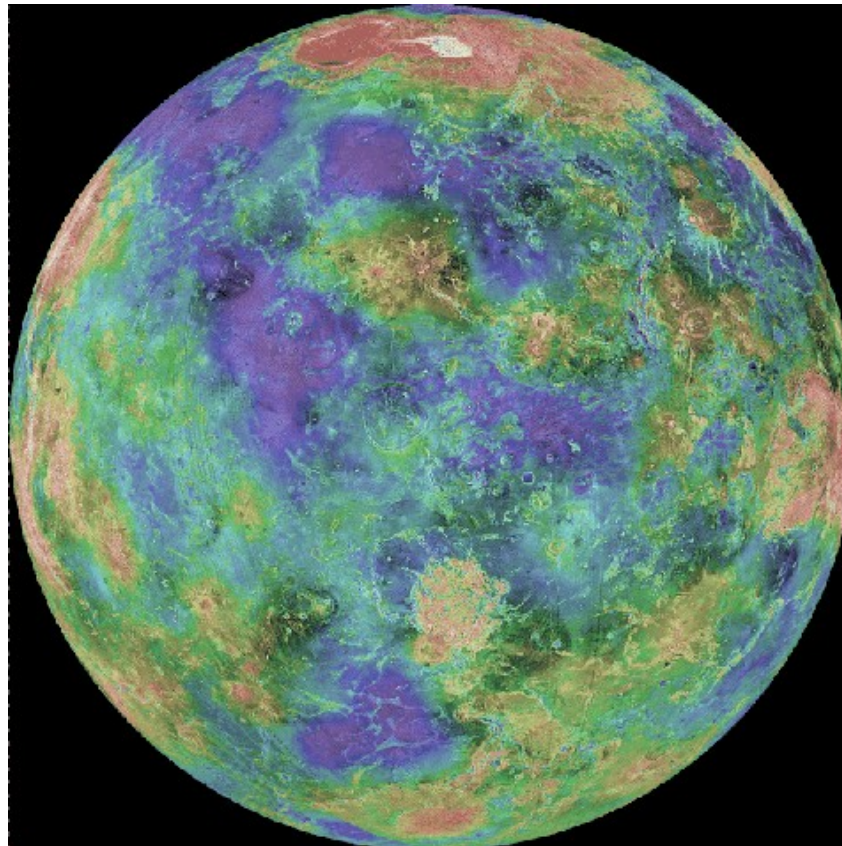
- range shift

$$\Delta r_2 = r_2 - r_{2ref} = f(r_1, \Delta z)$$

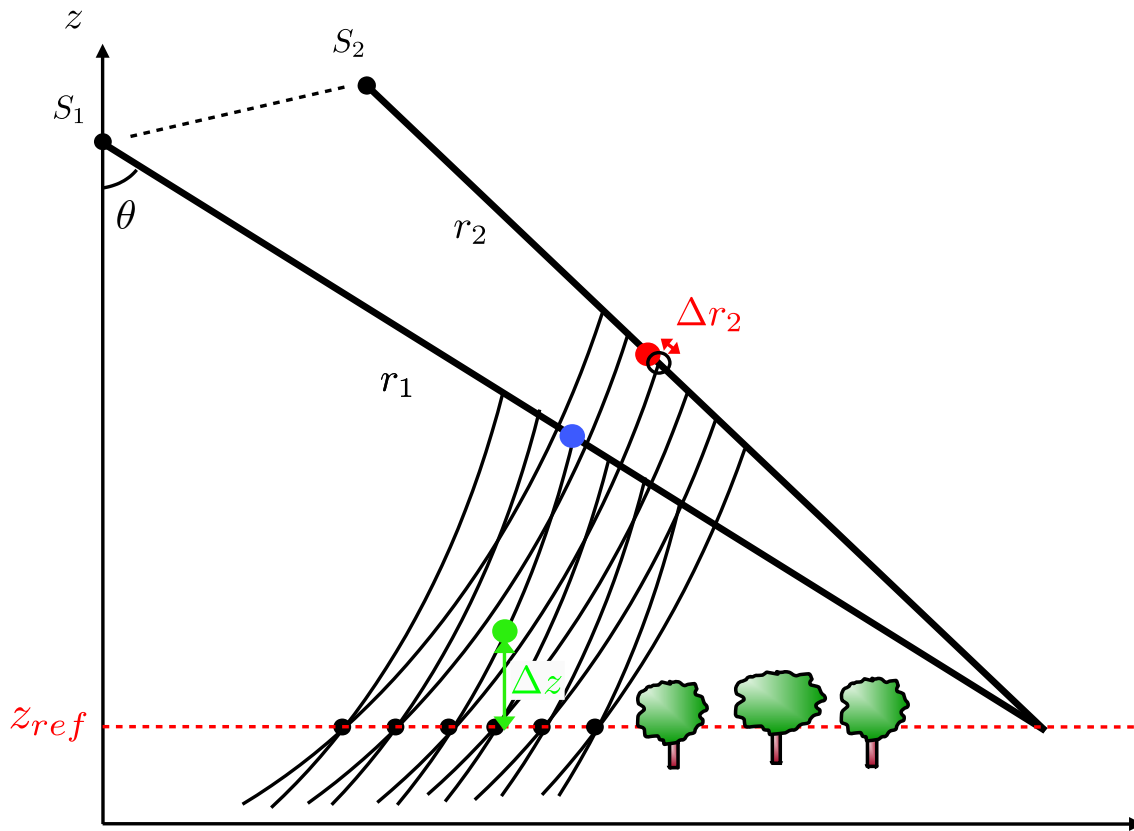
- Topography estimation
stereo radar grammetry (deformation mapping)
- Heavy processing, low-resolution & robust estimation

Stereo radar-grammetry

DEM of Venus measured by the Magellan probe



Retrieving topography from co-registered images



- Case 3
 - small $\Delta z \rightarrow$ no range shift
 $|\Delta r_2| \ll \delta r$
 - topography estimation by interferometry

$$s_i(x, r_{i_{ref}}) \approx a_c \exp(-jk_c r_i)$$

$$\begin{aligned} \Delta\phi &= \arg(s_1 s_2^*) = k_c(r_2 - r_1) \\ &= f(r_1, \Delta z) \end{aligned}$$

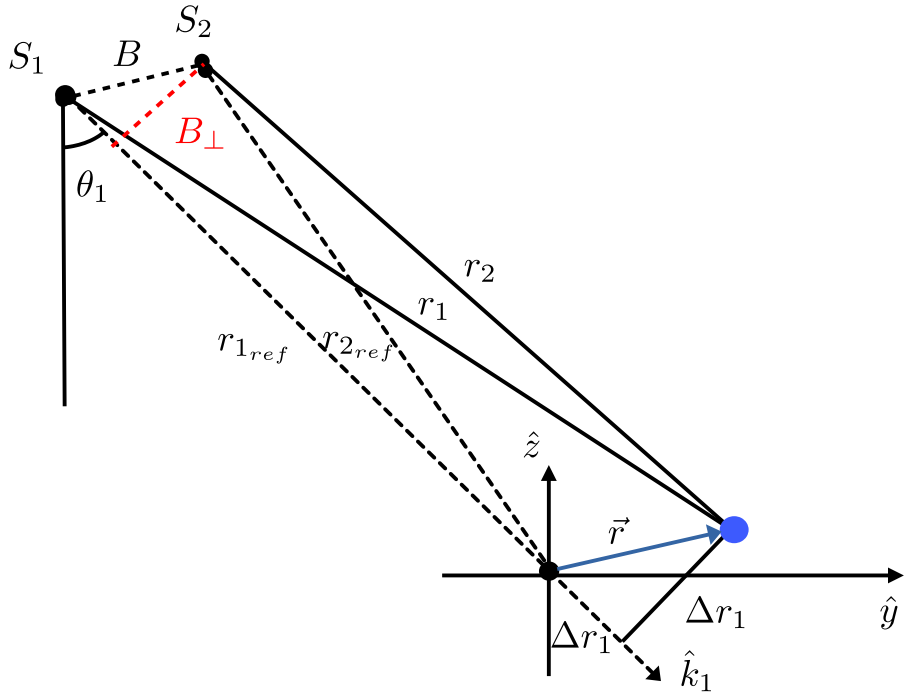
- High-resolution estimation

$$|\Delta\phi| < \pi \Rightarrow |\Delta r| < \frac{\lambda}{4}$$

- Validity requirement $|\Delta r_2| < \alpha \delta r$

SAR interferometry

Interferometric phase components



- Local plane wave approximation

$$r_i \approx r_{i_{ref}} + \Delta r_i, \quad \Delta r_i = \vec{r} \cdot \hat{k}_i, \quad \Delta \theta = \frac{B_{\perp}}{r_{1_{ref}}}$$

- Compound interferometric phase

$$\Delta \phi = k_c (r_2 - r_1) = \Delta \phi_{ref} + \Delta \phi_{topo}, \quad k_c = \frac{4\pi}{\lambda}$$

- Reference (or flat-earth) phase component

$$\Delta \phi_{ref} = k_c (r_{2_{ref}} - r_{1_{ref}})$$

- Topographic phase component

$$\Delta \phi_{topo} = -k_c \Delta \theta \nu$$

$$= k_z \Delta z, \quad k_z = -\frac{4\pi}{\lambda} \frac{B_{\perp}}{r_{1_{ref}} \sin \theta_1}$$

Interferometric phase flattening

Compound interferometric phase

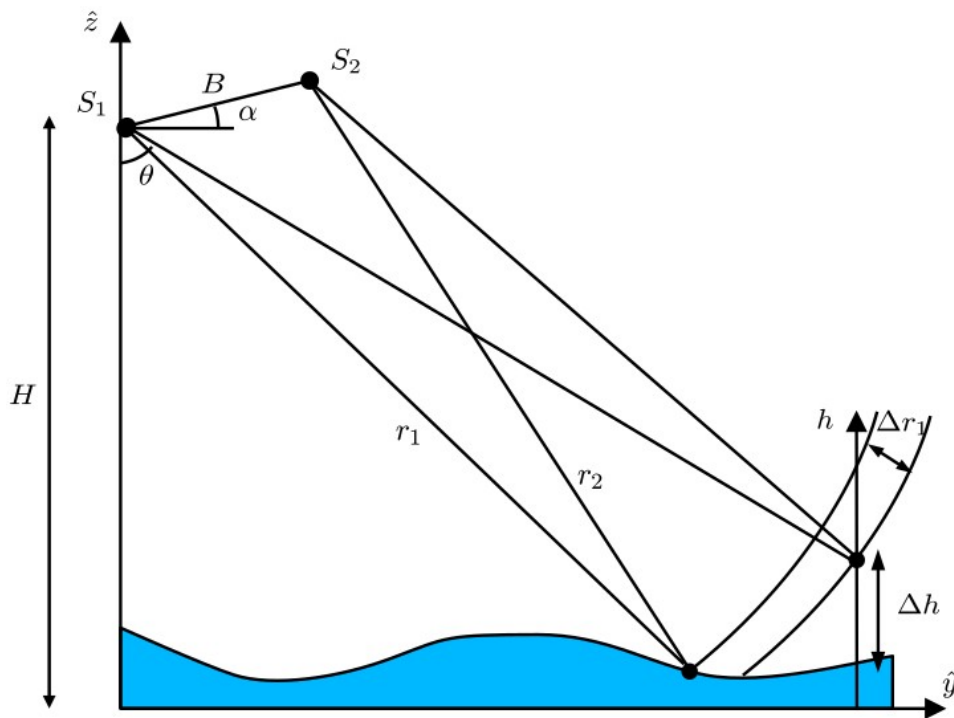
$$\Delta\phi = \Delta\phi_{ref} + \Delta\phi_{topo}$$

1) Compute reference phase (from acqu. Geometry)

$$\Delta\phi_{ref} = k_c(r_{2_{ref}} - r_{1_{ref}})$$

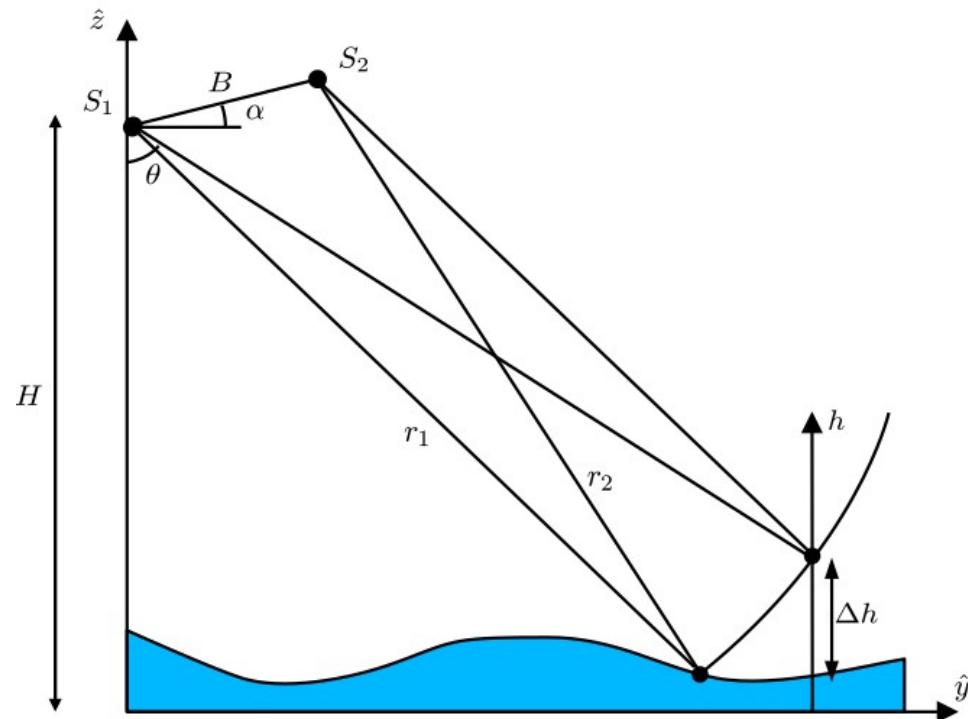
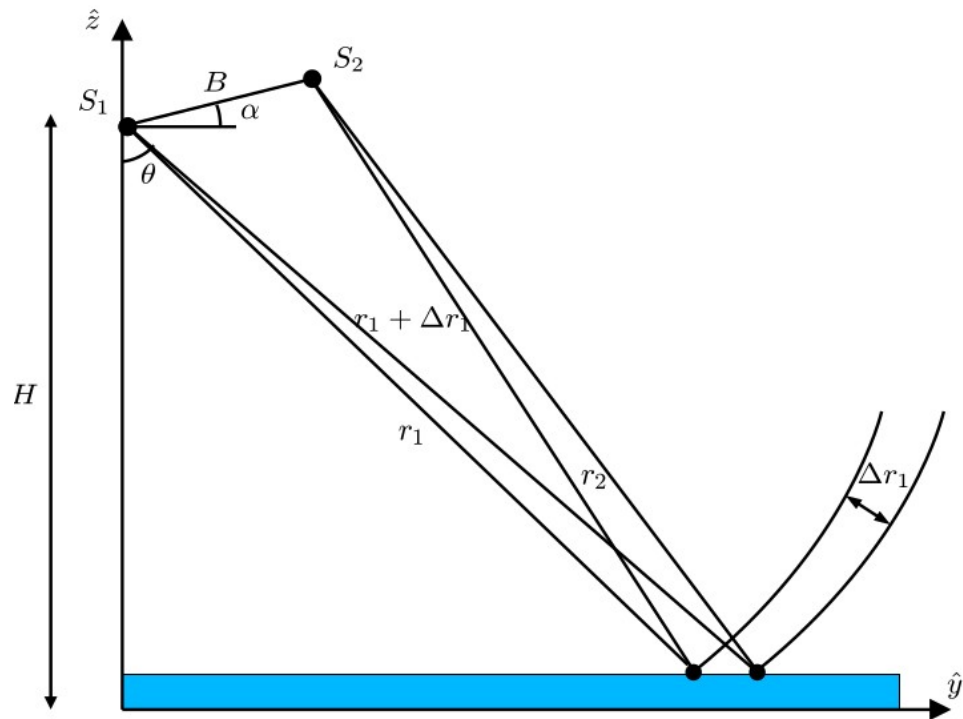
2) Flatten interferogram

$$\Delta\phi_{topo} = \arg(\exp(j(\Delta\phi - \Delta\phi_{ref})))$$



SAR interferometry

Topographic InSAR phase retrieval



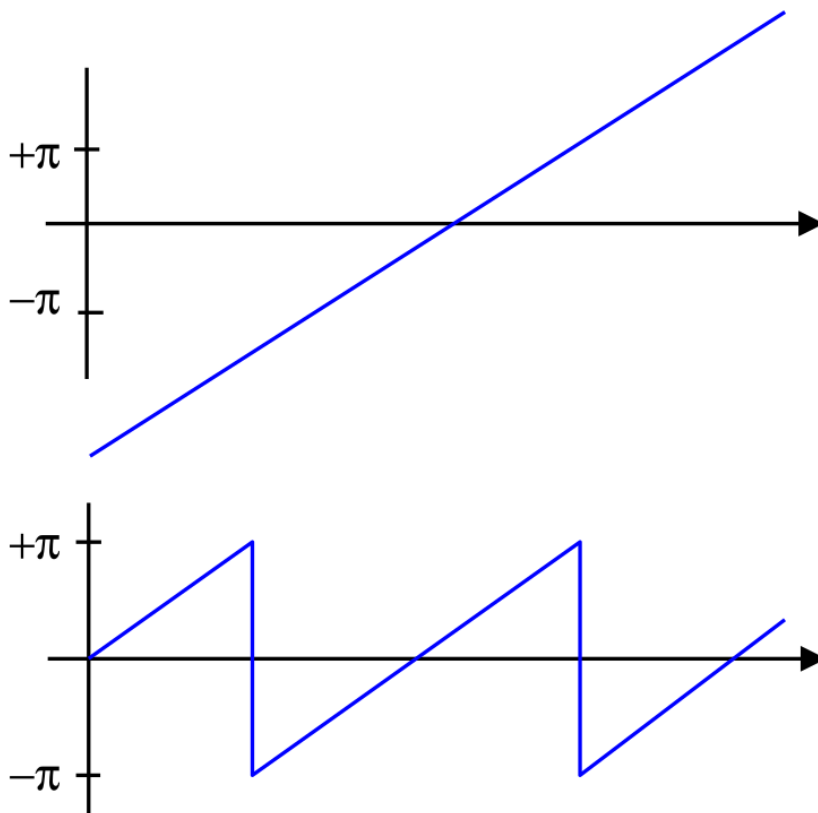
- Sensitivity to:
 - baseline
 - Frequency (wavelength)
 - Off-nadir angle

$$\Delta\phi_{topo} = k_z\Delta z, \quad k_z = -\frac{4\pi}{\lambda} \frac{B_{\perp}}{r_{1ref} \sin\theta_1}$$

- Ambiguous height h_{amb}

$$k_z(\Delta z + h_{amb}) - k_z\Delta z = 2\pi \quad \Rightarrow \quad \boxed{h_{amb} = \frac{2\pi}{k_z}}$$

InSAR phase wrapping



Interferometric phase computation

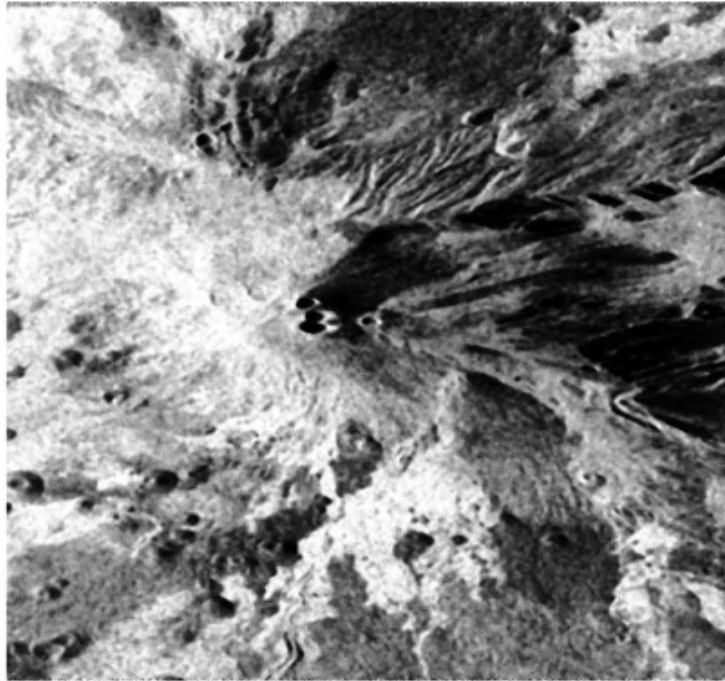
- ▶ $\Delta\phi_{12} = \arg(s_{12})$, with $s_{12} = s_1 s_2^*$
- ▶ Four-quadrant arc-tangent operator

$$\Delta\phi_{12_est} = \arctan(\Im(S_{12}), \Re(S_{12}))$$

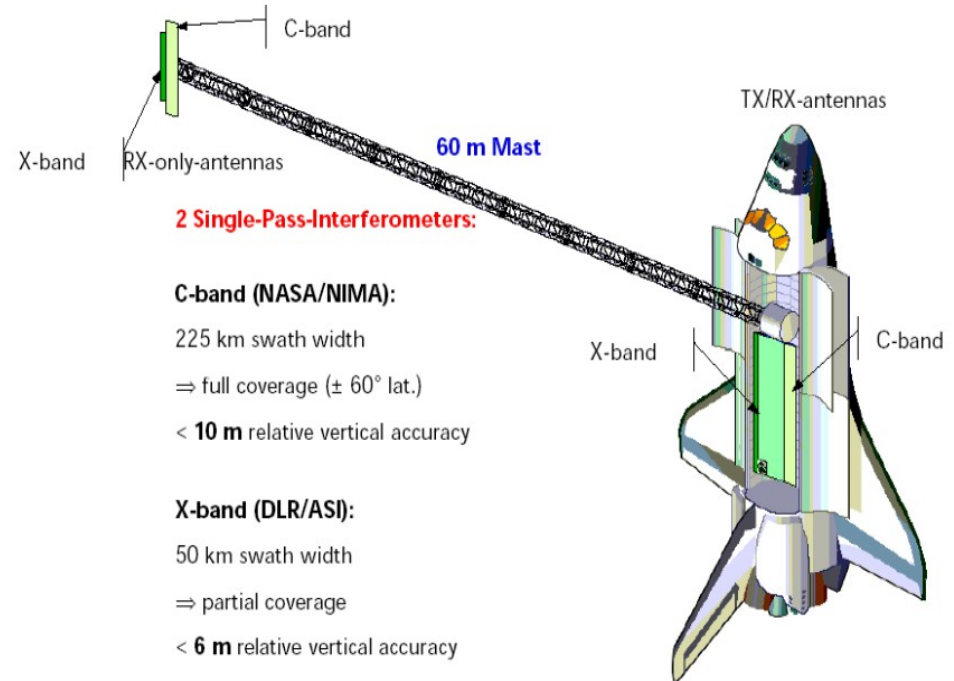
Interferometric phase computation

- ▶ $\Delta\phi_{12_est} \in] -\pi, \pi]$
- ▶ $\Delta\phi_{12_est} = \Delta\phi_{12} + k2\pi$, $k \in \mathbb{Z}$
- ▶ Phase wrapping effect : “fringes”

3D mapping using InSAR



Amplitude image

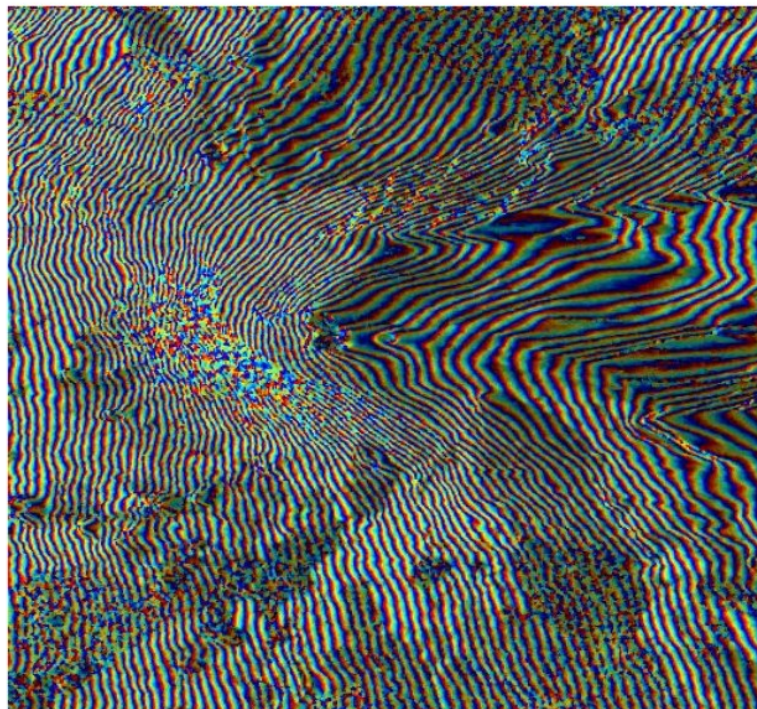


Absolute phase image

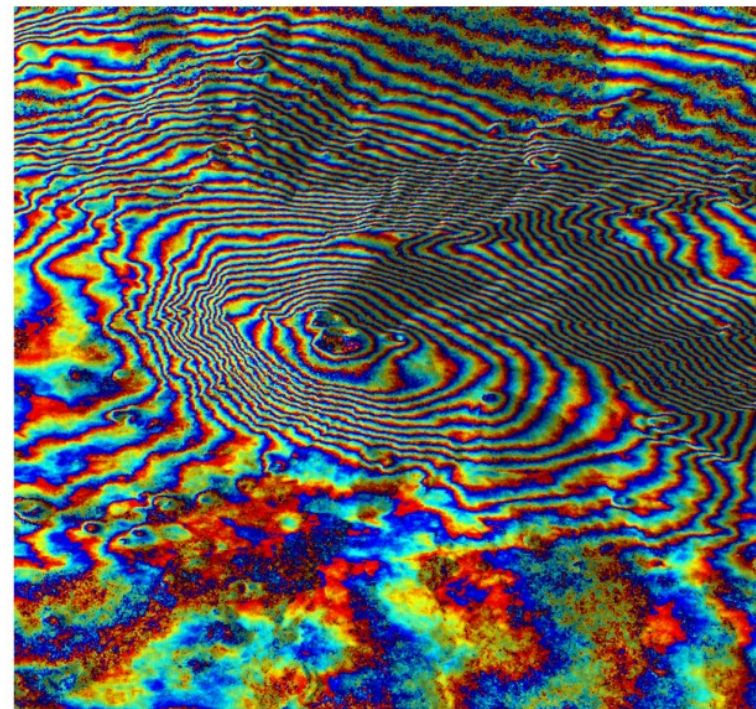
SIR-C, X-band, Mount Etna, Italy

3D mapping using InSAR

InSAR phase flattening



$$\Delta\phi$$

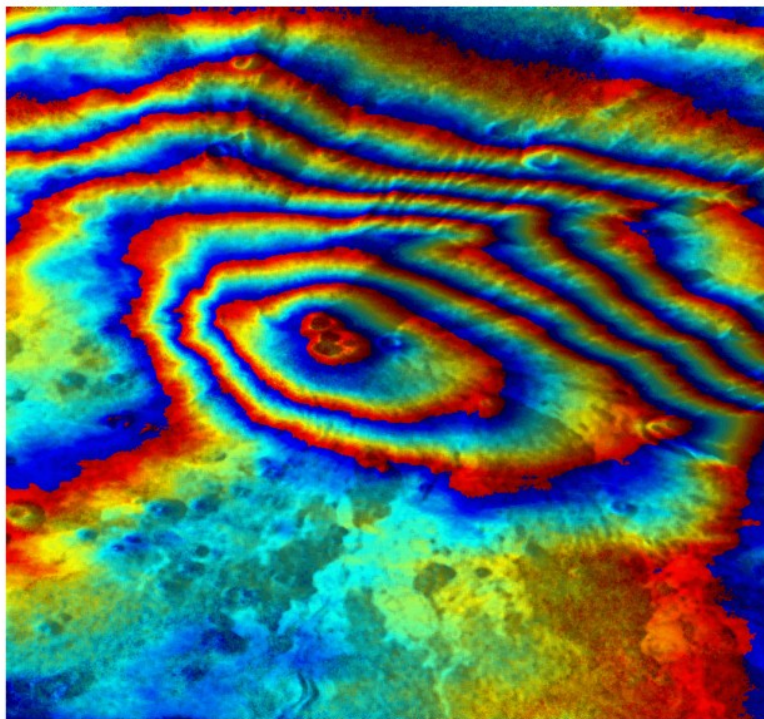


$$\Delta\phi_{topo}$$

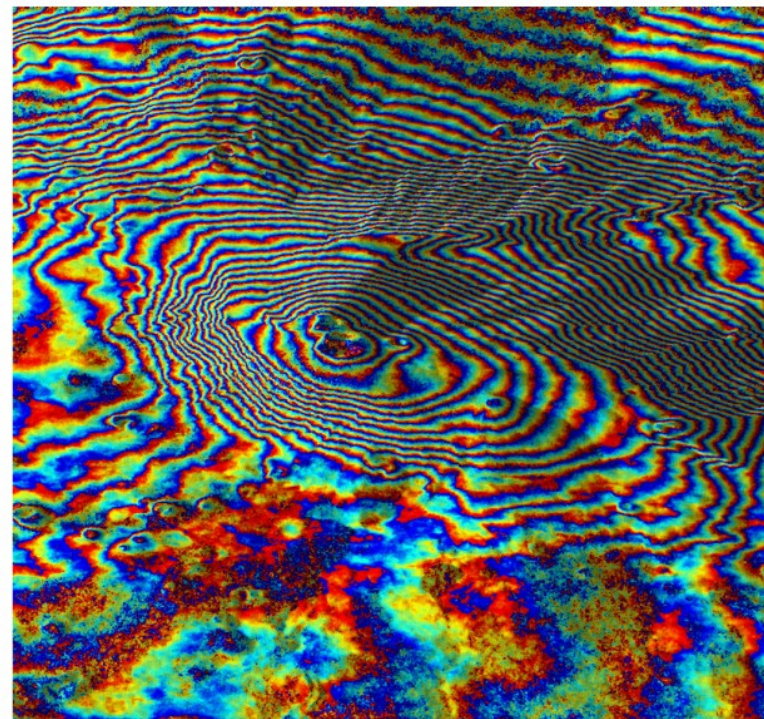
Radar look direction ?

3D mapping using InSAR

Sensitivity to baseline



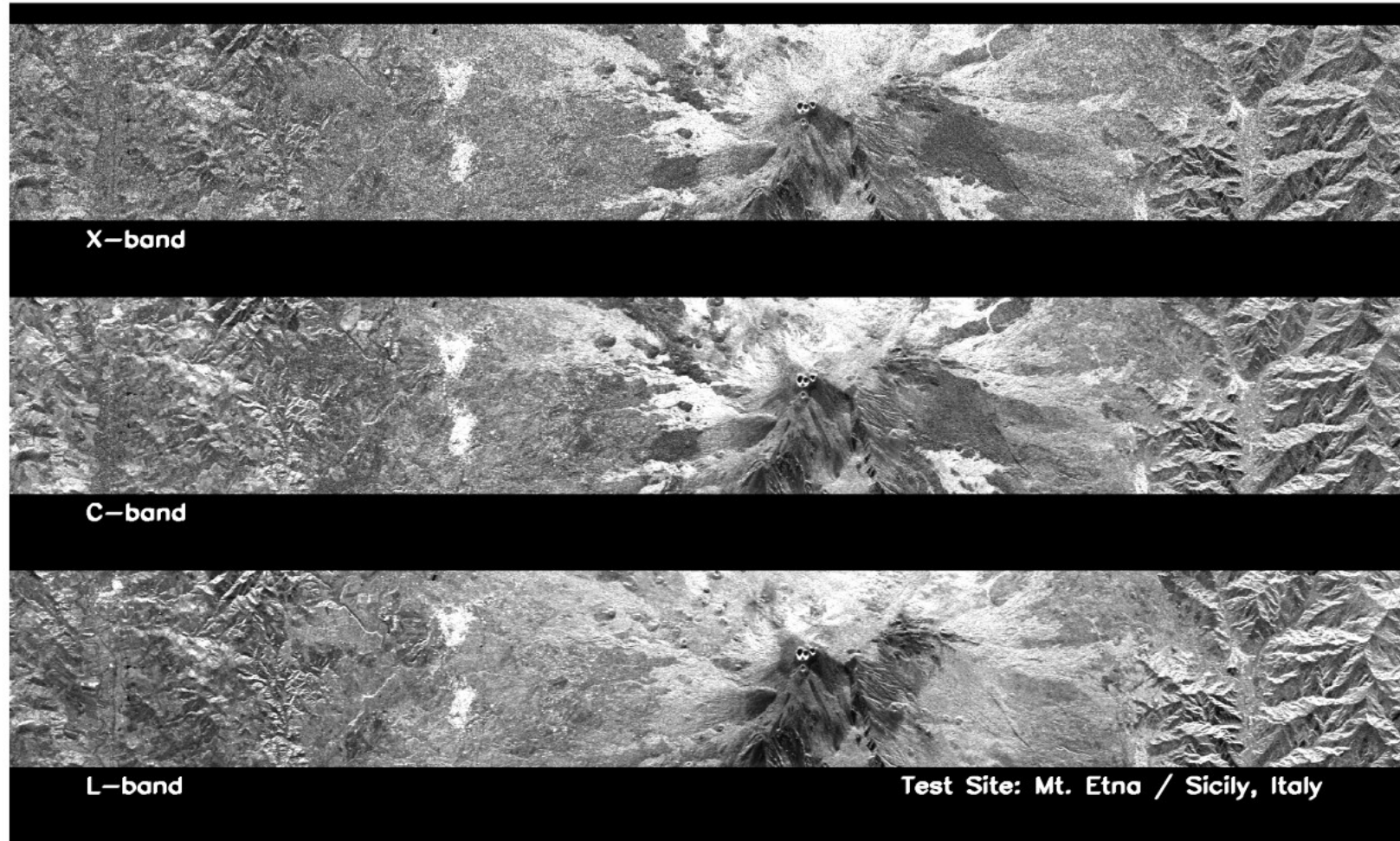
12m baseline



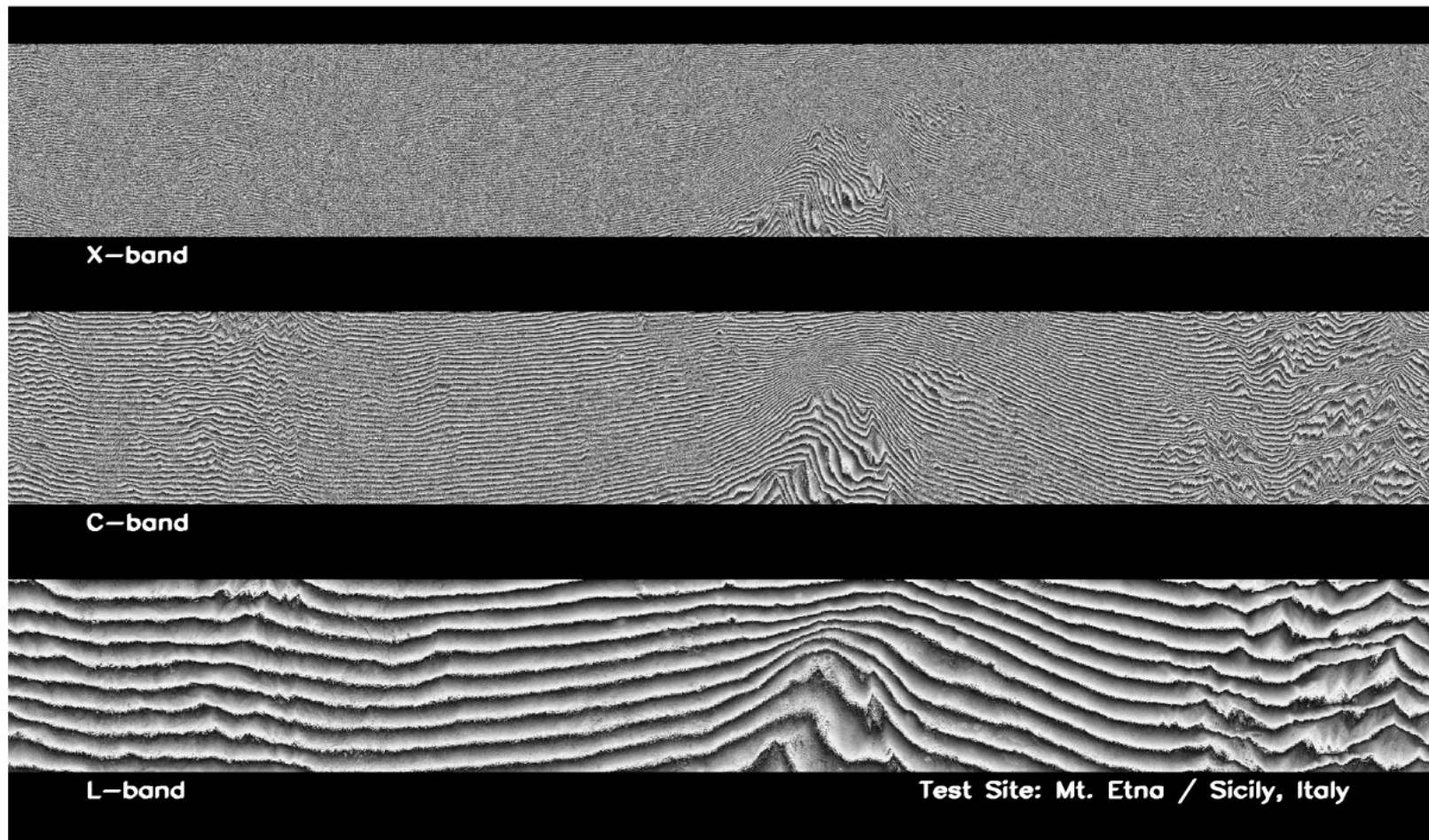
60m baseline

$$\Delta\phi_{topo} = -k_c \frac{B_{\perp}}{r_{1ref} \sin\theta_1}$$

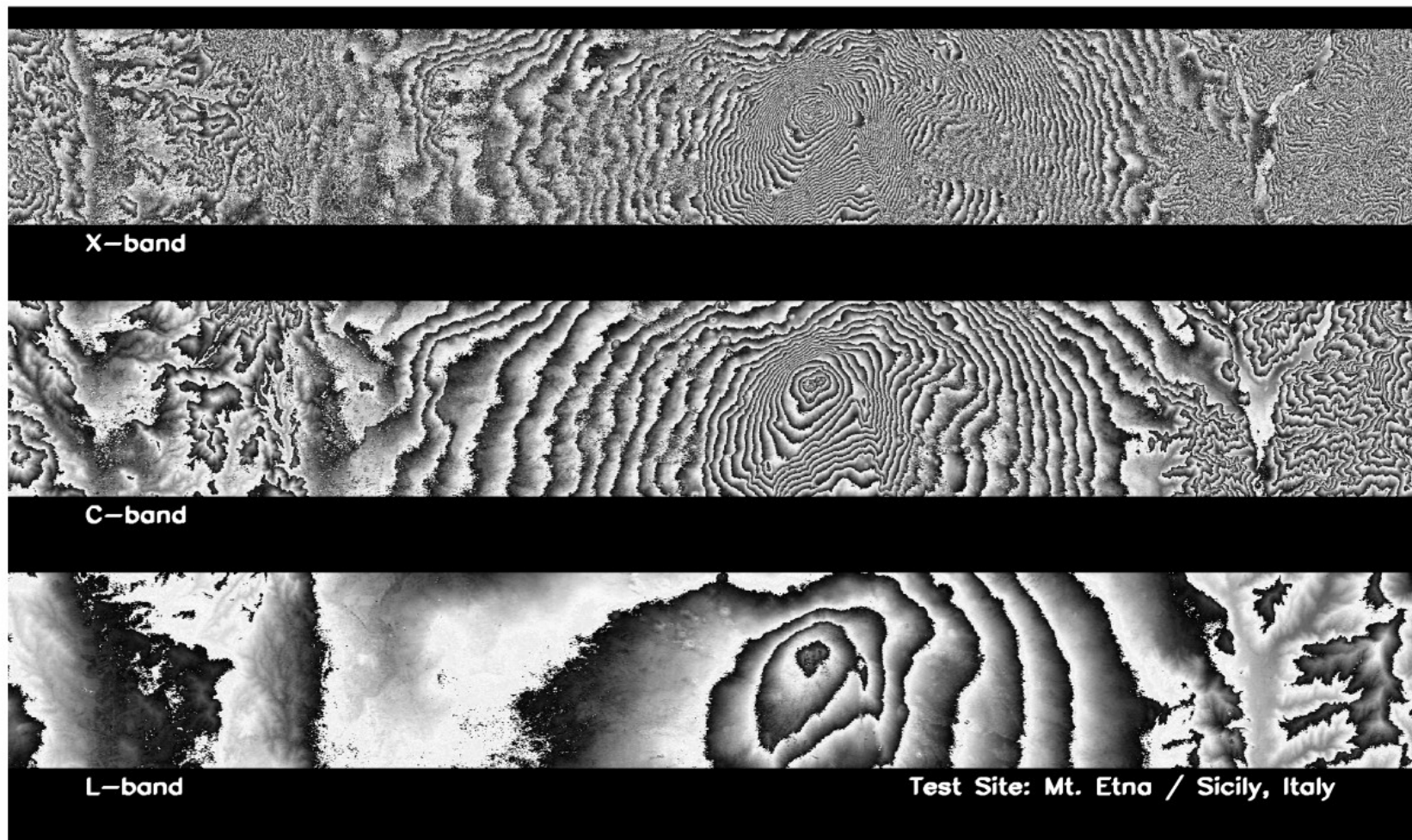
Sensitivity to frequency: reflectivity



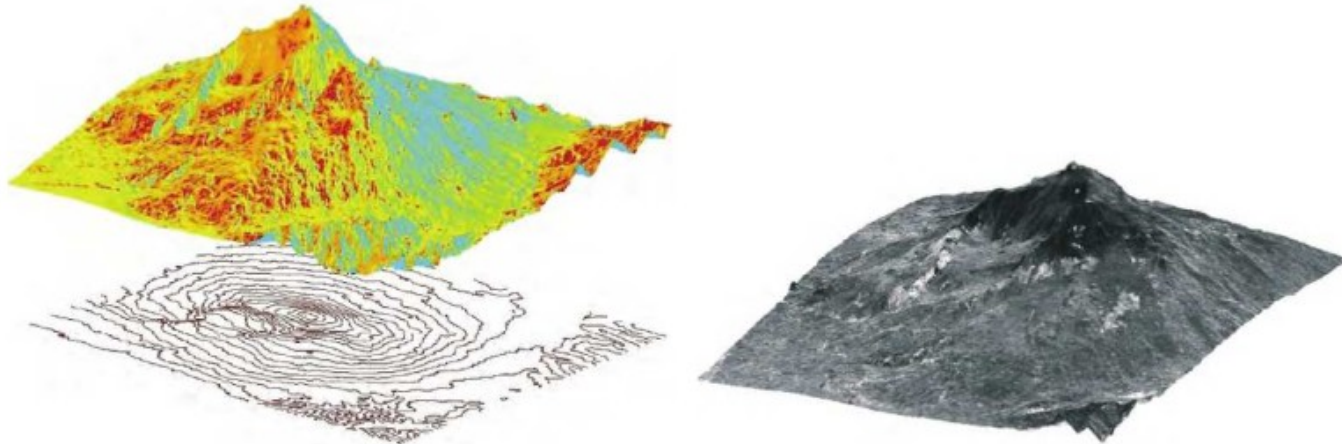
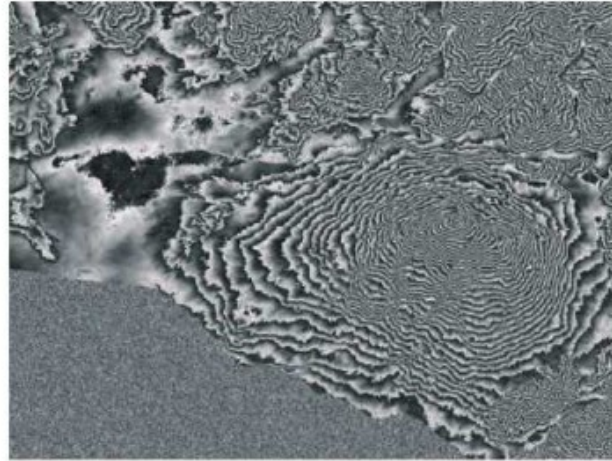
Sensitivity to frequency: raw phase



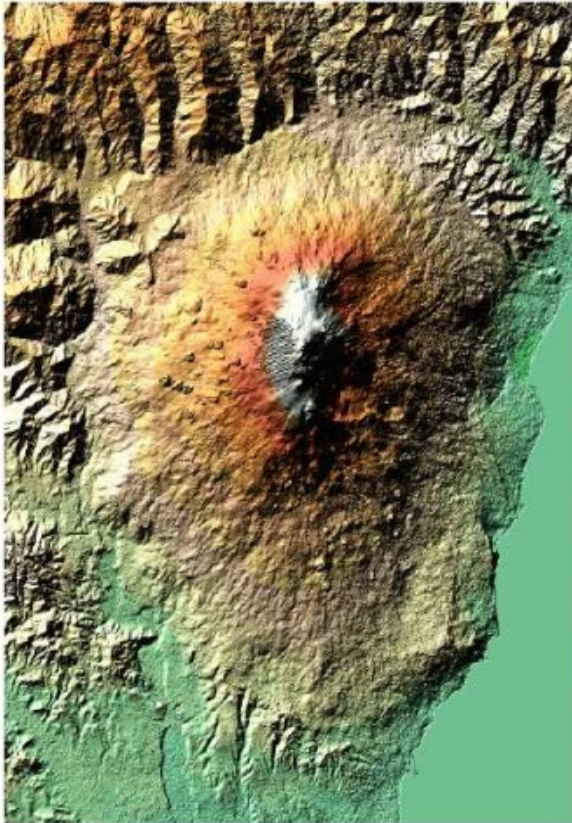
Sensitivity to frequency: flattened phase



Phase unwrapping

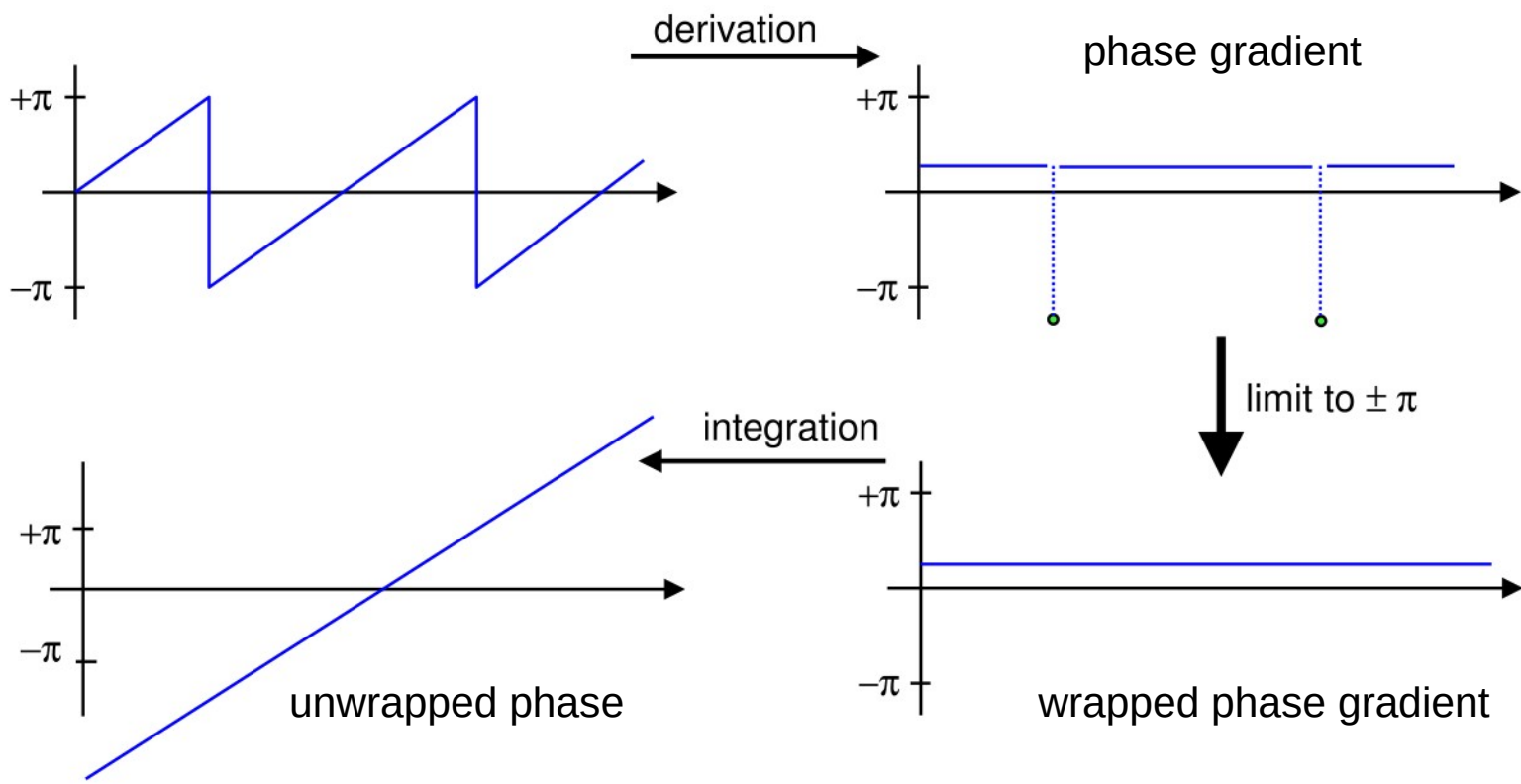


Need for phase unwrapping



Phase unwrapping principle in 1D

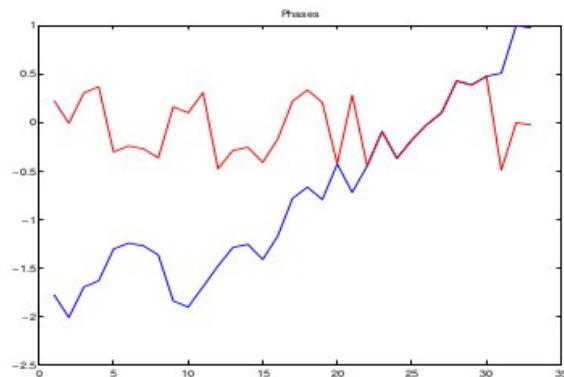
Itoh's method



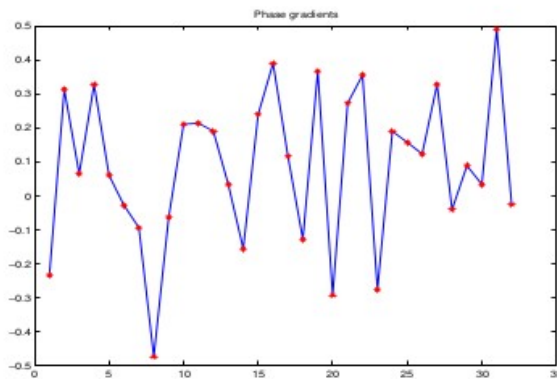
Aliasing-free phase unwrapping

Aliasing-free condition

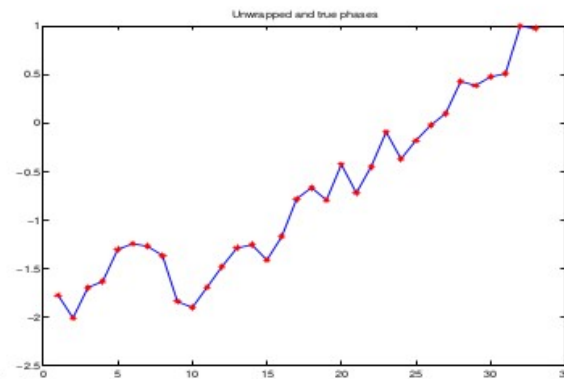
$$|\Delta\phi| < \pi$$



(a) $\phi/(2\pi), \psi/(2\pi)$



(b) $\Delta\phi/(2\pi), \mathcal{W}(\Delta\psi)/(2\pi)$

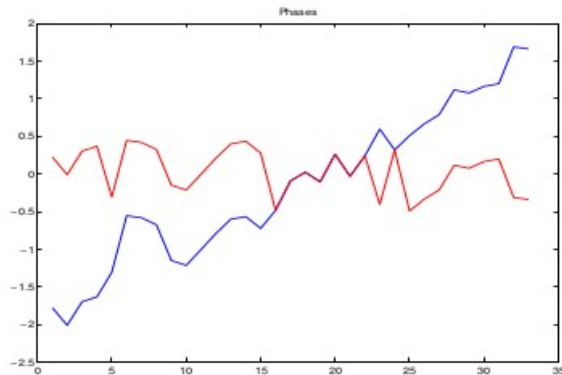


(c) $\phi/(2\pi), \hat{\phi}/(2\pi)$

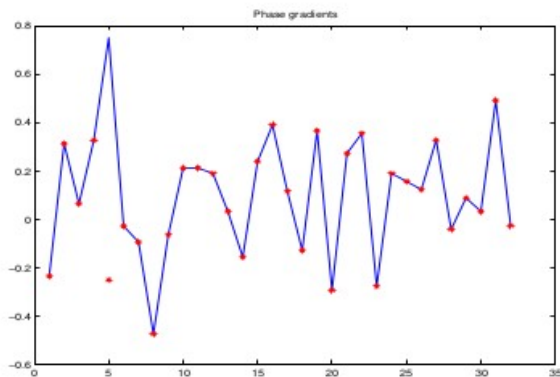
Aliasing-free phase unwrapping

Possible alias if

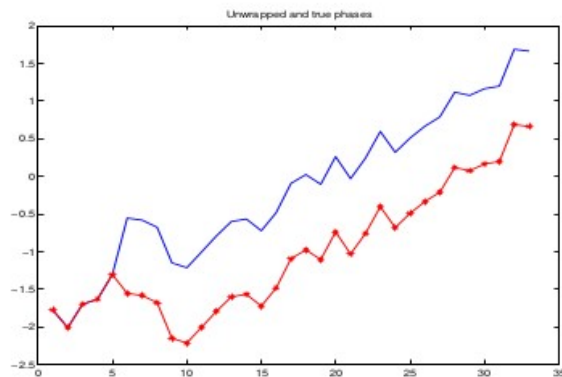
$$\exists |\Delta\phi| > \pi$$



(a) $\phi/(2\pi), \psi/(2\pi)$



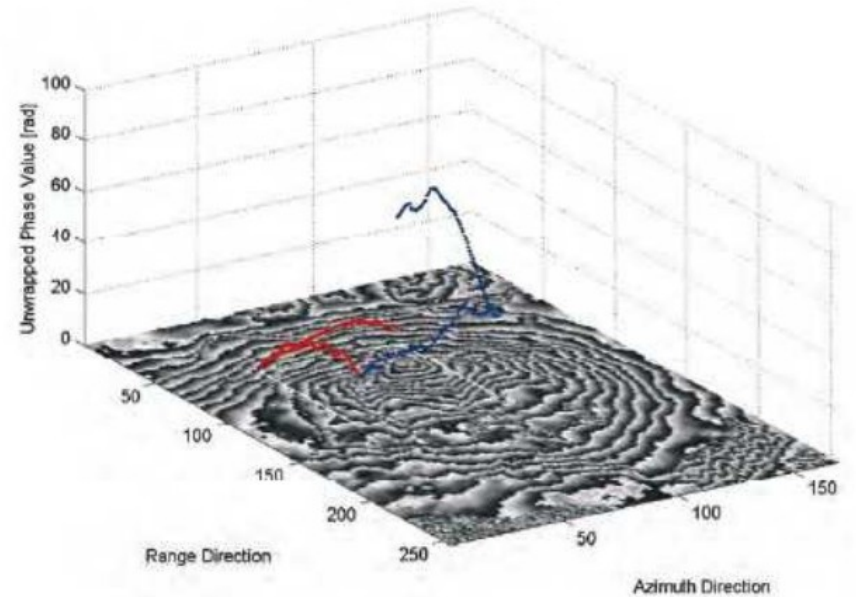
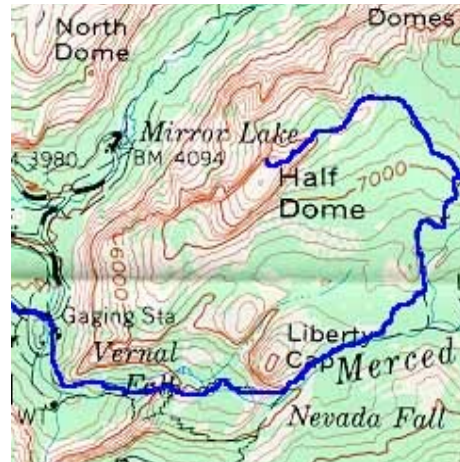
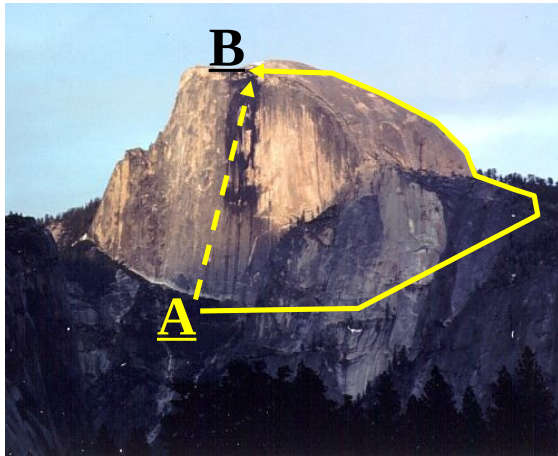
(b) $\Delta\phi/(2\pi), \mathcal{W}(\Delta\psi)/(2\pi)$



(c) $\phi/(2\pi), \hat{\phi}/(2\pi)$

2D phase unwrapping

Better conditioning through 2D integration path diversity



2D phase unwrapping

wrapped phase



residues



branches



2D phase unwrapping

2D phase unwrapping formalism

• Wrapped phase: $\psi(x, r) \rightarrow \boldsymbol{\psi} \in \mathbb{R}^{N_x N_r}$

• Gradient operator: $\Delta(\boldsymbol{\psi}) = \begin{bmatrix} \Delta_x(\boldsymbol{\psi}) \\ \Delta_r(\boldsymbol{\psi}) \end{bmatrix}$

$$\hat{\phi} = \arg \min_{\phi} \|\Delta\phi - \mathcal{W}(\Delta\boldsymbol{\psi})\|_p$$

• Unwrapped phase: ϕ

• L^p norm: $\|\mathbf{x}\|_p = \left(\frac{1}{N} \sum_i |x_i|^p \right)^{1/p}$

• Wrapping operator $\mathcal{W}(\phi) = \arg e^{j\phi}$

• L^2 solution:

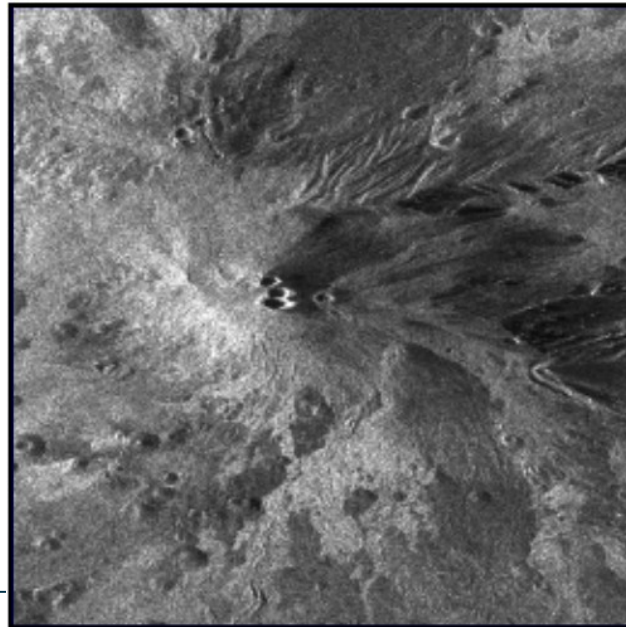
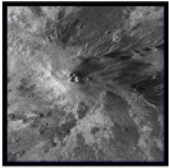
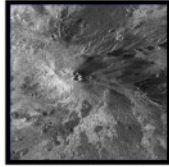
- very fast
- Inaccurate (smooth)

• L^1 solution

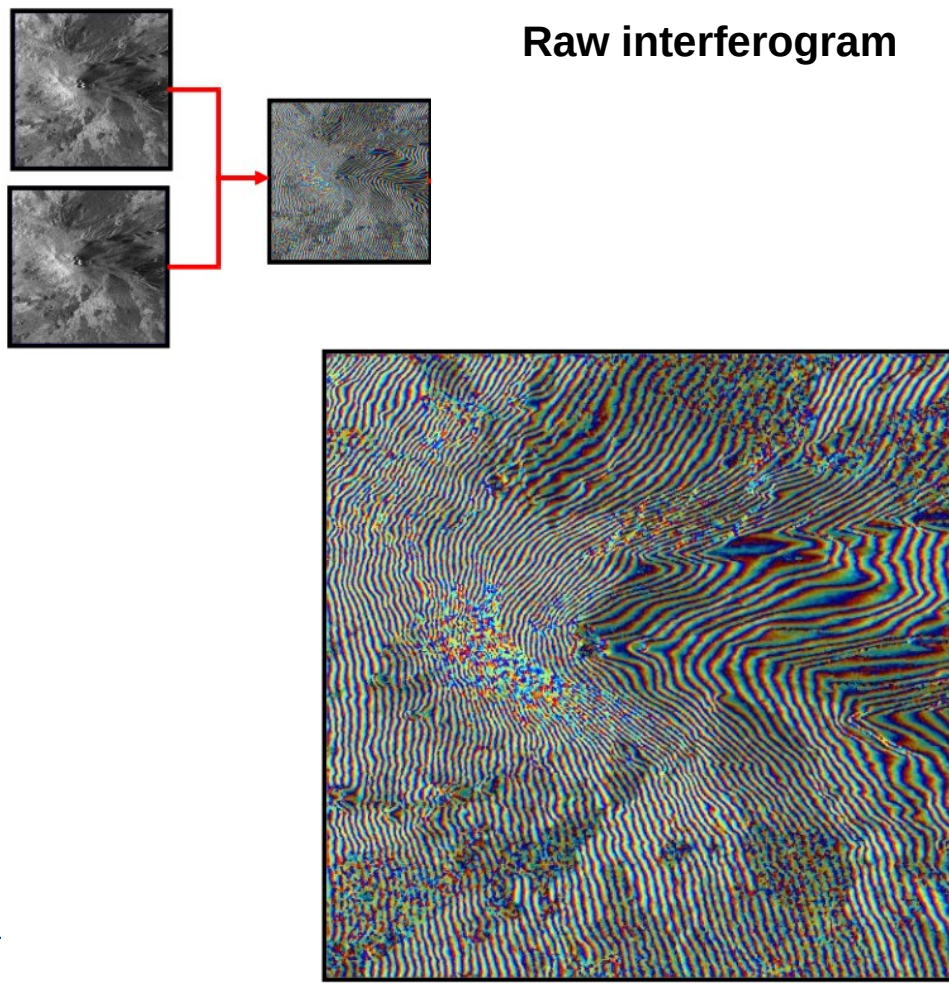
- slower
- very robust
- better performance

Interferometric SAR processing chain

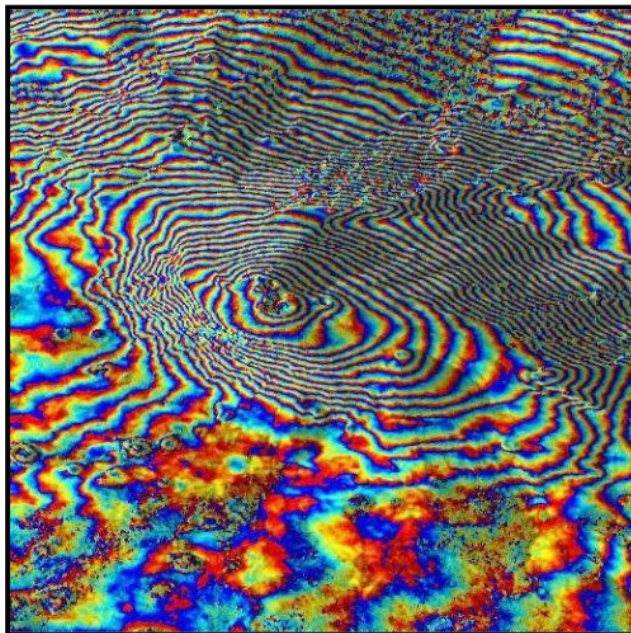
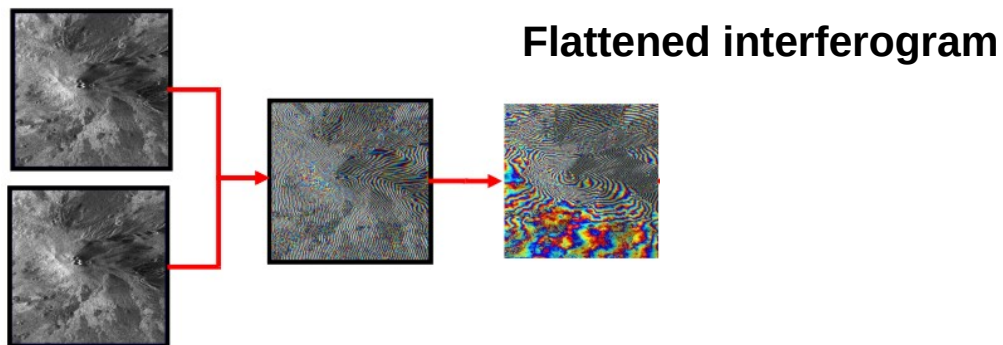
Acquired images



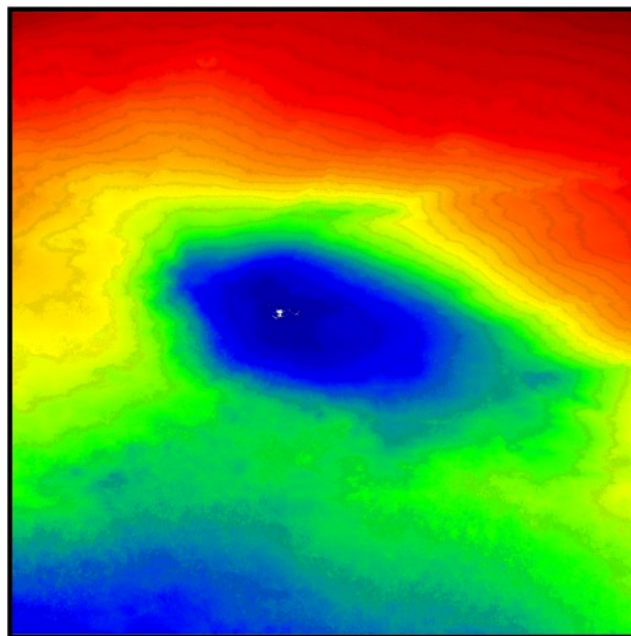
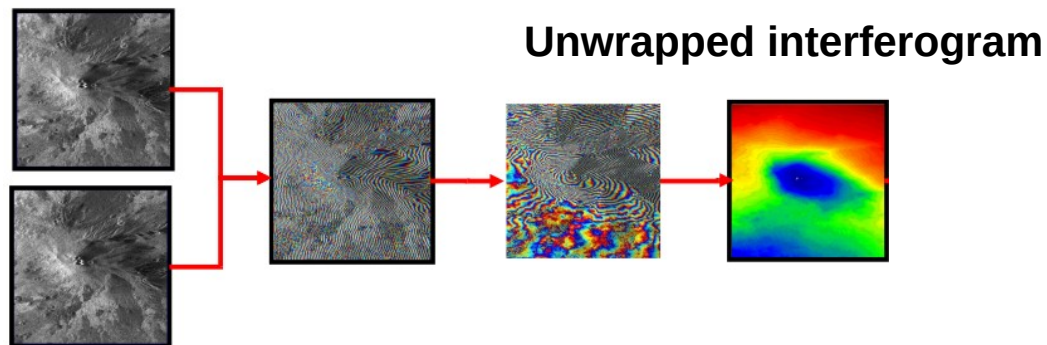
Interferometric SAR processing chain



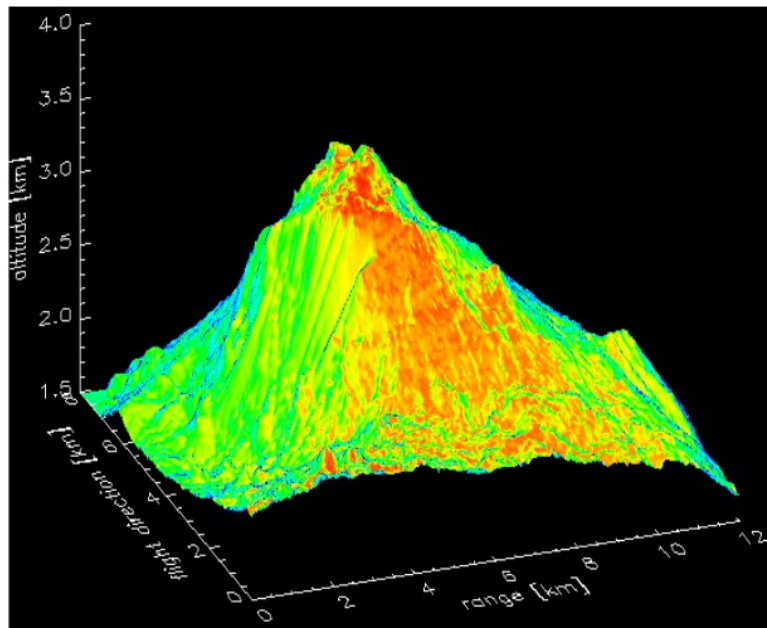
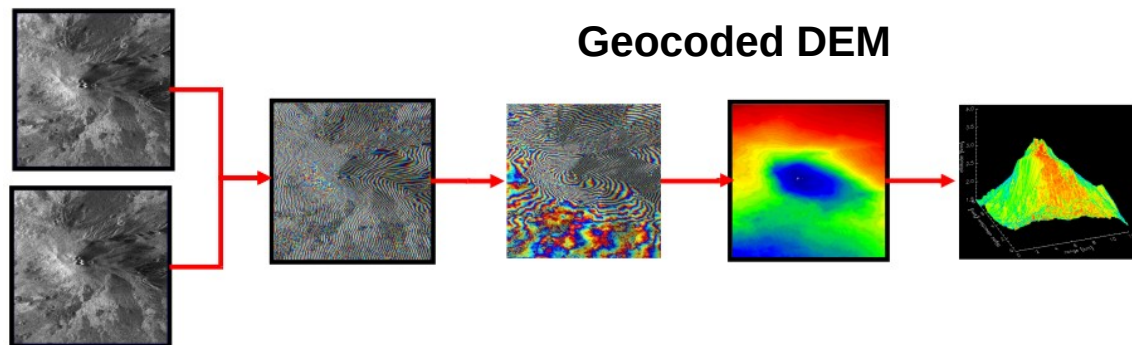
Interferometric SAR processing chain



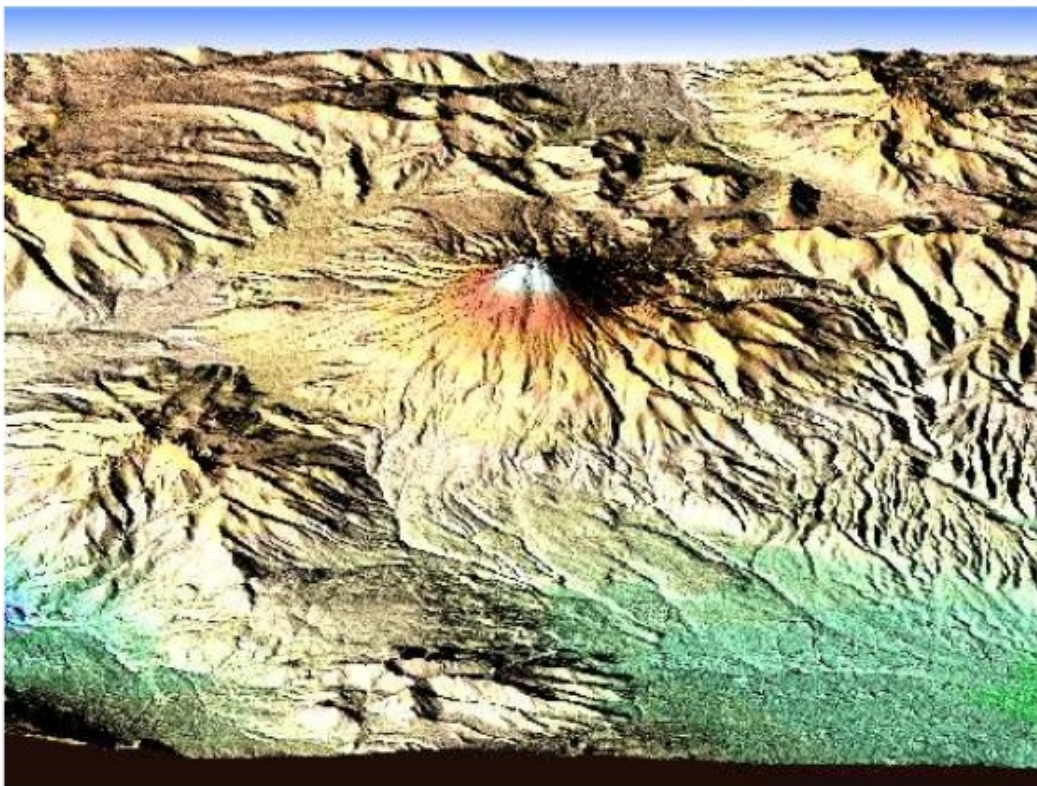
Interferometric SAR processing chain



Interferometric SAR processing chain



SRTM/X-SAR DEM, Mount Kotopaxi, Ecuador



High resolution elevation mapping worldwide

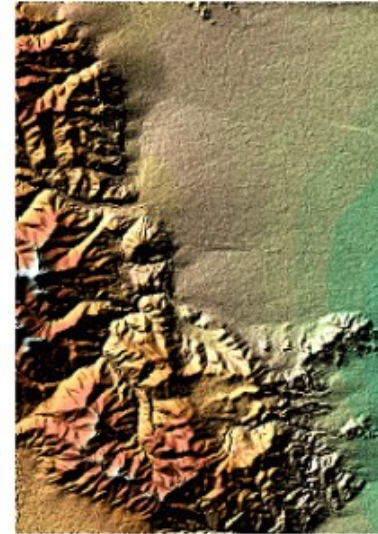


GTOP-030

Spatial Resolution: 30 Arc Sec = 1km
Height Accuracy : 100 m – 500 m

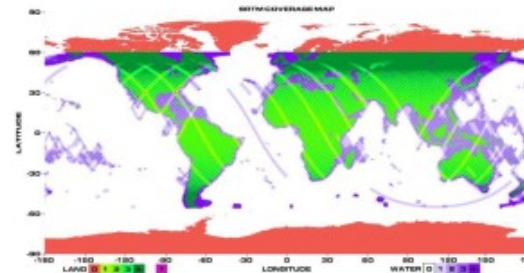


February 2000
Single Pol Channel

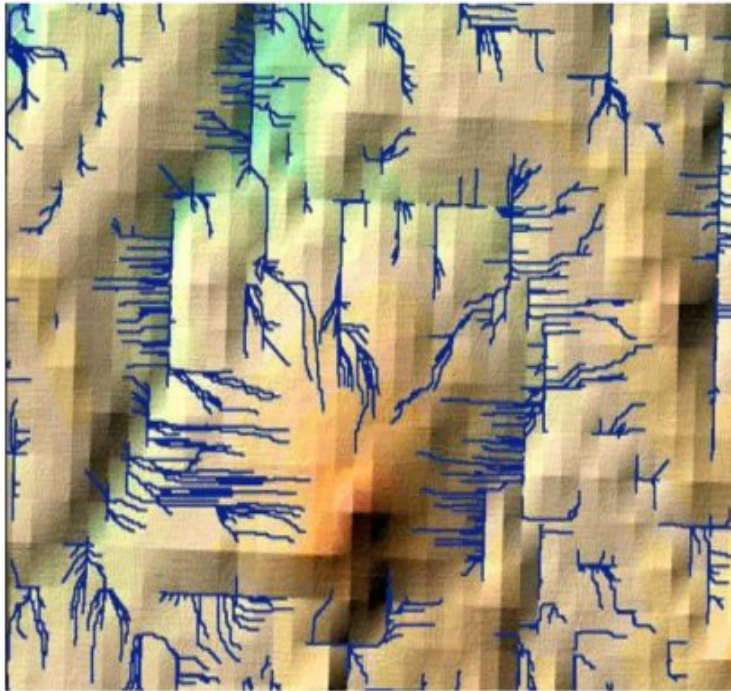


SRTM: X-SAR

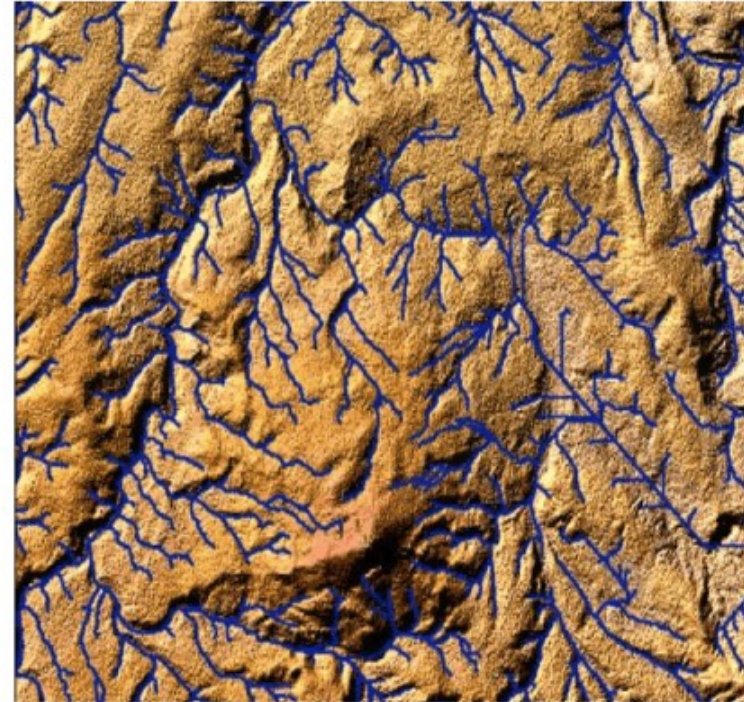
Spatial Resolution: 1 Arc Sec = 30m
Height Accuracy : 6-10 m



Hydrological water flow simulation



Globe-30 DEM



SRTM DEM

SAR Interferometry basics

A signal processing approach

Single SAR image statistics

S : Single Look Complex (SLC) image

Array of coherent coefficients $S_{(x,r)} = \rho_{(x,r)} e^{j\phi_{(x,r)}}$

Reflexion coefficient amplitude image

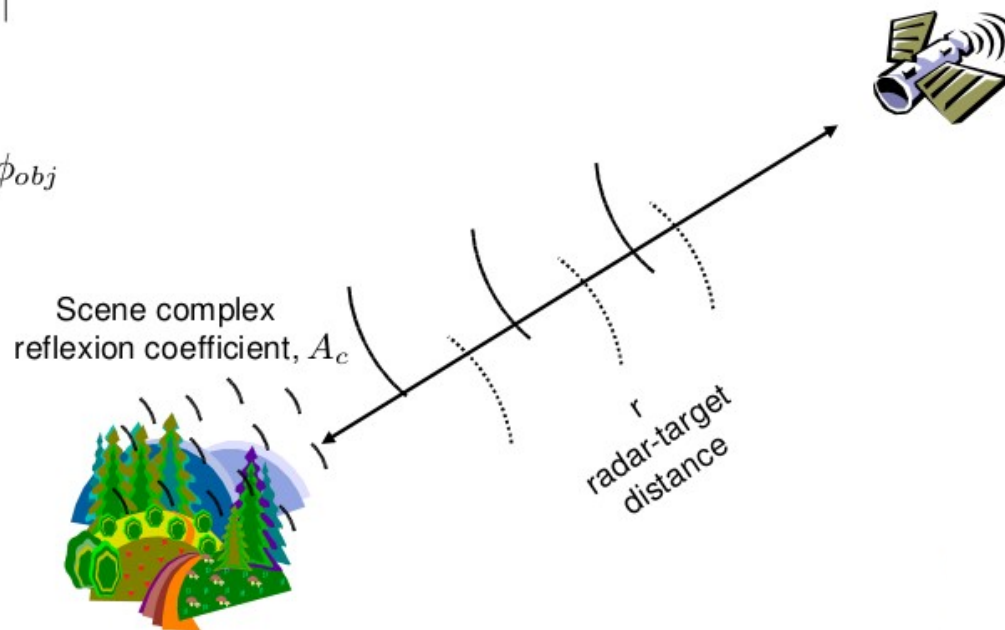
$$\rho_{(x,r)} \propto |A_c(x,r)|$$

Composite phase image

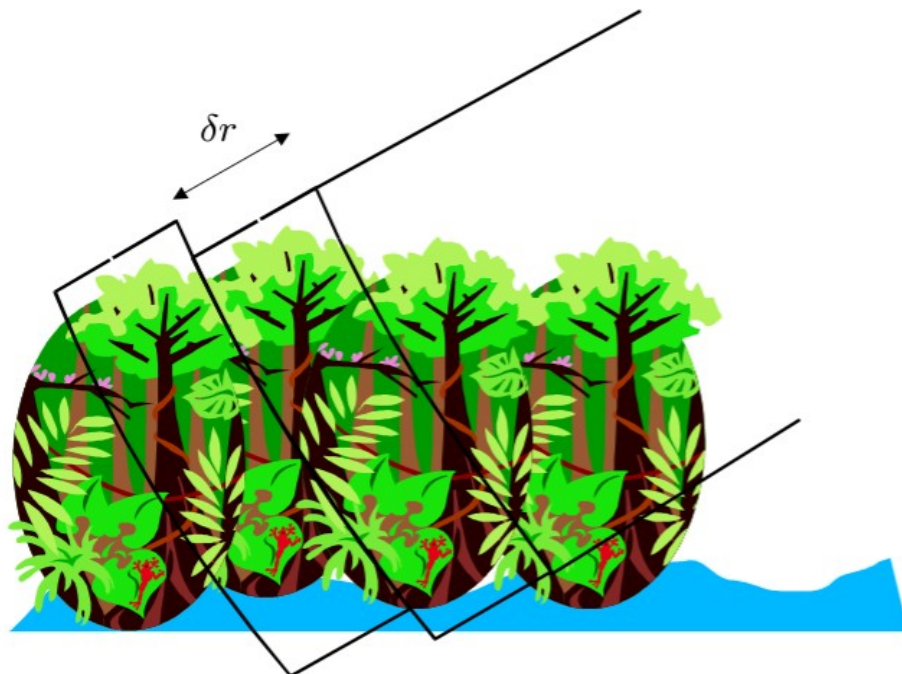
$$\phi_{(x,r)} = \phi_r + \phi_{obj} = kr + \phi_{obj}$$

ϕ_r deterministic component

ϕ_{obj} may be random

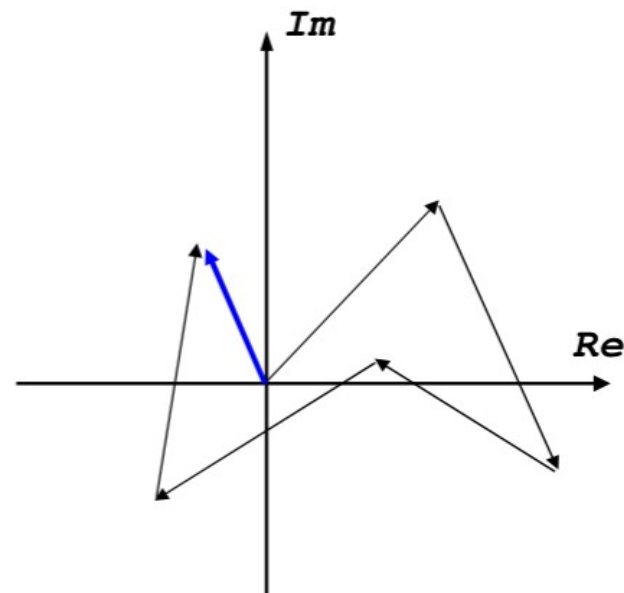


Speckle effect



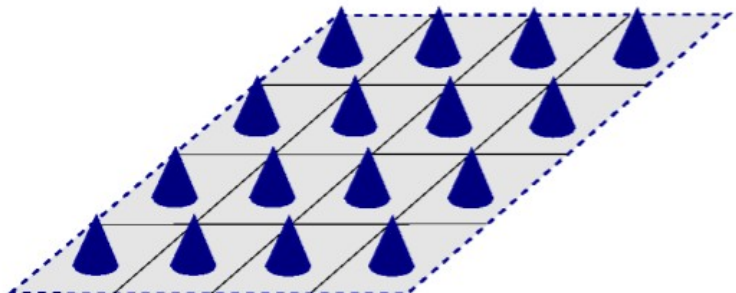
ϕ_{obj} is usually random

Coherent superposition
of a large number of contributions

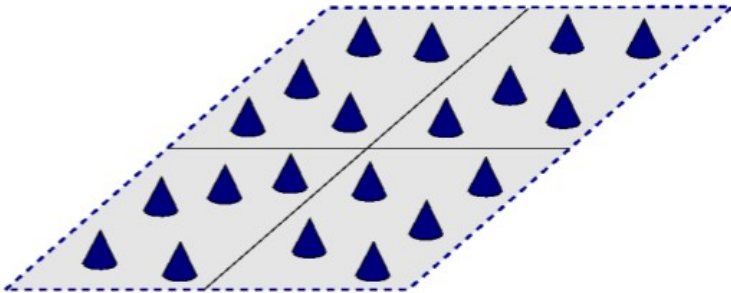


Deterministic vs random scattering

Point-like strong scatterers



Natural surfaces



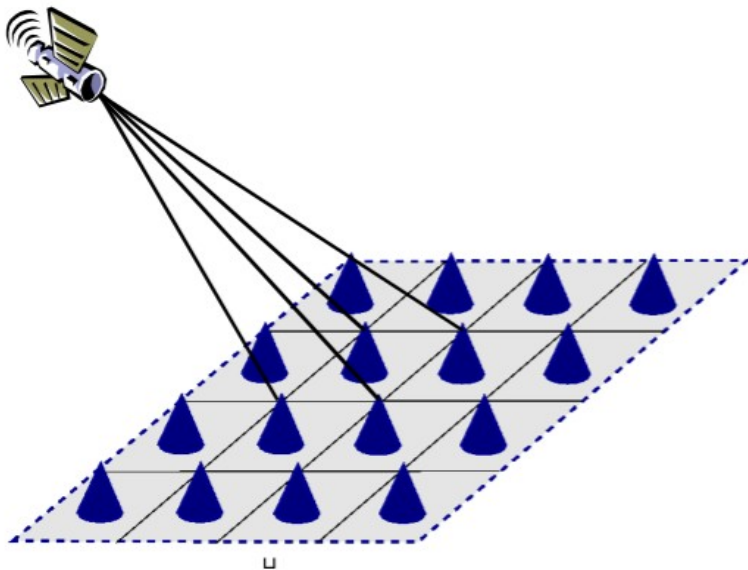
A single dominant response within a resolution cell

Volumetric media



Sum of a large number of contributions with random position, amplitude and phase \Rightarrow within a resolution cell

Deterministic scattering



A single dominant response within a resolution cell

SAR signal for a given resolution cell

$$s(x, r) = \int a_c(r') e^{-jkr'} h(x - x', r - r') dv'$$

For a point-like scatterer

$$a_c(r) = A e^{j\phi_0} \delta(r - r_0) + \epsilon \approx A e^{j\phi_0} \delta(r - r_0)$$

$$s(x, r) = A e^{j\phi_0} e^{-jkr_0} h(x - x_0, r - r_0)$$

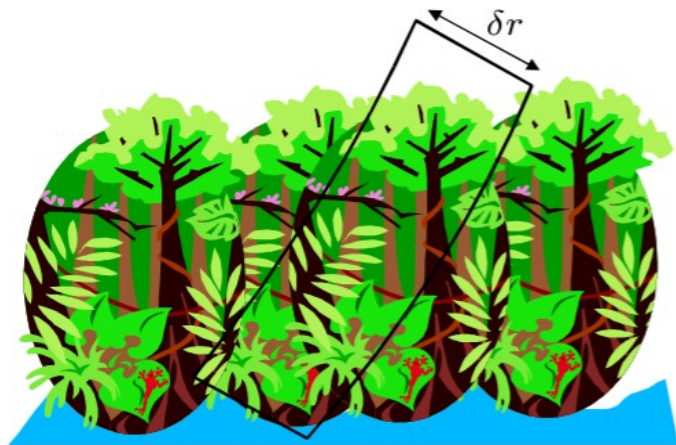
Deterministic signal components

$$s(x, r) = \sqrt{I} e^{j\phi}$$

- intensity: $I = |s(x, r)|^2$

phase: $\phi = \phi_0 - kr_0 = \arg s(x, r)$

Random scattering



↑
Distributed medium
with numerous random contributions
within a resolution cell

SAR signal for a given resolution cell

$$s(x, r) = \int a_c(r') e^{-jkr'} h(x - x', r - r') dv'$$

For a set of random scatterers

Complex amplitude density $a_c(r) = a_c(r) e^{j\phi_0(r)}$

Coherent integration

⇒ random behavior : « **speckle effect** »

Speckle product model

$$s(x, r) = \sqrt{I} e^{j\phi} = \sqrt{I_{eq}} e^{j\phi_{eq}} n_c$$

- random intensity: $I = |s(x, r)|^2$

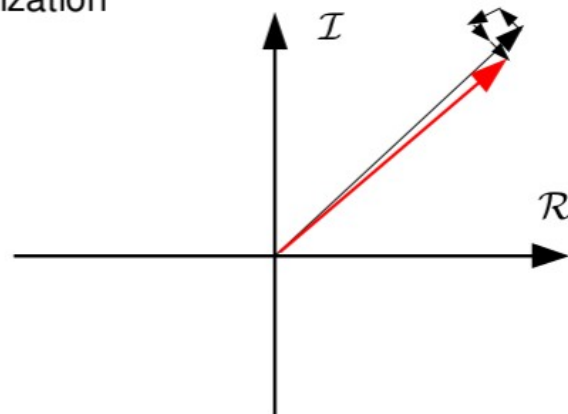
random phase: $\phi = \arg(s(x, r))$

speckle component: n_c

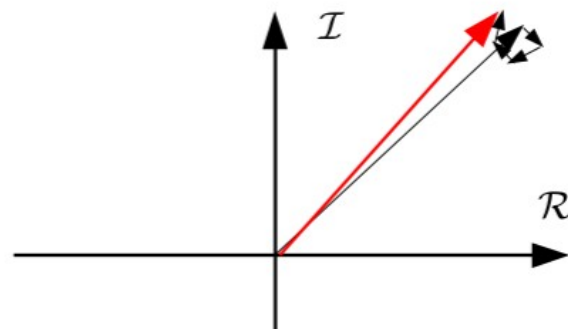
Deterministic scattering



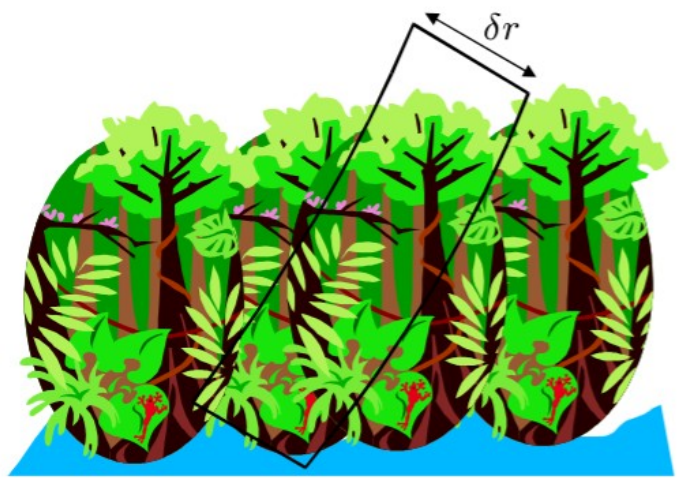
• One realization



• Another one

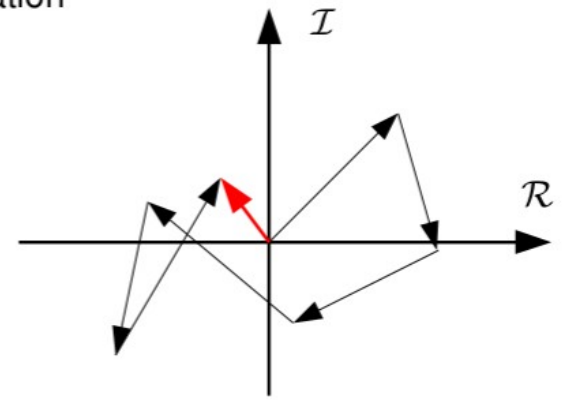


Random scattering

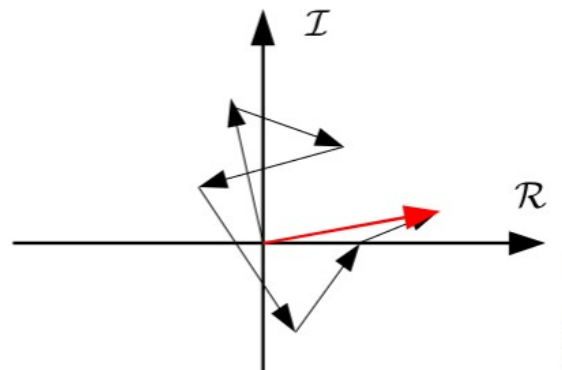


↑
Distributed medium
with numerous random contributions
within a resolution cell

- One realization



- Another one



Fully developed speckle statistics

Central limit theorem

Assumptions :

- no dominant scatterer
- uniformly distributed object phase
- uncorrelated scatters
- large number of scatterers



s follows a complex normal distribution

$E(s) = 0$ uniform phase argument

$\text{var}(s) = E(I) = \bar{I}$ « true » reflectivity

Complex signal distribution $s = \sqrt{\bar{I}} e^{j\bar{\phi}} n_c$

Multiplicative speckle component with circular Gaussian distribution

$$n_c = \mathcal{R}(n_c) + j\mathcal{I}(n_c) \sim \mathcal{N}_c(0, 1)$$

Single-look complex (SLC) data

Complex circular normal distribution $s = \sqrt{\bar{I}}e^{j\bar{\phi}}n \sim \mathcal{N}_C(0, \bar{I})$

Intensity

Exponential distribution $\hat{I} = |s|^2 \sim \mathcal{E}(\bar{I})$

Unbiased reflectivity estimate $E(\hat{I}) = \bar{I} = \text{var}(s)$

Extremely high dispersion $\text{var}(\hat{I}) = \bar{I}^2$

Phase

Uniform distribution → mean value cannot be retrieved

Multi-looked data statistics

Single-look intensity

Exponential distribution

$$\hat{I} = |s|^2 \sim \mathcal{E}(\bar{I})$$

Unbiased reflectivity estimate

$$\mathbb{E}(\hat{I}) = \bar{I} = \text{var}(s)$$

Extremely high dispersion

$$\text{var}(\hat{I}) = \bar{I}^2$$

Multi-look intensity

L independent realizations (or looks)

$$\{I(l) = |s(l)|^2\}_{l=1}^L$$

Multi-look intensity estimate

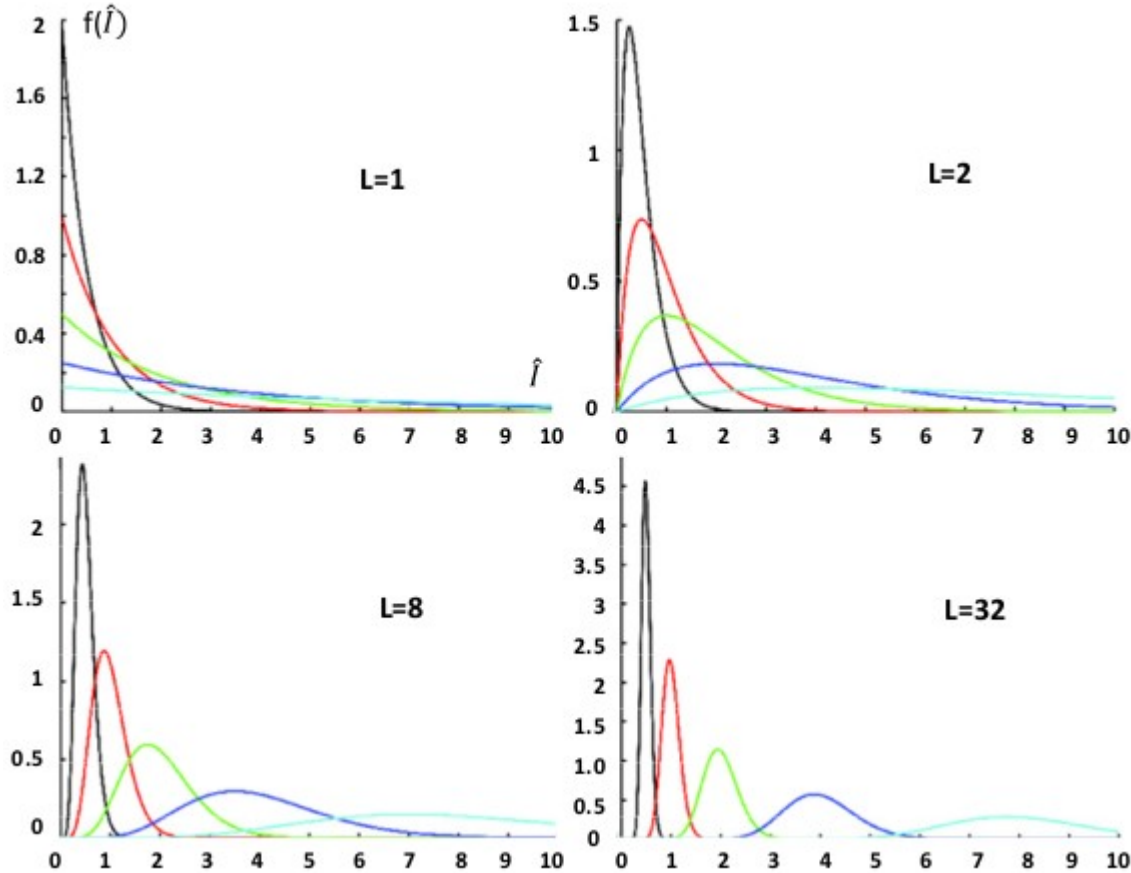
$$\hat{I} = \frac{1}{L} \sum_l I(l)$$

Bias, variance

$$\mathbb{E}(\hat{I}) = \bar{I} = \text{var}(s)$$

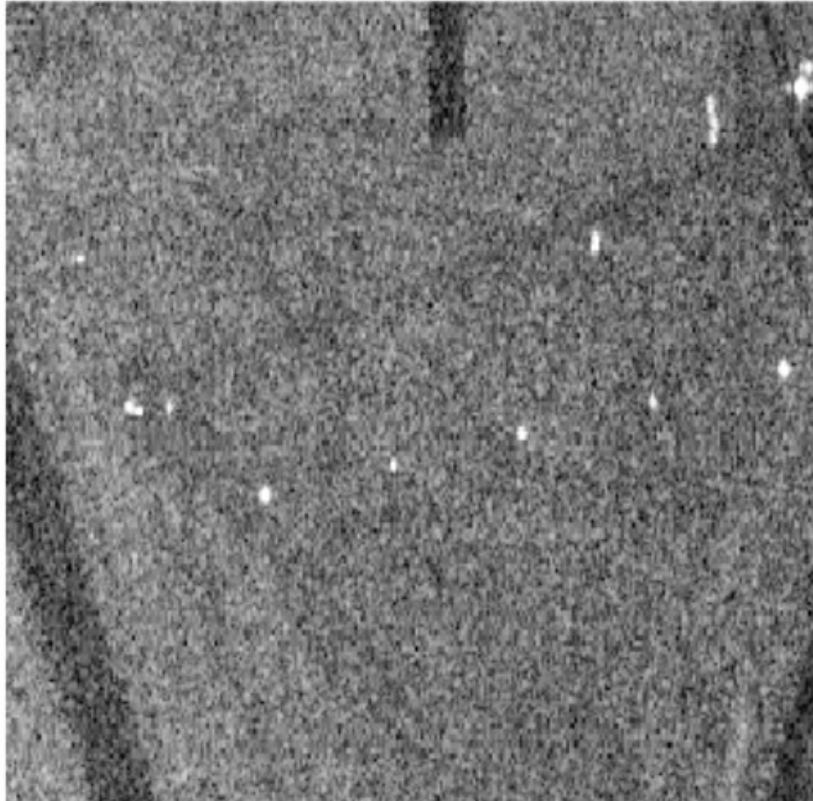
$$\text{var}(\hat{I}) = \frac{\bar{I}^2}{L}$$

Multilooking

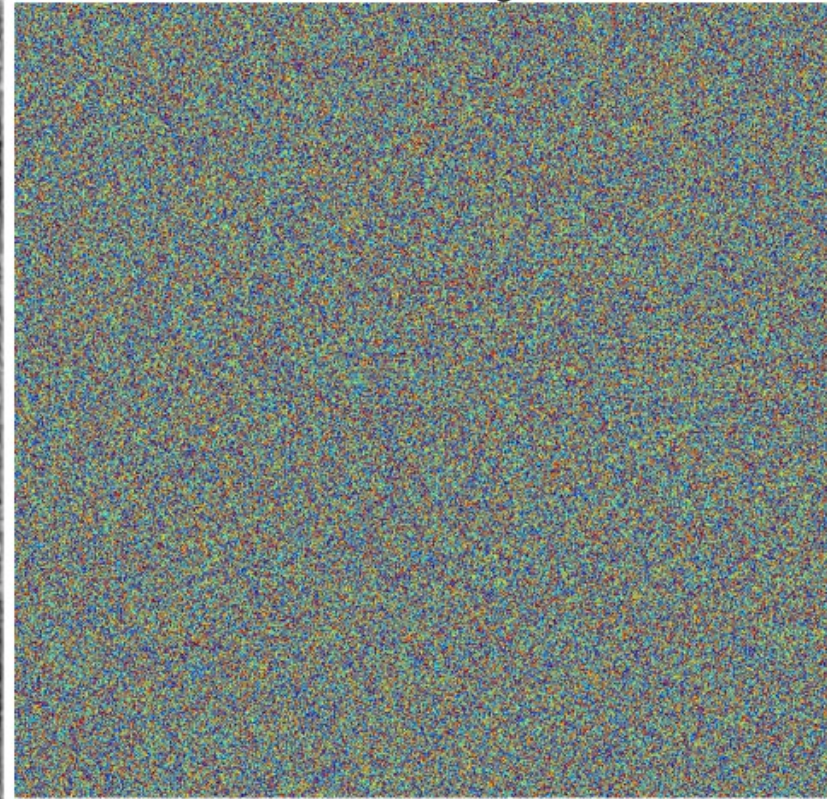


Single Look Complex (SLC) SAR image

Intensity image



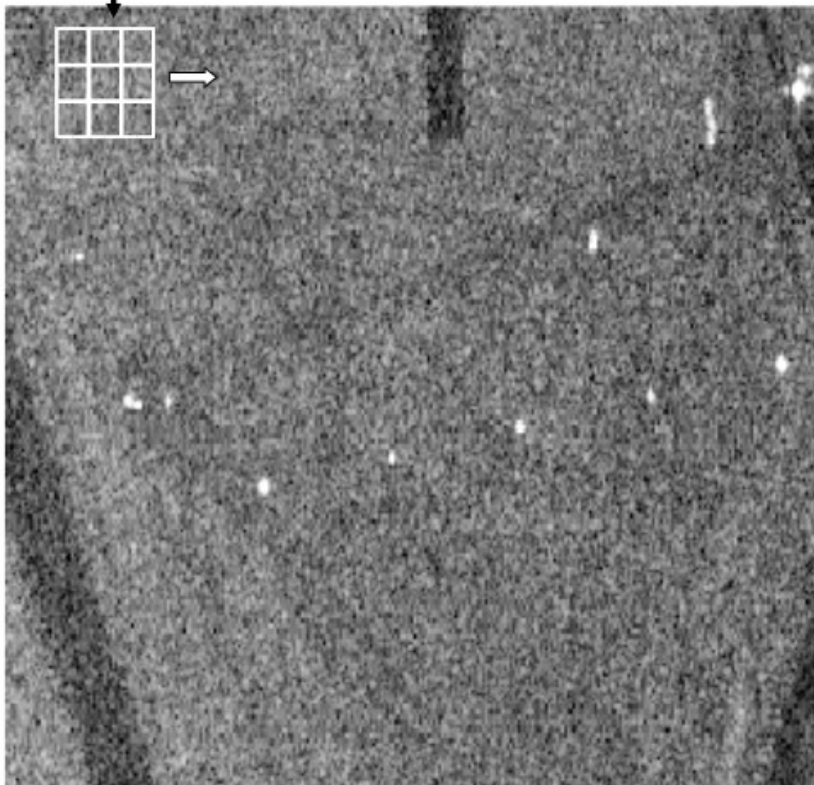
Phase image



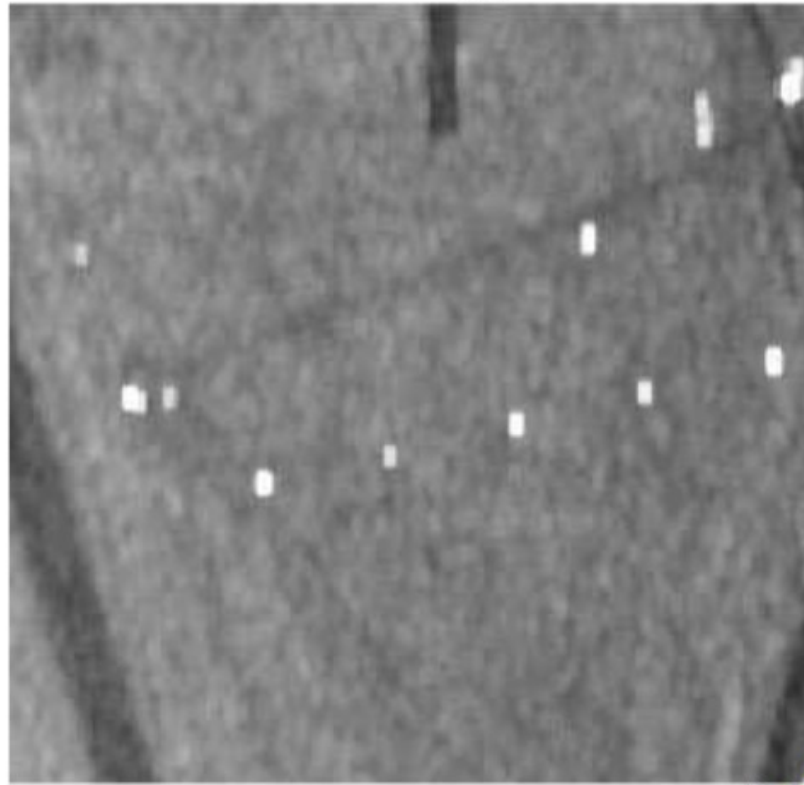
Single Look Complex (SLC) SAR image

$$\hat{I} = \frac{1}{L} \sum_L I_i$$

Single look image

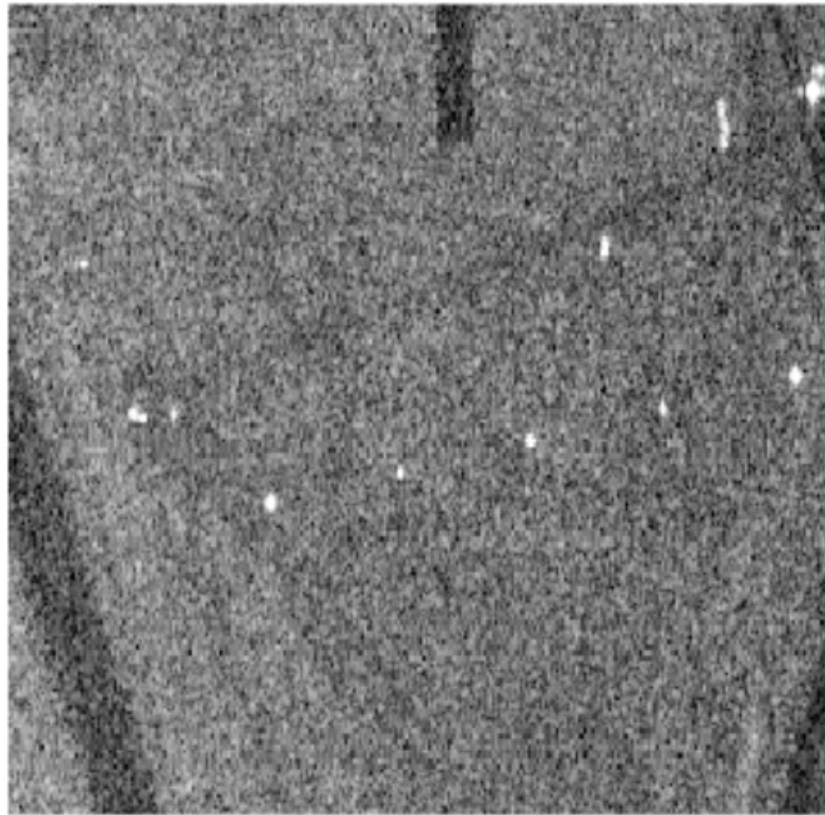


After spatial filtering (N*N boxcar)

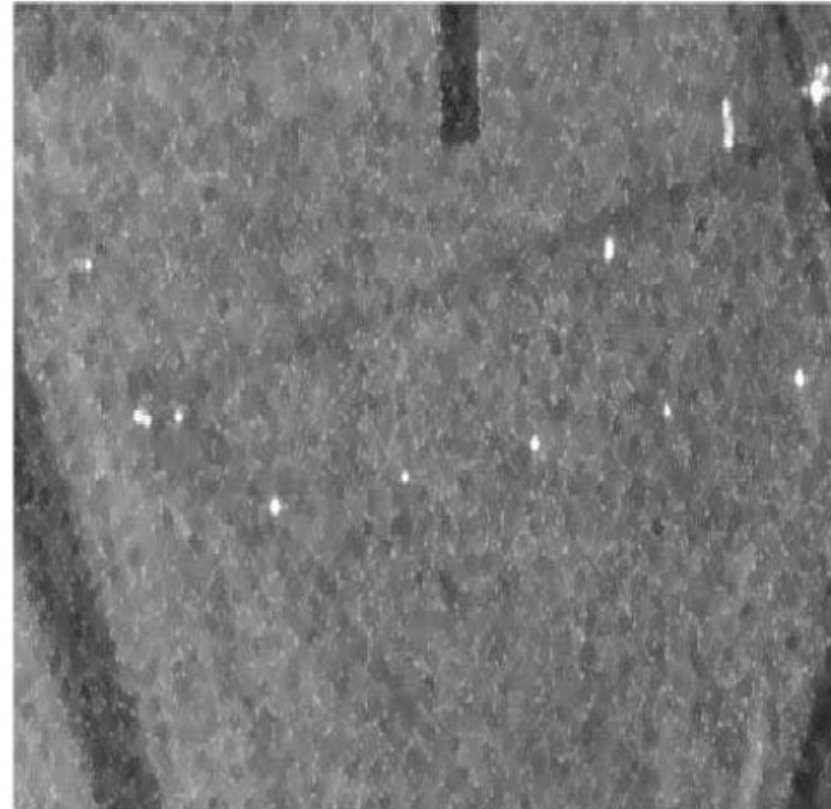


Speckle filtering

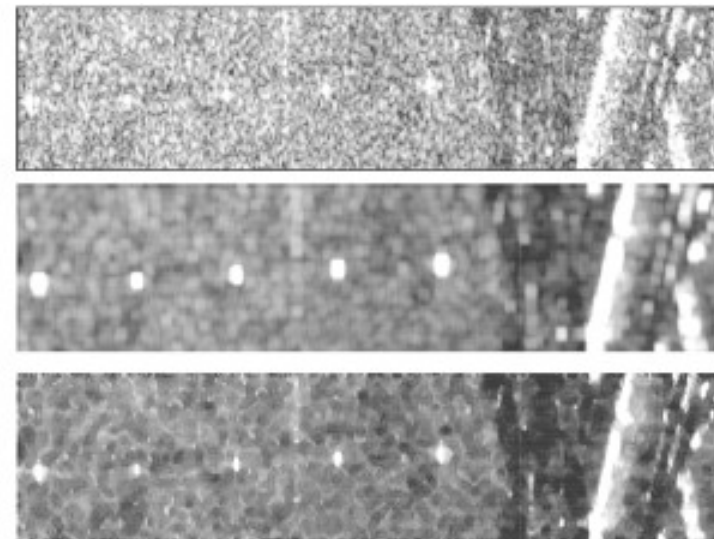
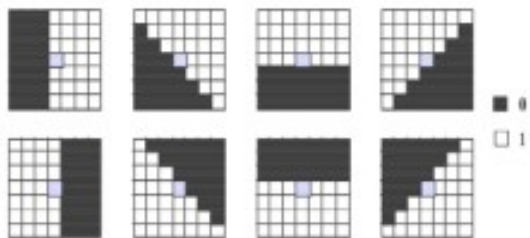
Single look image



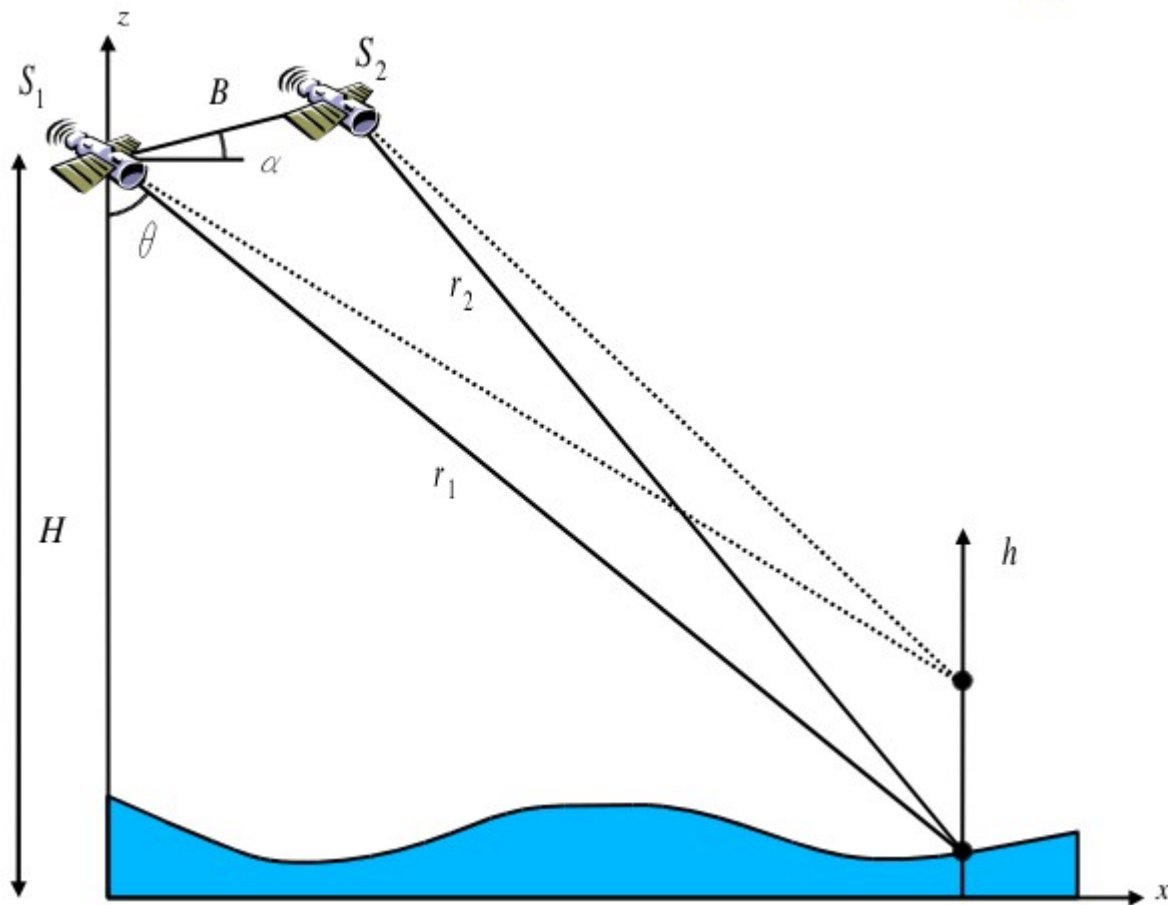
After spatial filtering (N*N Lee filter)



Single Look Complex (SLC) SAR image



Interferometric SAR image statistics



Coherent SAR image pair

$$s_1 = \sqrt{I_1} e^{j\phi_1} = \sqrt{I_1} e^{j(-kr_1 + \phi_{obj1})}$$

$$s_2 = \sqrt{I_2} e^{j\phi_2} = \sqrt{I_2} e^{j(-kr_2 + \phi_{obj2})}$$

Assumptions

$$I_1 \approx I_2 \quad \text{and} \quad \phi_{obj1} \approx \phi_{obj2}$$

Interferometric phase difference

$$\Delta\phi_{12} = \arg(s_1 s_2^*)$$

Interferometric SAR image statistics

Joint interferometric representation

$$\mathbf{k} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \sim \mathcal{N}_c(\mathbf{0}, \mathbf{C})$$

$$\text{with } \mathbf{C} = \mathbf{E}(\mathbf{k}\mathbf{k}^\dagger) = \begin{bmatrix} \mathbf{E}(s_1 s_1^*) & \mathbf{E}(s_1 s_2^*) \\ \mathbf{E}(s_1^* s_2) & \mathbf{E}(s_2 s_2^*) \end{bmatrix} = \begin{bmatrix} \overline{I_1} & \gamma \sqrt{\overline{I_1} \overline{I_2}} \\ \gamma^* \sqrt{\overline{I_1} \overline{I_2}} & \overline{I_2} \end{bmatrix}$$

Interferometric coherence : normalized correlation coefficient

$$\gamma = \frac{\mathbf{E}(s_1 s_2^*)}{\sqrt{\overline{I_1} \overline{I_2}}} = |\gamma| e^{j\phi} \quad |\gamma| \leq 1 \quad \text{Cauchy-Schwarz inequality}$$

$|\gamma| = 1 \Rightarrow \phi = \Delta\phi_{12}$ interferometric assumptions are fulfilled

$|\gamma| = 0 \Rightarrow \phi = ?$ interferometric images are totally uncorrelated

$|\gamma|$ is an indicator of the interferometric information (and phase) quality

Multi-looked InSAR data statistics

Multi-look covariance matrix

L independent realizations (or looks) $\{\mathbf{k}(l)\}_{l=1}^L$

Multi-look covariance estimate

$$\hat{\mathbf{C}} = \frac{1}{L} \sum_l \mathbf{k}(l) \mathbf{k}^{*t}(l)$$

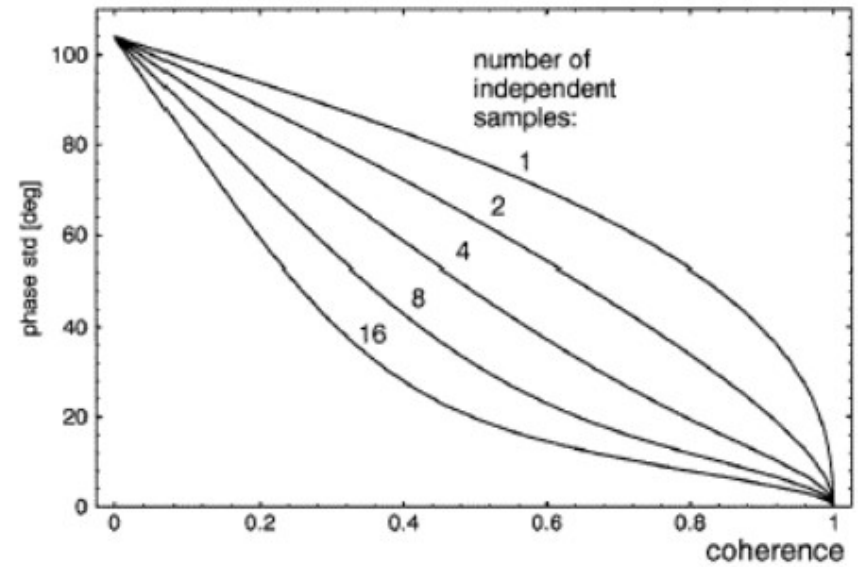
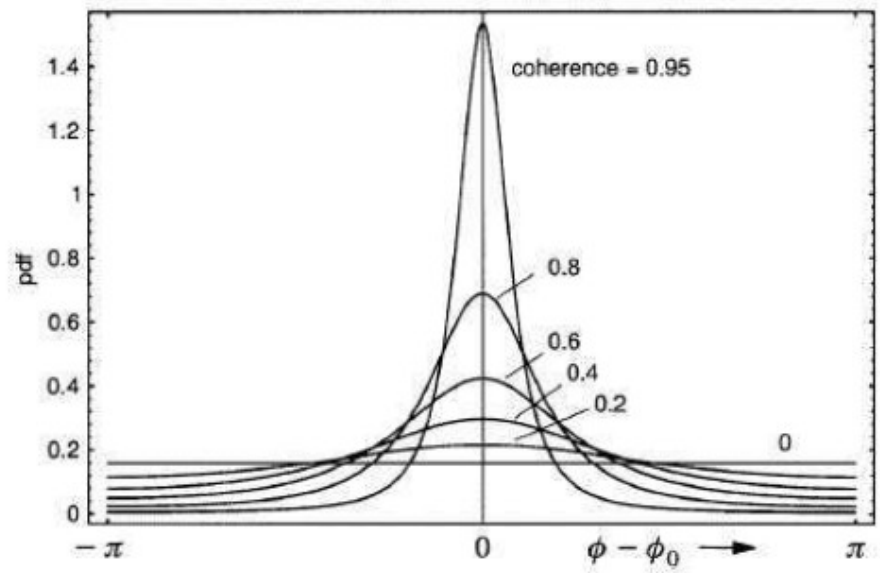
Unbiased

$$E(\hat{\mathbf{C}}) = \mathbf{C}$$

Multi-look coherence estimate

$$\hat{\gamma} = \frac{\sum_l s_1(l) s_2^*(l)}{\sqrt{(\sum_l |s_1(l)|^2)(\sum_l |s_2(l)|^2)}}$$

Multilook InSAR coherence statistics



$$\hat{\phi} = \arg \hat{\gamma}$$

$$E(\hat{\phi}) = \phi$$

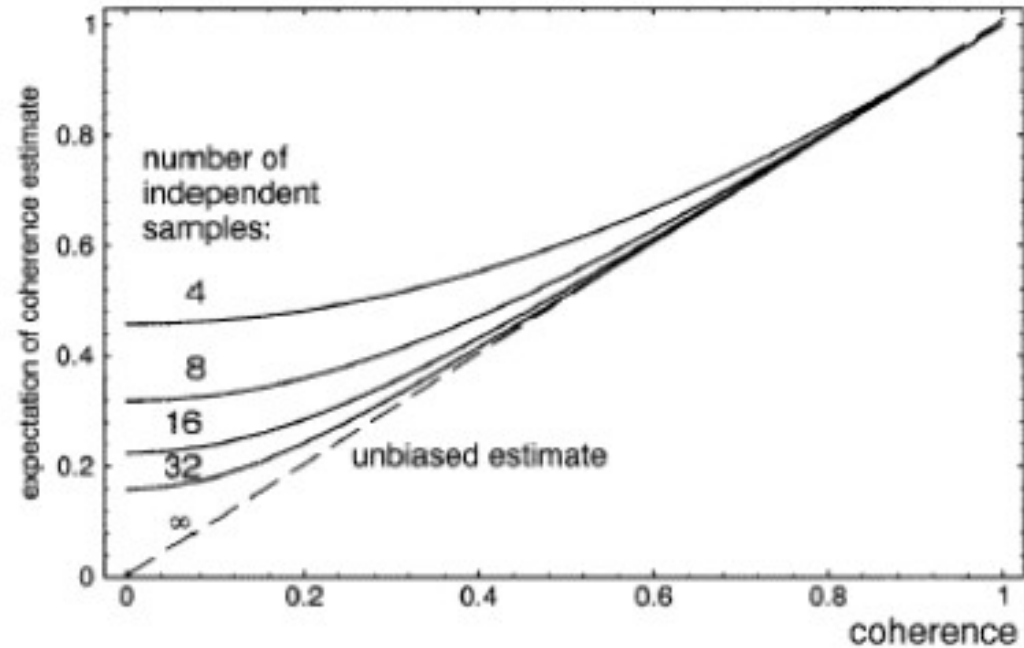
$$\text{var}(\hat{\phi}) = f(L, |\gamma|)$$

Multilook InSAR coherence statistics

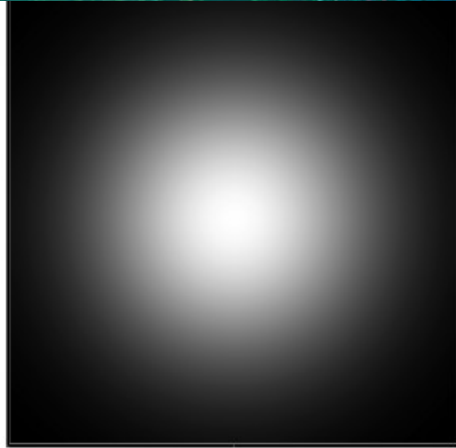
$$\hat{\rho} = |\hat{\gamma}|$$

$$E(\hat{\rho}) \neq |\gamma|$$

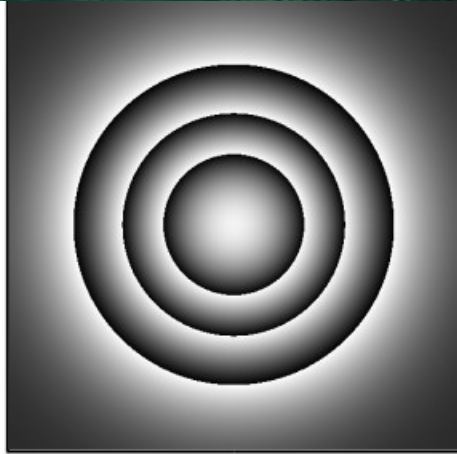
$$\lim_{L \rightarrow +\infty} \hat{\rho} = |\gamma|$$



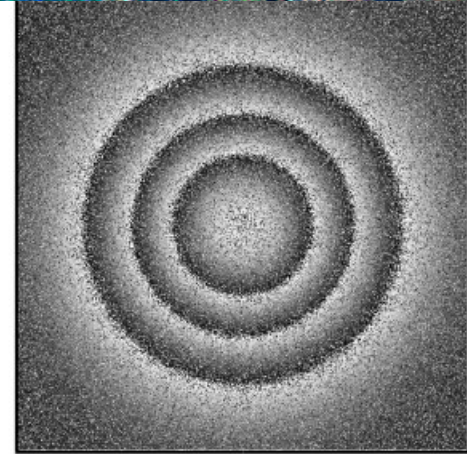
Multilook InSAR coherence



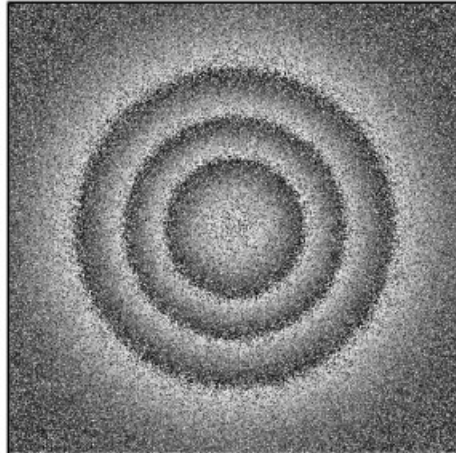
Absolute «True»
Phase



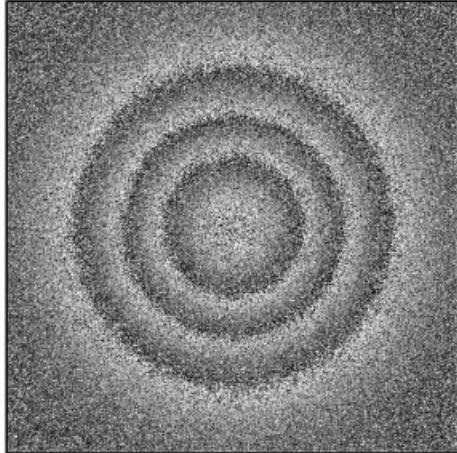
Coherence=1.0 L=1



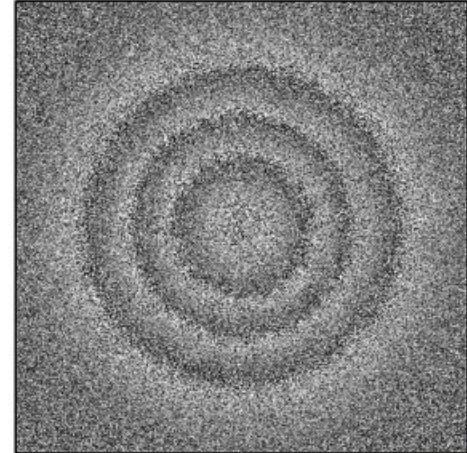
Coherence=0.8 L=1



Coherence=0.6 L=1



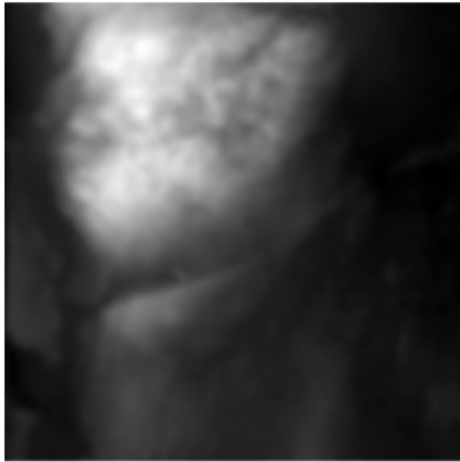
Coherence=0.4 L=1



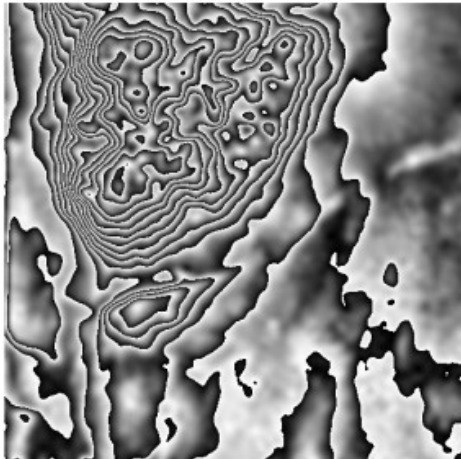
Coherence=0.2 L=1



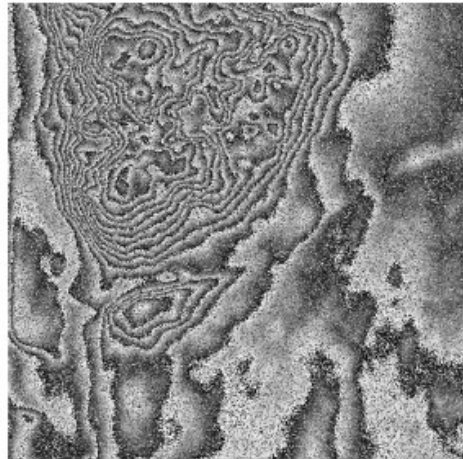
Multilook InSAR coherence



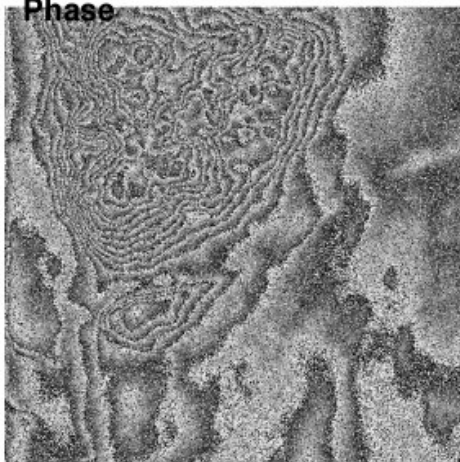
Absolute «True»
Phase



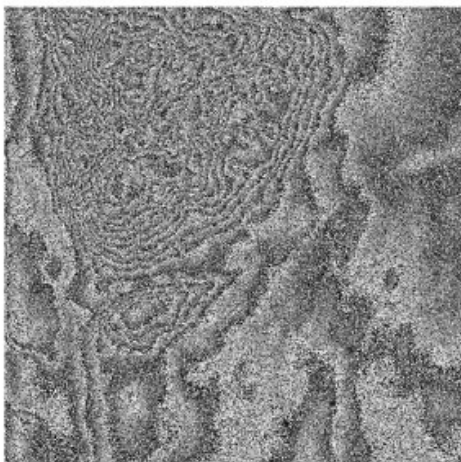
Coherence=1.0 L=1



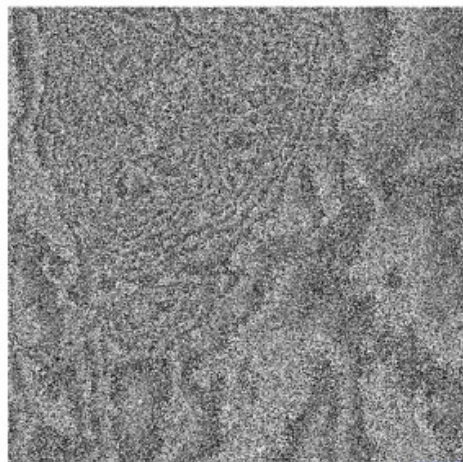
Coherence=0.8 L=1



Coherence=0.6 L=1



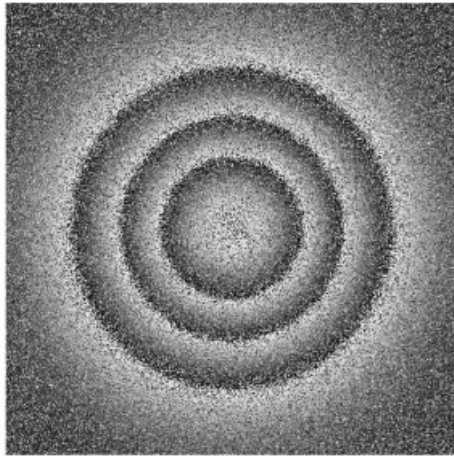
Coherence=0.4 L=1



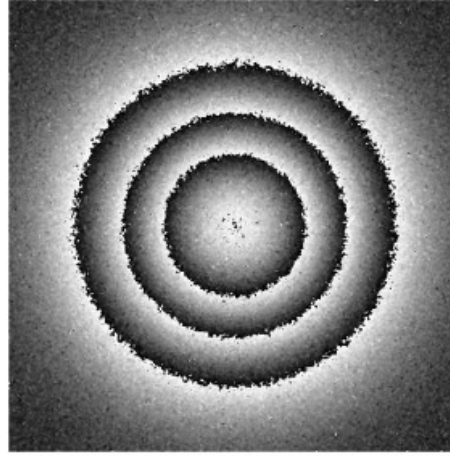
Coherence=0.2 L=1



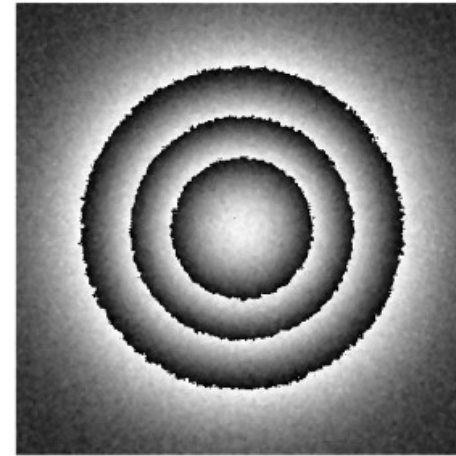
Multilook InSAR coherence



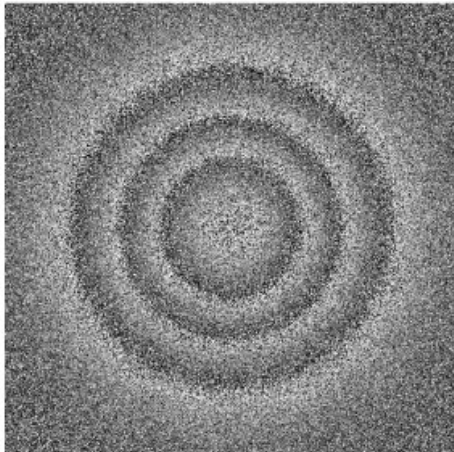
Coherence = 0.7 L=1



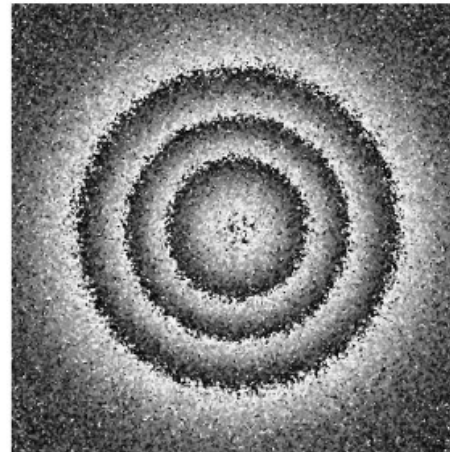
Coherence = 0.7 L=8



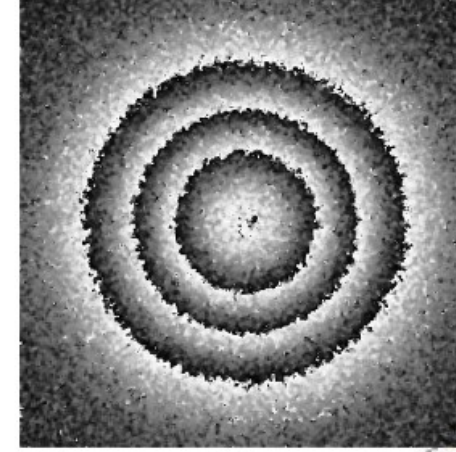
Coherence = 0.7 L=16



Coherence = 0.3 L=1

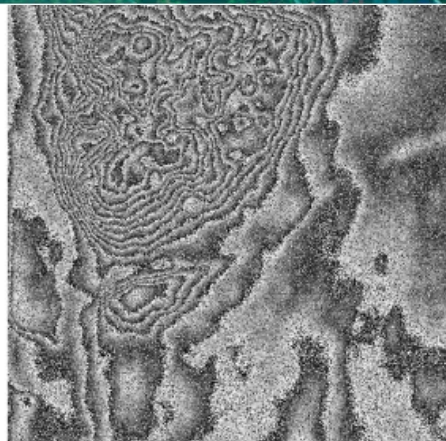


Coherence = 0.3 L=8

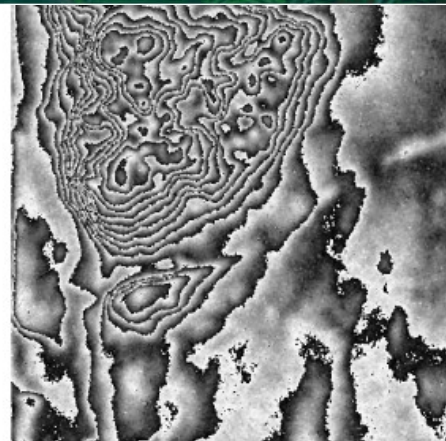


Coherence = 0.3 L=16

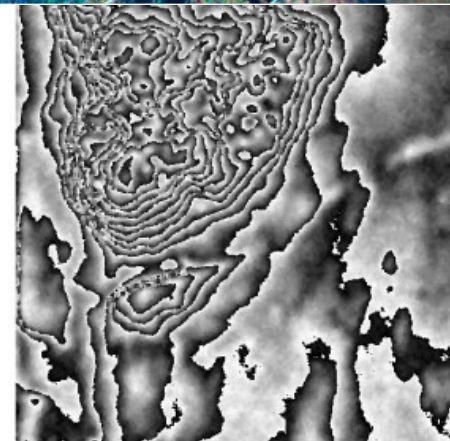
Multilook InSAR coherence



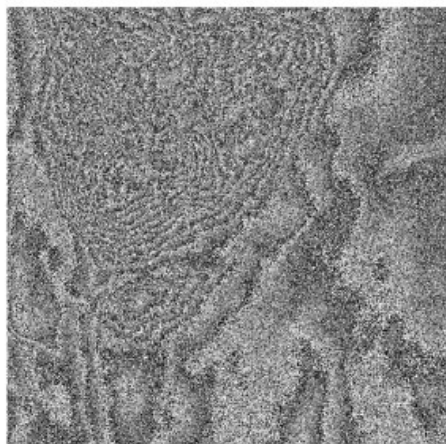
Coherence = 0.7 L=1



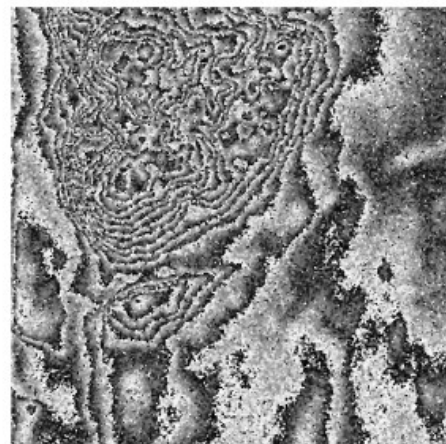
Coherence = 0.7 L=8



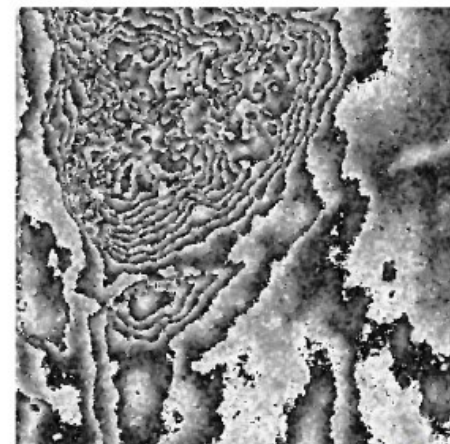
Coherence = 0.7 L=16



 Coherence = 0.3 L=1



Coherence = 0.3 L=8



Coherence = 0.3 L=16

The true value of the coherence, γ , is fixed by a set of external sources :

Thermal or system noise : SAR amplifiers, ADC, antennas ...

Geometric decorrelation : Baseline, squint ...

Volume decorrelation : Volumetric media e.g. forest ...

Temporal variations : wind, flowing or plowing, building ...

Processing errors : coregistration, interpolation ...

$$\gamma = \gamma_{th} \cdot \gamma_{geom} \cdot \gamma_{vol} \cdot \gamma_{temp} \cdot \gamma_{proc}$$

- Signal plus system noise

Synthesized noisy signal $s_{th_i} = s_i + n_i$

Uncorrelated additive white noise terms

$$E(n_i) = 0, \text{var}(n_i) = \sigma_{N_e}^2 \quad E(n_1 n_2^*) = 0, E(n_i s_j) = 0$$

- Multiplicative coherence decomposition

$$\begin{aligned} \gamma &= \frac{E(s_{th_1} s_{th_2}^*)}{\sqrt{E(|s_{th_1}|^2) E(|s_{th_2}|^2)}} = \frac{1}{1 + \frac{\sigma_{N_e}^2}{E(|s_{th_i}|^2)}} \frac{E(s_1 s_2^*)}{\sqrt{E(|s_1|^2) E(|s_2|^2)}} \\ &= \gamma_{th} \gamma_{rem} \quad (\text{rem : remaining}) \end{aligned}$$

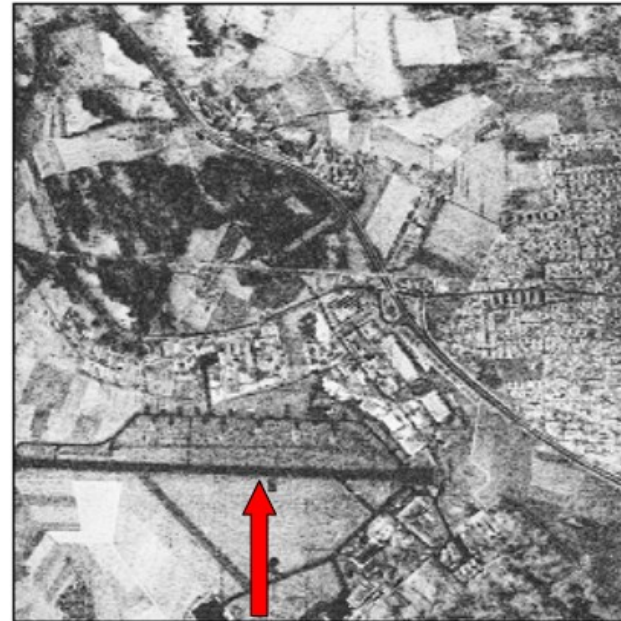
$$\gamma_{th} = \frac{1}{1 + SNR^{-1}} \quad \text{with} \quad SNR = \frac{E(|s_i|^2)}{E(|n_i|^2)}$$

Thermal (SNR) decorrelation

Intensity image



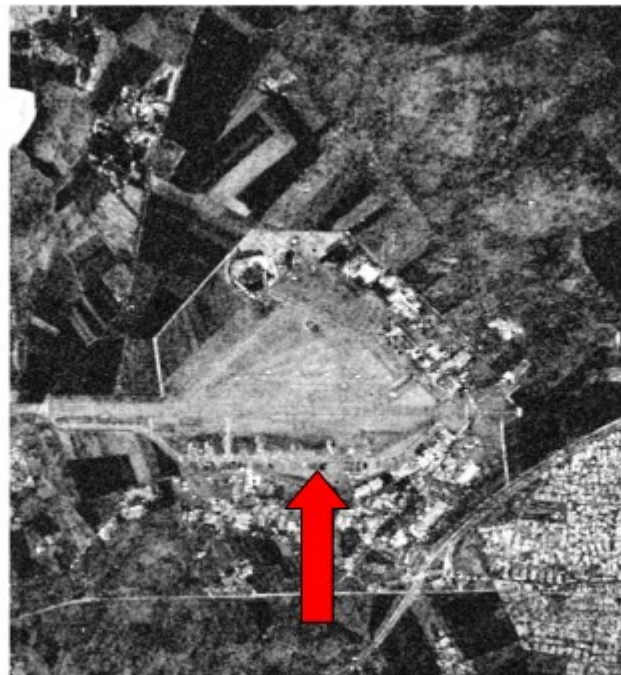
Coherence image



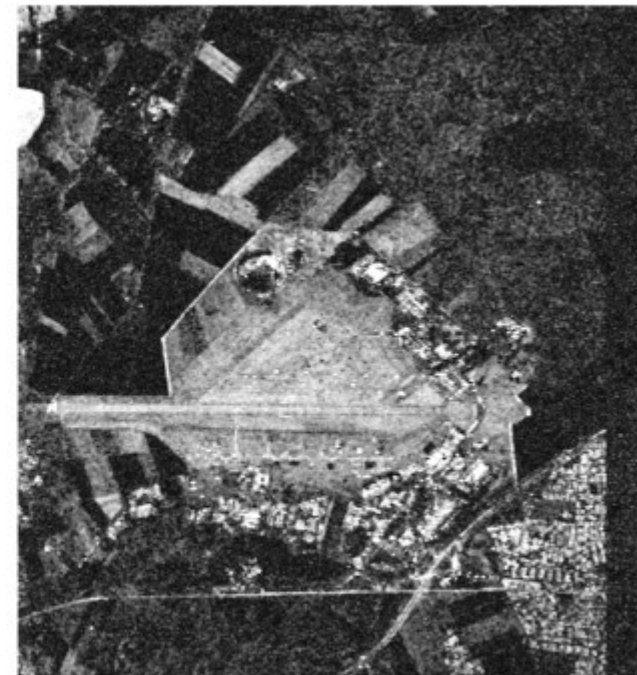
Temporal decorrelation



1 hour, 20m baseline



3 months, 0m baseline

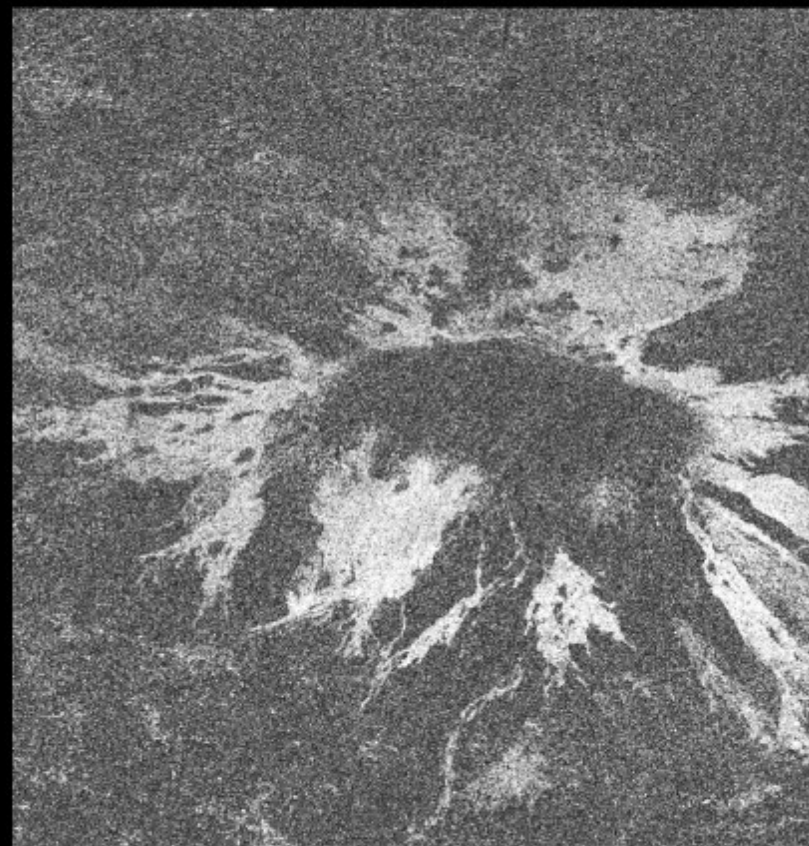


1 year, 0m baseline

Temporal decorrelation



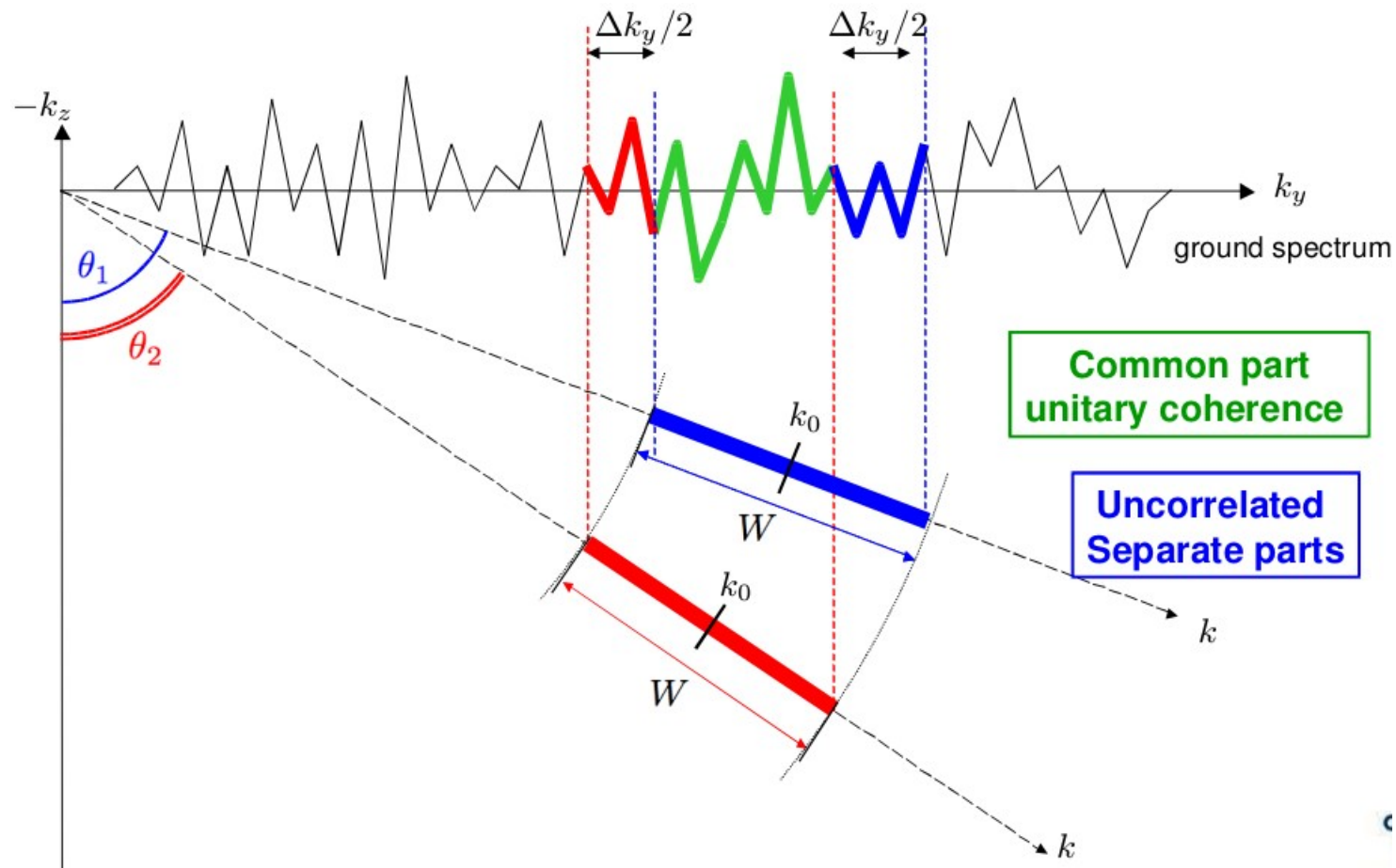
1 day ERS-1/ERS-2



70 days ERS-1/ERS-1

Test Site: Mt. Etna/Italy

Range decorrelation (spectral shift)

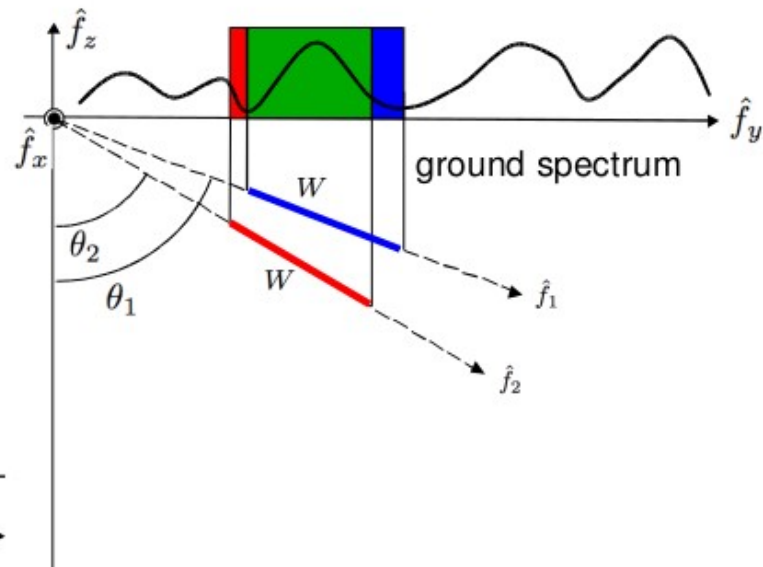
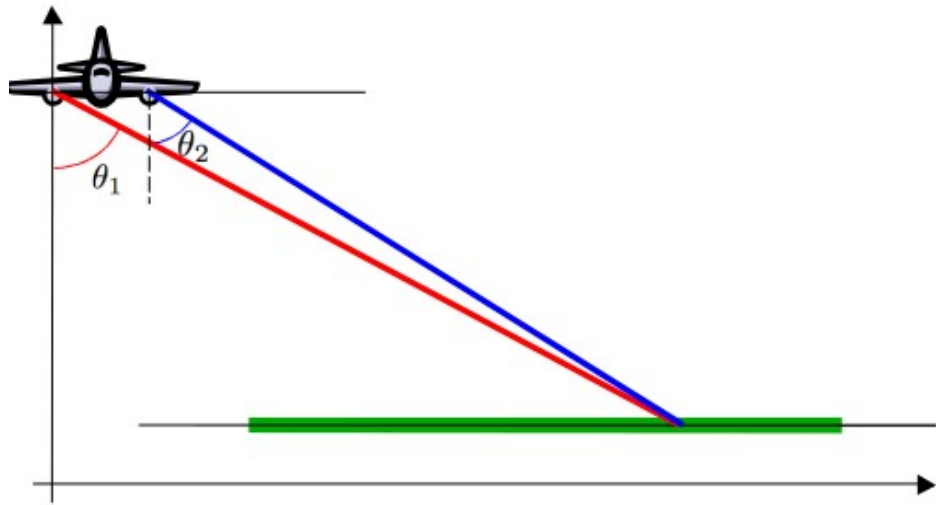


$$\gamma_{ss} = 1 - \frac{B_{\perp}}{B_{\perp}^{crit}}$$

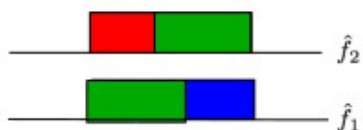
$$B_{\perp}^{crit} = \frac{W}{k_0} r_1 \tan(\theta - \alpha)$$

α , ground slope

Range decorrelation (spectral shift)



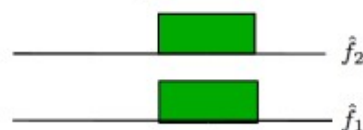
original spectra



spectral shift



bandpass filtering



Range decorrelation (spectral shift)

DLR E-SAR images, L band



with flat earth phase



without flat earth phase



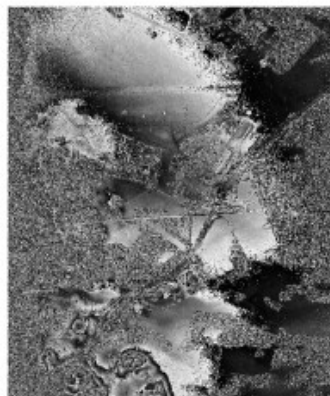
after range filtering

Range decorrelation (spectral shift)

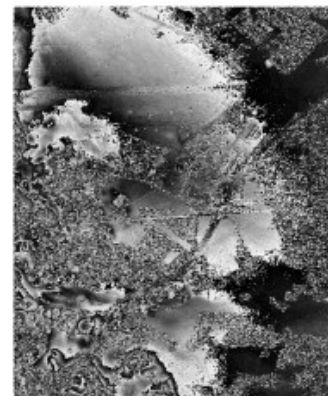
125m baseline
interferograms



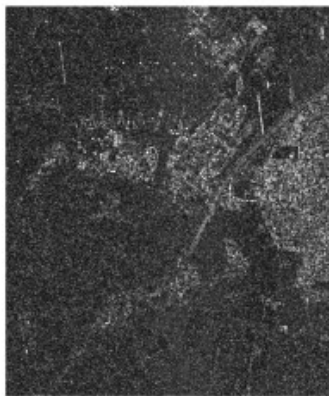
Interferometric phase



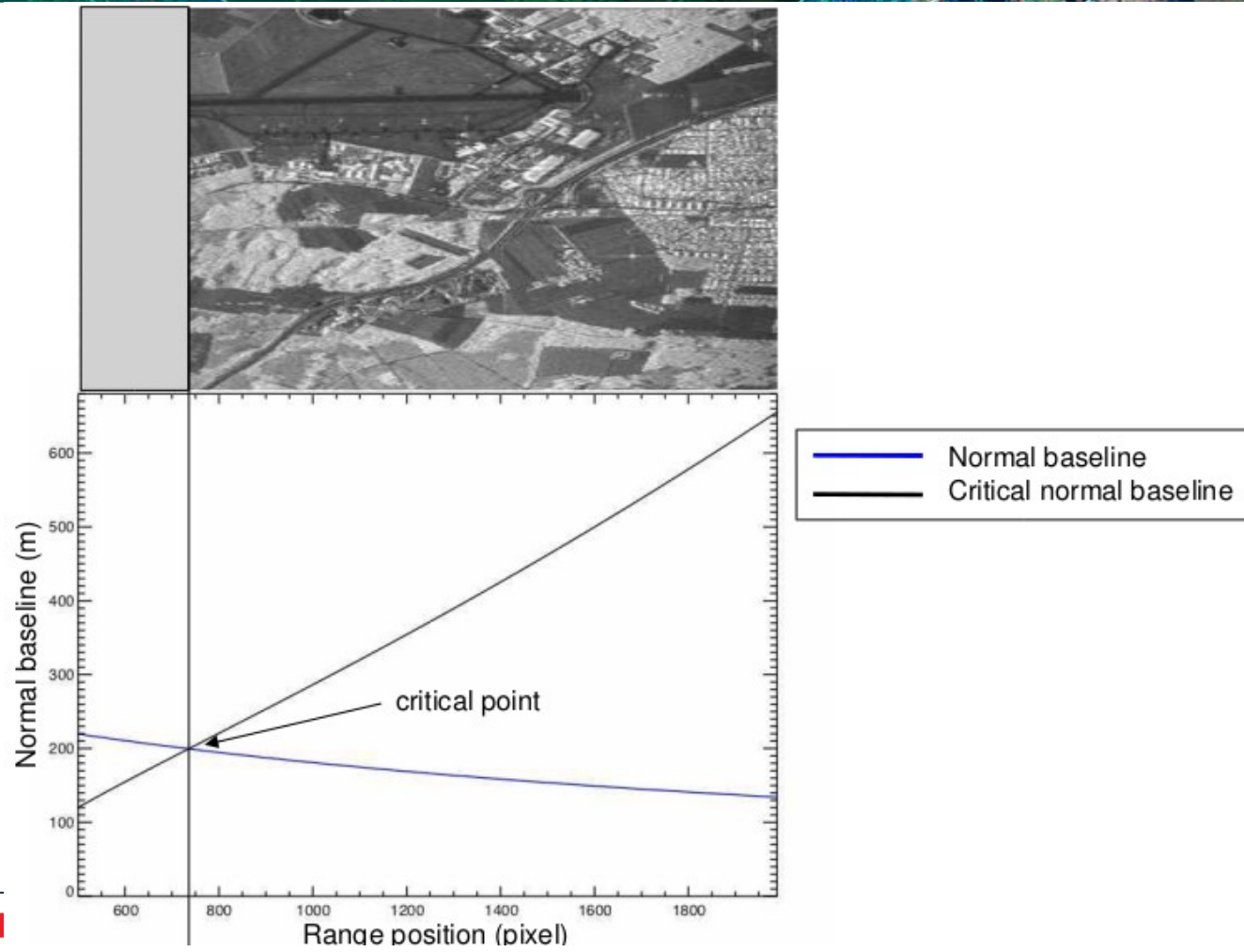
flat earth phase correction



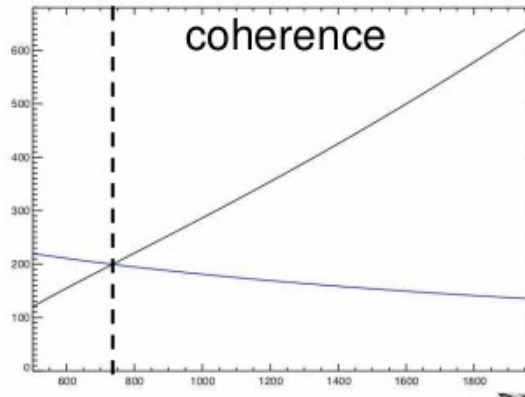
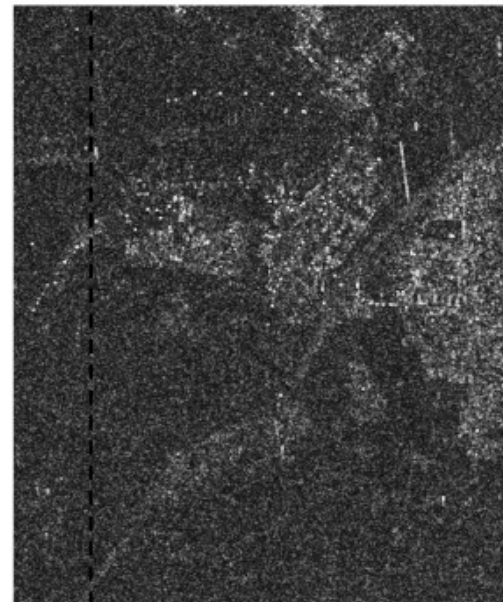
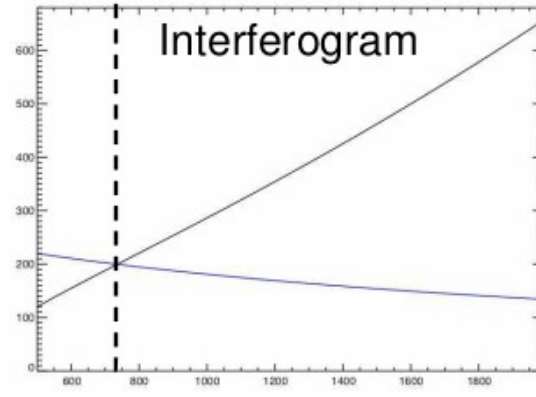
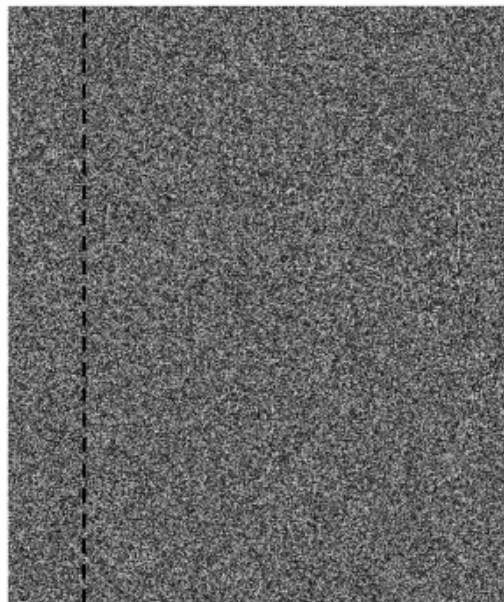
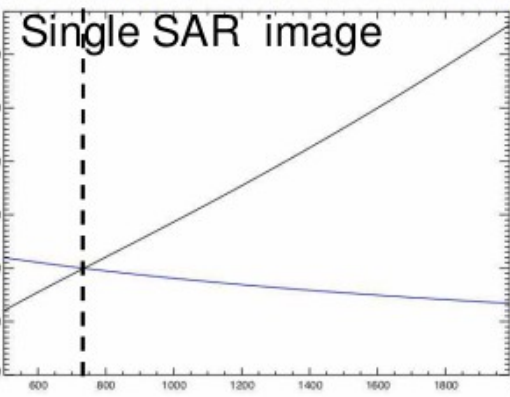
adaptive range spectral filtering



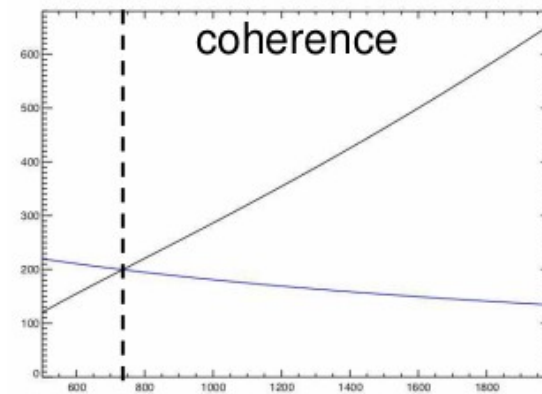
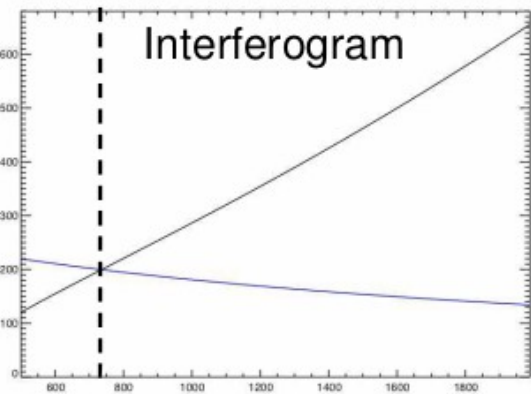
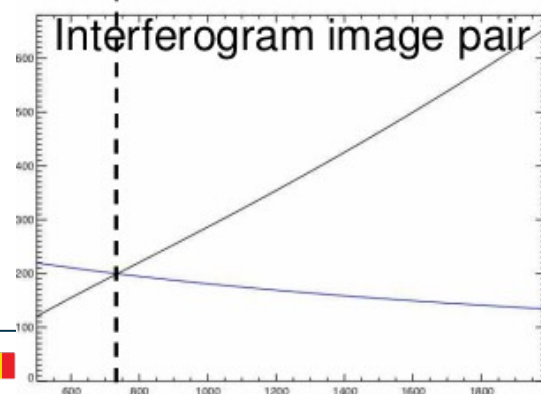
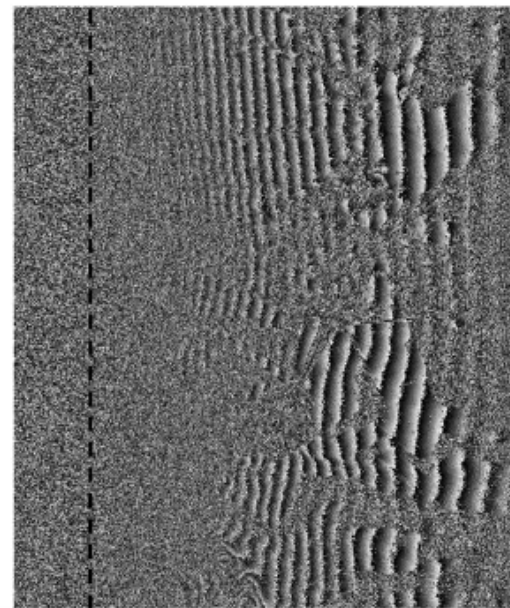
Range decorrelation: critical baseline



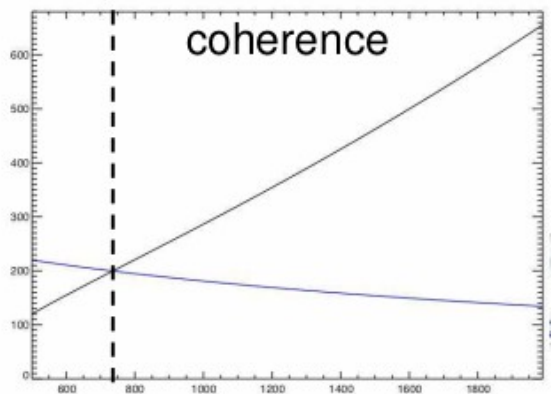
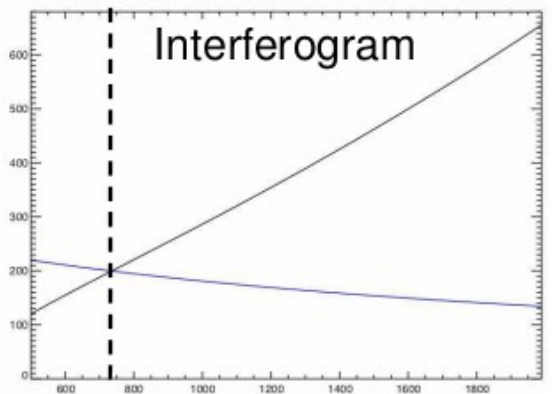
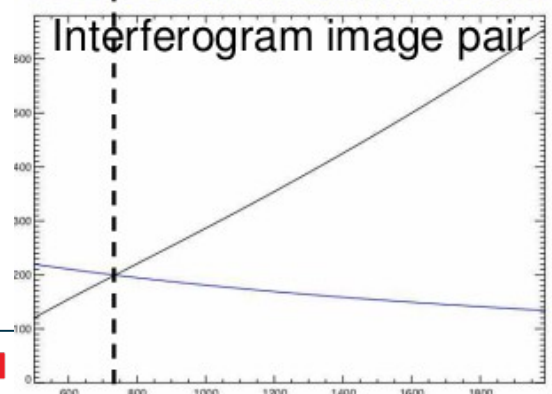
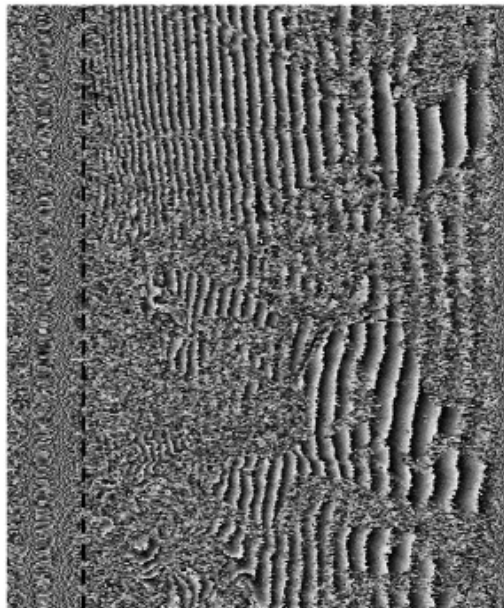
Range decorrelation: critical baseline

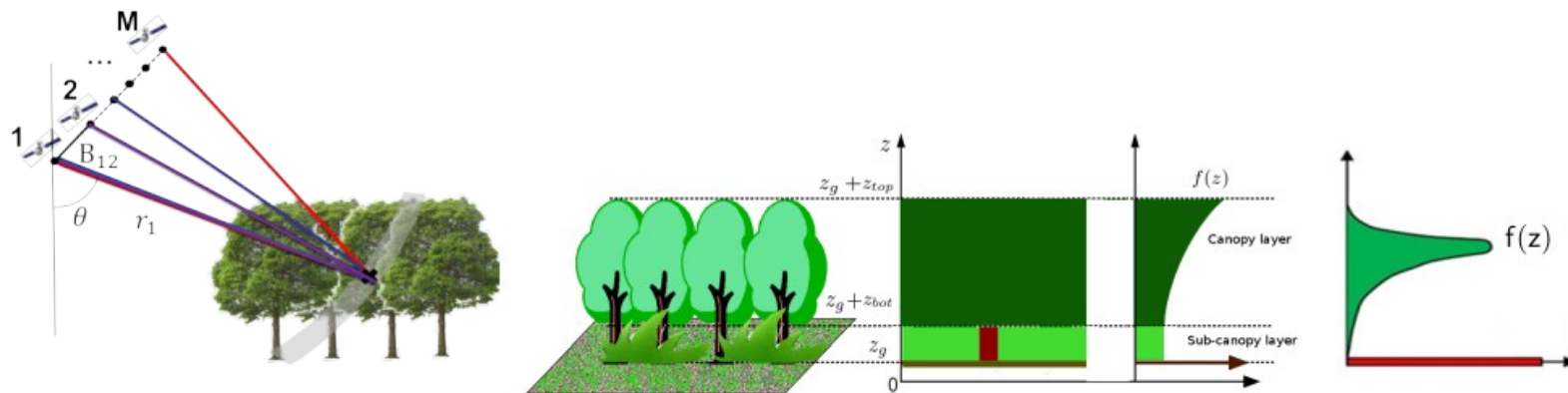


Range decorrelation: critical baseline



Range decorrelation: critical baseline





Born approx. at order 1

$$s_i \approx \int_{C(x_0, r_0)} a_c(z) e^{-jk_{z_i} z} dz,$$

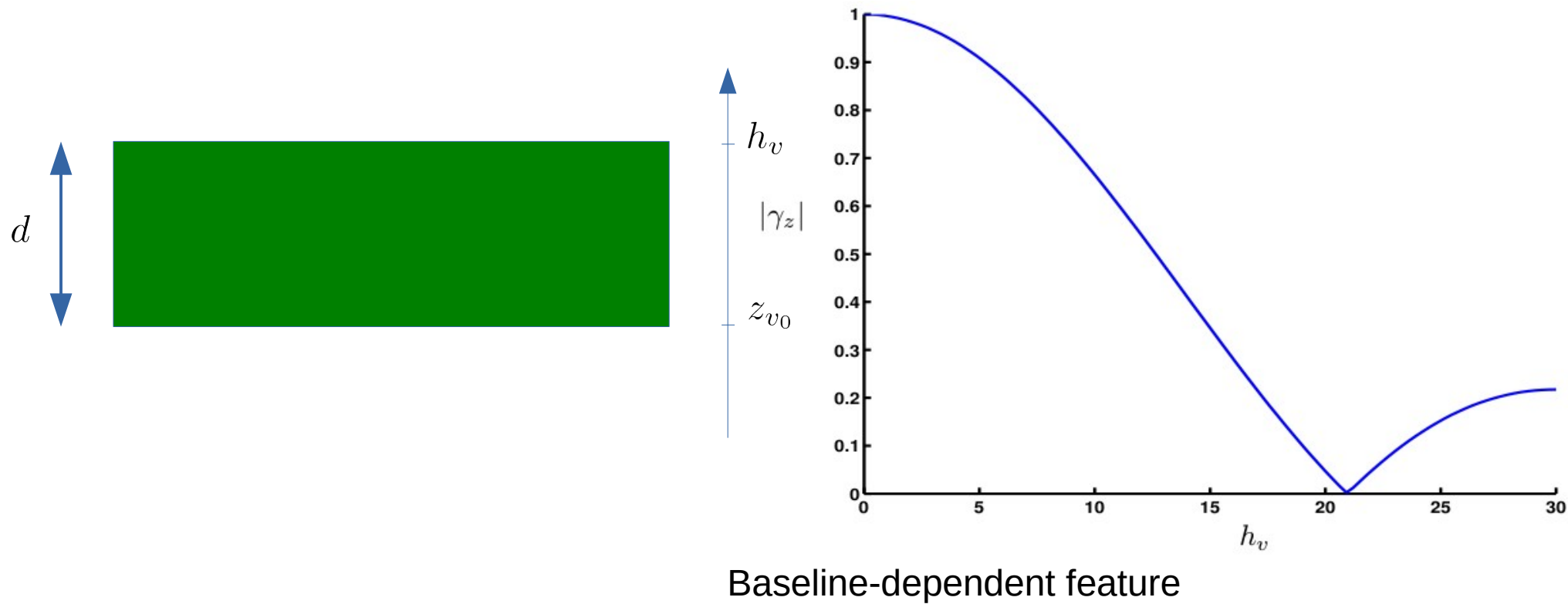
Uncorrelated random reflectivity \rightarrow

$$I_i = \mathbb{E}(|s_i|^2) = \int_{C(x_0, r_0)} f(z) dz, \quad f(z) = \mathbb{E}(|a_c(z)|^2)$$

Volumetric decorrelation

$$\gamma_z = \frac{\int f(z) e^{-jk_z z} dz}{\int f(z) dz}$$

Single SAR image statistics



Baseline-dependent feature

Volume decorrelation

1 hour, 20m baseline, L band



Volume decorrelation & frequency

